FINITE-ALPHABET NOMA FOR TWO-USER UPLINK CHANNEL

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ABSTRACT

We consider the non-orthogonal multiple access (NOMA) design for a classical two-user multiple access channel (MAC) with finite-alphabet inputs. In contrast to the majority of existing NOMA schemes using continuous Gaussian distributed inputs, we consider practical quadrature amplitude modulation (QAM) constellations at both transmitters, whose sizes are not necessarily the same. By adjusting the scaling factors (i.e., instantaneous transmitting powers) of both users, we aim to maximize the minimum Euclidean distance of the received sum-constellation for a maximum likelihood (ML) receiver. The formulated problem is a *mixed continuous-discrete* optimization problem and in general it is nontrivial to resolve. By carefully examining the structure of the objective function, we discover that Farey sequence can be employed to tackle the formulated problem. However, the existing Farey sequence is not applicable when the constellation sizes of the two users are different. To address this challenge, we define a new type of Farey sequence, termed *punched Farey sequence*. Based on this new definition and its properties, we manage to attain a closed-form optimal solution to the original problem by first dividing the entire feasible region into a finite number of Farey intervals and then taking the maximum over all the subintervals. Finally, computer simulations are carried out to verify our theoretical analysis, and to demonstrate the advantages of the proposed NOMA over known orthogonal and non-orthogonal designs.

1. INTRODUCTION

Non-orthogonal multiple access (NOMA) has recently emerged as a key enabling radio access technology to meet the unprecedented requirements of the fifth generation (5G) cellular networks, due to its inherent advantages of high spectral efficiency, massive connectivity, and low transmission latency [1–3]. The basic principle of NOMA is to serve more than one user with distinct channel conditions simultaneously in the same orthogonal resource block along the time, frequency, or code axes. This can be achieved by applying the superposition coding (SC) at the transmitter side and multiuser detector (e.g., successive interference cancellation (SIC)) at the receiver side to distinguish the co-channel users. By taking practical constraints on user fairness and/or radio resource management into consideration, NOMA has been intensively investigated in various wireless communication systems [4–9].

Up to now, we note that the vast majority of existing NOMA designs assumed the use of Gaussian input signals [4–8, 10–19]. Although the Gaussian input is of great significance both theoretically and practically, its implementation in reality will be built on huge storage capacity, unaffordable computational complexity and extremely long decoding delay [20, Ch. 9]. More importantly, the actual transmitted signals in real communication systems are drawn from finite-alphabet constellations, such as pulse amplitude modulation (PAM), quadrature amplitude modulation (QAM), and phase-shift keying (PSK) [21, Ch. 5]. Besides, applying the results derived from the Gaussian inputs to the signals with finite-alphabet inputs can lead to significant performance loss [22]. In this sense, Gaussian input serves mostly as the theoretical benchmark. By contrast, the NOMA design with finite-alphabet inputs is of utmost practical importance and has attracted considerable efforts, see e.g., [23–28] and references therein. The main principle of these efforts is to ensure that the signal originated from each user can be uniquely decoded from the received sum-signal at the receiver side. However, all NOMA designs provided in [23–27] used mutual information as the performance measure, where the solutions were numerical and limited insights on the relationship between the sum-constellation and each user's constellation can thus be drawn from the obtained solutions.

Inspired by the aforementioned work, in this paper we target a *closed-form* NOMA design for a classical two-user Gaussian multiple access channel (MAC) with finite-alphabet inputs and an optimal maximum likelihood (ML) detector at the receiver, where the two users are allowed to transmit simultaneously in the same frequency band. We note that the optimal power control scheme for the Gaussian MAC with finite-alphabet inputs is still an open problem and only numerical solutions are available [23, 24, 29, 30]. To fill this gap, we investigate, for the first time, the optimal power control problem for the two-user Gaussian MAC with *finite square QAM constellations* that maximizes the minimum Euclidean distance of the received signals with the maximum likelihood (ML) detector. Note that QAM signaling is more spectrally efficient than other commonly-used constellations such as PSK signaling. The main contributions of this paper can be summarized as follows:

1. We develop a practical NOMA design for the classical two-user Gaussian MAC, where the two users are allowed to adopt not necessarily the same QAM constellations.
2. Our Farey sequence-based design framework developed in [28] can no longer be applied here due to the fact that the two users may use different QAM constellations. To address this challenging problem, we define a new type of

The work of Z. Dong and L. Huang was supported by the National Natural Science Foundation of China under Grants U1713217, U1501253, and 61501485, and the Natural Science foundation of Guangdong Province under Grant 2015A030311030. The work of H. Chen and B. Vucetic was supported by the Australian Research Council under Grant FL160100032 and Grant DP150104019. The work of J.-K. Zhang was supported by the Natural Sciences and Engineering Research Council of Canada.

Farey sequence, termed *punched Farey sequence*, which is essential for our NOMA design with not necessarily the same QAM constellations. This concept is even mathematically new to the best of our knowledge [31]. Based on the punched Farey sequence and its properties, we manage to resolve the *mixed continuous-discrete* optimization problem by providing a neat closed-form optimal solution.

2. SYSTEM MODEL AND PROBLEM FORMULATION

2.1. Two-User Real Gaussian Multiple Access Channel

We note that each complex Gaussian MAC with square QAM constellations can be splitted into two identical parallel real-scalar Gaussian MACs with PAM constellations, which are called the in-phase and quadrature components [21, 23]. This fact motivates us to consider a real-scalar Gaussian MAC with PAM constellations directly, given as follows:

$$y = |h_1|w_1 s_1 + |h_2|w_2 s_2 + n, \tag{1}$$

where $|h_k|$ denotes the real channel coefficient between the k-th transmitter and the access point and it is known perfectly by all the nodes; $n \sim \mathcal{N}(0, \sigma^2)$ is a real additive Gaussian noise term; the information-bearing symbol $s_k \in \{\pm(2\ell - 1)\}_{\ell=1}^{M_k/2}$, $k = 1, 2$ is drawn from a standard PAM constellation with equal probability; w_1 and w_2 are the real non-negative scalars determining the minimum Euclidean distance of the actual transmitted PAM constellation sets, which are referred to as the *weighting coefficients*. We assume that the transmitted signals are subject to average power constraints such that $\mathbb{E}[w_1^2|s_1|^2] \le P_1/2$ and $\mathbb{E}[w_2^2|s_2|^2] \le P_2/2$.

2.2. The Weighting Coefficients Design Problem

In this section, we consider the weighting coefficient design problem. For notation simplicity, we set $|\tilde{h}_1| = \sqrt{\frac{3P_1}{2(M_1^2-1)}}|h_1|$, $|\tilde{h}_2| = \sqrt{\frac{3P_2}{2(M_2^2-1)}}|h_2|$, $\tilde{w}_1 = \sqrt{\frac{2(M_1^2-1)}{3P_1}}w_1$, $\tilde{w}_2 = \sqrt{\frac{2(M_2^2-1)}{3P_2}}w_2$, such that $0 < \tilde{w}_1 \le 1$ and $0 < \tilde{w}_2 \le 1$. The received signal in (1) can thus be re-written as:

$$y = |\tilde{h}_1|\tilde{w}_1 s_1 + |\tilde{h}_2|\tilde{w}_2 s_2 + n. \tag{2}$$

We assume that a coherent maximum-likelihood (ML) detector is used by the access point to estimate the transmitted signals in a symbol-by-symbol fashion. Mathematically, the estimated signals can be expressed as $(\hat{s}_1, \hat{s}_2) = \arg\min_{(s_1,s_2)} \big|y - (|\tilde{h}_1|\tilde{w}_1 s_1 + |\tilde{h}_2|\tilde{w}_2 s_2)\big|$.

By applying the nearest neighbour approximation method [21, Ch.6.1.4] at high SNRs for ML receiver, the average error rate is dominated by the minimum Euclidean distance of the received constellation points owing to the exponential decaying of the Gaussian distribution. As such, in this paper, we aim to devise the optimal value of $(\tilde{w}_1, \tilde{w}_2)$ to maximize the minimum Euclidean distance of constellation points of the received signal. The Euclidean distance between the two received signals $y(s_1, s_2)$ and $y(\tilde{s}_1, \tilde{s}_2)$ at the receiver for (s_1, s_2) and $(\tilde{s}_1, \tilde{s}_2)$ in the noise-free case is given by $|y(s_1, s_2) - y(\tilde{s}_1, \tilde{s}_2)| = \big||\tilde{h}_1|\tilde{w}_1(s_1 - \tilde{s}_1) - |\tilde{h}_2|\tilde{w}_2(\tilde{s}_2 - s_2)\big|$.

Note that s_1, $\tilde{s}_1$, s_2 and $\tilde{s}_2$ are all odd numbers, and thus we can let $s_1 - \tilde{s}_1 = 2n$ and $\tilde{s}_2 - s_2 = 2m$, in which $n \in \mathbb{Z}_{M_1-1}$ and $m \in \mathbb{Z}_{M_2-1}$ with $\mathbb{Z}_N \triangleq \{0, \pm1, \cdots, \pm N\}$ denoting the set containing all the possible differences. Similarly, we also define $\mathbb{Z}^2_{(M_1-1,M_2-1)} \triangleq \{(a, b) : a \in \mathbb{Z}_{M_1-1}, b \in \mathbb{Z}_{M_2-1}\}$, and $\mathbb{N}^2_{(M_1-1,M_2-1)} \triangleq \{(a, b) : a \in \mathbb{N}_{M_1-1}, b \in \mathbb{N}_{M_2-1}\}$ where $\mathbb{N}_N \triangleq \{0, 1, \cdots, N\}$. From the definitions above, $(s_1, s_2) \ne (\tilde{s}_1, \tilde{s}_2)$ is equivalent to $(m, n) \ne (0, 0)$ (i.e., $m \ne 0$ or $n \ne 0$). To proceed, we define

$$d(m, n) = \frac{1}{2}|y(s_1, s_2) - y(\tilde{s}_1, \tilde{s}_2)| = \big||\tilde{h}_1|\tilde{w}_1 n - |\tilde{h}_2|\tilde{w}_2 m\big|,$$
$$(m, n) \in \mathbb{Z}^2_{(M_1-1,M_2-1)} \setminus \{(0, 0)\}, \tag{3}$$

where $\mathcal{A}\setminus\mathcal{B} \triangleq \{x \in \mathcal{A} \text{ and } x \notin \mathcal{B}\}$. We are at a point to formally formulate the following max-min optimization problem,

Problem 1 *Find the optimal $(\tilde{w}_1^*, \tilde{w}_2^*)$ subject to the individual average power constraint such that the minimum Euclidean distance d^* of the received constellation points is maximized, i.e.,*

$$(\tilde{w}_1^*, \tilde{w}_2^*) = \arg\max_{(\tilde{w}_1,\tilde{w}_2)} \min_{(m,n)\in\mathbb{Z}^2_{(M_1-1,M_2-1)}\setminus\{(0,0)\}} d(m, n)$$
$$\text{s.t. } 0 < \tilde{w}_1 \le 1 \text{ and } 0 < \tilde{w}_2 \le 1. \tag{4}$$

Note that the inner optimization variable of finding the minimum Euclidean distances is discrete, while the outer one $(\tilde{w}_1, \tilde{w}_2)$ is continuous. In other words, Problem 1 is a *mixed continuous-discrete* optimization problem and it is in general hard to solve. To the best of our knowledge, only numerical solutions to such kind of problems are available in the open literature [23, 24, 29, 30]. To optimally and systematically solve this problem, we now develop a design framework based on the *Farey sequence* [31], in which the entire feasible region of $(\tilde{w}_1, \tilde{w}_2)$ is divided into a finite number of mutually exclusive sub-regions. Then, for each sub-region, the formulated optimization problem can be solved optimally with a closed-form solution, and subsequently the overall maximum value of Problem 1 can be attained by taking the maximum value of the objective function among all the possible sub-regions. We first consider the inner optimization problem in (4) given by:

Problem 2 Finding differential pairs with the minimum Euclidean distance:

$$\min_{(m,n)\in\mathbb{Z}^2_{(M_1-1,M_2-1)}\setminus\{(0,0)\}} d(m, n)$$
$$= \min_{(m,n)\in\mathbb{Z}^2_{(M_1-1,M_2-1)}\setminus\{(0,0)\}} \big||\tilde{h}_1|\tilde{w}_1 n - |\tilde{h}_2|\tilde{w}_2 m\big|. \tag{5}$$

We should point out that finding the closed-form solution to the optimal (m, n) for (5) is not trivial since the solution depends on the values of $|\tilde{h}_1|$ and $|\tilde{h}_2|$, which can span the whole positive real axis. Moreover, the values of $\tilde{w}_1$ and $\tilde{w}_2$ will be optimized later and cannot be determined beforehand. It is worth mentioning here that a similar optimization problem was formulated and resolved for a Gaussian Z channel in our previous work [28]. In [28], we resorted to the existing Farey sequence to solve the formulated problem. However, due to the inherent symmetric structure between numerators and denominators of the conventional Farey sequence, our results presented in [28] refers only to the case where both transmitters need to use exactly *identical* constellation size (i.e., the same transmission rate) and thus cannot be applied to the problem in this paper with M_1 and M_2 not necessarily the same. Motivated by this, in this paper we define a new type of Farey sequence, termed punched Farey sequence. In the subsequent section, we will introduce the definition and some important properties of the original Farey sequence and the developed punched Farey sequence.

2.3. Punched Farey Sequence

We now propose a new definition in number theory called *Punched Farey sequence* which characterizes the relationship between two positive integers as follows:

Definition 1 *The punched Farey sequence $\mathfrak{P}_K^L$ is the ascending sequence of irreducible fractions whose denominators are no greater than K and numerators are no greater than L.*

Example 1 *$\mathfrak{P}_5^2$ is the ordered sequence* $\left(\frac{0}{1}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{2}{3}, \frac{1}{1}, \frac{2}{1}, \frac{1}{0}\right)$.

We now develop some elementary properties of the punched Farey sequence in line with Farey sequences [31] and the proof can be found in [32] which is omitted due to space limitation. It is worth pointing out that, although for some properties, we can find the counterparts in conventional Farey sequences, the extension to the punched Farey sequences is non-trivial and the following results are new.

Property 1

1. *If $\frac{n_1}{m_1}$ and $\frac{n_2}{m_2}$ are two adjacent terms in $\mathfrak{P}_K^L$ ($\min\{K,L\}\geq 2$) such that $\frac{n_1}{m_1} < \frac{n_2}{m_2}$, then, 1) $\frac{n_1+n_2}{m_1+m_2} \in \left(\frac{n_1}{m_1}, \frac{n_2}{m_2}\right)$, $\frac{m_1+m_2}{n_1+n_2} \in \left(\frac{m_2}{n_2}, \frac{m_1}{n_1}\right)$; 2) $m_1n_2 - m_2n_1 = 1$; 3) If $n_1+n_2 \leq L$, then $m_1+m_2 > K$ and if $m_1+m_2 \leq K$, then $n_1+n_2 > L$; 4) $n_1+n_2 \geq 1$ where the equality is attained if and only if $\frac{n_1}{m_1} = \frac{0}{1}$ and $\frac{n_2}{m_2} = \frac{1}{K}$. Likewise, $m_1+m_2 \geq 1$ where the equality is attained if and only if $\frac{n_1}{m_1} = \frac{L}{1}$ and $\frac{n_2}{m_2} = \frac{1}{0}$.*
2. *If $\frac{n_1}{m_1}$, $\frac{n_2}{m_2}$ and $\frac{n_3}{m_3}$ are three consecutive terms in $\mathfrak{P}_K^L$ with $\min\{K,L\} \geq 2$ such that $\frac{n_1}{m_1} < \frac{n_2}{m_2} < \frac{n_3}{m_3}$, then $\frac{n_2}{m_2} = \frac{n_1+n_3}{m_1+m_3}$.*
3. *Let $\frac{n_1}{m_1}, \frac{n_2}{m_2}, \frac{n_3}{m_3}, \frac{n_4}{m_4} \in \mathfrak{P}_K^L$ with $\min\{K,L\} \geq 3$. If $\frac{n_1}{m_1} < \frac{n_2}{m_2} < \frac{n_3}{m_3} < \frac{n_4}{m_4}$, where $\frac{n_2}{m_2}, \frac{n_3}{m_3}$ are successive in $\mathfrak{P}_K^L$, then $\frac{n_1+n_3}{m_1+m_3} \leq \frac{n_2}{m_2}$ and $\frac{n_3}{m_3} \leq \frac{n_2+n_4}{m_2+m_4}$.*

2.4. The Minimum Euclidean Distance of the Received Signal

We are now ready to solve Problem 2 to find the differential pairs (m,n) having the minimum Euclidean distance. To this end, we first introduce the following preliminary propositions.

Proposition 1

1. *Let $\mathbb{F}^2_{(M_1-1,M_2-1)} = \{(m,n) : \frac{n}{m} \in \mathfrak{P}_{M_2-1}^{M_1-1}\}$. Then, we have $\min_{(m,n)\in\mathbb{Z}^2_{(M_1-1,M_2-1)}\setminus\{(0,0)\}} d(m,n) = \min_{(m,n)\in\mathbb{F}^2_{(M_1-1,M_2-1)}} d(m,n)$;*
2. *Let $\frac{n_1}{m_1}$ and $\frac{n_2}{m_2}$ be two terms of $\mathfrak{P}_{M_2-1}^{M_1-1}$ such that $\frac{n_1}{m_1} < \frac{n_2}{m_2}$. Then, for $\frac{|\tilde{h}_2|\tilde{w}_2}{|\tilde{h}_1|\tilde{w}_1} \in \left(\frac{n_1}{m_1}, \frac{n_2}{m_2}\right)$ and $d(m,n) = \left||\tilde{h}_1|\tilde{w}_1 n - |\tilde{h}_2|\tilde{w}_2 m\right|$, we have 1) If $\frac{|\tilde{h}_2|\tilde{w}_2}{|\tilde{h}_1|\tilde{w}_1} = \frac{n_1+n_2}{m_1+m_2}$, then $d(m_1,n_1) = d(m_2,n_2)$; 2) If $\frac{|\tilde{h}_2|\tilde{w}_2}{|\tilde{h}_1|\tilde{w}_1} \in \left(\frac{n_1}{m_1}, \frac{n_1+n_2}{m_1+m_2}\right)$, then $d(m_1,n_1) < d(m_2,n_2)$; 3) If $\frac{|\tilde{h}_2|\tilde{w}_2}{|\tilde{h}_1|\tilde{w}_1} \in \left(\frac{n_1+n_2}{m_1+m_2}, \frac{n_2}{m_2}\right)$, then $d(m_2,n_2) < d(m_1,n_1)$;*
3. *For any $\frac{n_1}{m_1}, \frac{n_2}{m_2}, \frac{n_3}{m_3}, \frac{n_4}{m_4} \in \mathfrak{P}_{M_2-1}^{M_1-1}$ with $|\mathfrak{P}_{M_2-1}^{M_1-1}| \geq 4$, if $\frac{n_1}{m_1} < \frac{n_2}{m_2} < \frac{n_3}{m_3} < \frac{n_4}{m_4}$, where $\frac{n_2}{m_2}, \frac{n_3}{m_3}$ are successive in $\mathfrak{P}_{M_2-1}^{M_1-1}$, we have 1) $\min_{(m,n)\in\mathbb{F}^2_{(M_1-1,M_2-1)}} d(m,n) = d(m_2,n_2) = |\tilde{h}_2|\tilde{w}_2 m_2 - |\tilde{h}_1|\tilde{w}_1 n_2$, if $\frac{|\tilde{h}_2|\tilde{w}_2}{|\tilde{h}_1|\tilde{w}_1} \in \left(\frac{n_2}{m_2}, \frac{n_2+n_3}{m_2+m_3}\right)$; 2) $\min_{(m,n)\in\mathbb{F}^2_{(M_1-1,M_2-1)}} d(m,n) = d(m_3,n_3) = |\tilde{h}_1|\tilde{w}_1 n_3 - |\tilde{h}_2|\tilde{w}_2 m_3$, if $\frac{|\tilde{h}_2|\tilde{w}_2}{|\tilde{h}_1|\tilde{w}_1} \in \left(\frac{n_2+n_3}{m_2+m_3}, \frac{n_3}{m_3}\right)$.*

2.5. Closed-Form Optimal Solution to Problem 1

With the help of Proposition 1 presented in the previous subsection, we now can solve Problem 1 by restricting $\frac{|\tilde{h}_2|\tilde{w}_2}{|\tilde{h}_1|\tilde{w}_1}$ into a certain punched Farey interval determined by the corresponding Farey pair where a closed-form solution is attainable. More specifically, we consider the punched Farey sequence given by $\mathfrak{P}_{M_2-1}^{M_1-1} = \left(\frac{b_1}{a_1}, \frac{b_2}{a_2}, \cdots, \frac{b_C}{a_C}\right)$, where $C = |\mathfrak{P}_{M_2-1}^{M_1-1}|$. Now, assume that $\frac{|\tilde{h}_2|\tilde{w}_2}{|\tilde{h}_1|\tilde{w}_1} \in \left(\frac{b_k}{a_k}, \frac{b_{k+1}}{a_{k+1}}\right)$ where $\left(\frac{b_k}{a_k}, \frac{b_{k+1}}{a_{k+1}}\right)$ is the k-th punched Farey interval for $k = 1, \ldots, C-1$, and we aim to find the optimal $(\tilde{w}_1^*(k), \tilde{w}_2^*(k))$ such that

$$g\Big(\frac{b_k}{a_k}, \frac{b_{k+1}}{a_{k+1}}\Big) = \max_{(\tilde{w}_1,\tilde{w}_2)} \min_{(m,n)\in\mathbb{F}^2_{(M_1-1,M_2-1)}} d(m,n)$$

$$\text{s.t. } \frac{b_k}{a_k} < \frac{|\tilde{h}_2|\tilde{w}_2}{|\tilde{h}_1|\tilde{w}_1} \leq \frac{b_{k+1}}{a_{k+1}}, 0 < \tilde{w}_1 \leq 1 \text{ and } 0 < \tilde{w}_2 \leq 1.$$

By applying Proposition 1, we can obtain:

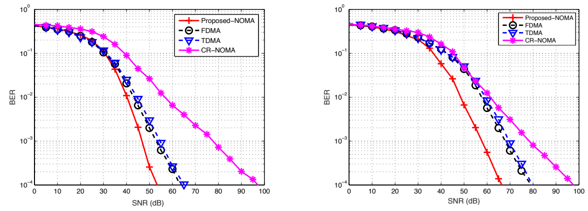
Lemma 1 *The optimal solution to Problem 2 is given as follows:*

- *If $\frac{|\tilde{h}_2|}{|\tilde{h}_1|} \leq \frac{b_k+b_{k+1}}{a_k+a_{k+1}}$, then $g\big(\frac{b_k}{a_k}, \frac{b_{k+1}}{a_{k+1}}\big) = \frac{|\tilde{h}_2|}{b_k+b_{k+1}}$ and $(\tilde{w}_1^*(k), \tilde{w}_2^*(k)) = \big(\frac{|\tilde{h}_2|(a_k+a_{k+1})}{|\tilde{h}_1|(b_k+b_{k+1})}, 1\big)$;*
- *If $\frac{|\tilde{h}_2|}{|\tilde{h}_1|} > \frac{b_k+b_{k+1}}{a_k+a_{k+1}}$, then $g\big(\frac{b_k}{a_k}, \frac{b_{k+1}}{a_{k+1}}\big) = \frac{|\tilde{h}_1|}{a_k+a_{k+1}}$ and $(\tilde{w}_1^*(k), \tilde{w}_2^*(k)) = \big(1, \frac{|\tilde{h}_1|(b_k+b_{k+1})}{|\tilde{h}_2|(a_k+a_{k+1})}\big)$.*

Now, we are ready to present the closed-form optimal solution to Problem 1 in terms of (w_1^*, w_2^*) which maximizes the minimum Euclidean distance of the sum-constellation, denoted by d_{noma}, over the entire feasible region.

Theorem 1 Closed-form optimal weighting coefficients: *The optimal solution to Problem 1 in terms of (w_1^*, w_2^*) is given by:*

1. *If $\frac{|h_2|}{|h_1|} \leq \sqrt{\frac{P_1(M_2^2-1)}{P_2M_2^2(M_1^2-1)}}$, then $(w_1^*, w_2^*) = \Big(\sqrt{\frac{3P_2M_2^2}{2(M_2^2-1)}}\frac{|h_2|}{|h_1|}, \sqrt{\frac{3P_2}{2(M_2^2-1)}}\Big)$, $d_{\text{noma}} = \sqrt{\frac{3P_2}{2(M_2^2-1)}}|h_2|$;*
2. *If $\sqrt{\frac{P_1(M_2^2-1)}{P_2M_2^2(M_1^2-1)}} < \frac{|h_2|}{|h_1|} \leq \sqrt{\frac{P_1M_1^2(M_2^2-1)}{P_2M_2^2(M_1^2-1)}}$, then $(w_1^*, w_2^*) = \Big(\sqrt{\frac{3P_1}{2(M_1^2-1)}}, \sqrt{\frac{3P_1}{2M_2^2(M_1^2-1)}}\frac{|h_1|}{|h_2|}\Big)$, $d_{\text{noma}} = \sqrt{\frac{3P_1}{2M_2^2(M_1^2-1)}}|h_1|$;*
3. *If $\sqrt{\frac{P_1M_1^2(M_2^2-1)}{P_2M_2^2(M_1^2-1)}} < \frac{|h_2|}{|h_1|} \leq \sqrt{\frac{P_1M_1^2(M_2^2-1)}{P_2(M_1^2-1)}}$, then $(w_1^*, w_2^*) = \Big(\sqrt{\frac{3P_2}{2M_1^2(M_2^2-1)}}\frac{|h_2|}{|h_1|}, \sqrt{\frac{3P_2}{2(M_2^2-1)}}\Big)$, $d_{\text{noma}} = \sqrt{\frac{3P_2}{2M_1^2(M_2^2-1)}}|h_2|$;*



(a) (b)

Fig. 1. Comparison between the Proposed-NOMA, CR-NOMA, TDMA and FDMA methods where 64-QAM is used for our case and 64-PSK is used for CR-based method: (a) $(\delta_1^2, \delta_2^2) = (1, 1)$, (b) $(\delta_1^2, \delta_2^2) = (1, 1/64)$.

4. *If* $\sqrt{\frac{P_1 M_1^2 (M_2^2-1)}{P_2 (M_1^2-1)}} < \frac{|h_2|}{|h_1|}$, *then* $(w_1^*, w_2^*) = \left(\sqrt{\frac{3P_1}{2(M_1^2-1)}}, \sqrt{\frac{3P_1 M_1^2}{2(M_1^2-1)}}\frac{|h_1|}{|h_2|}\right)$, $d_{\text{noma}} = \sqrt{\frac{3P_1}{2(M_1^2-1)}}|h_1|$.

3. SIMULATION RESULTS AND DISCUSSIONS

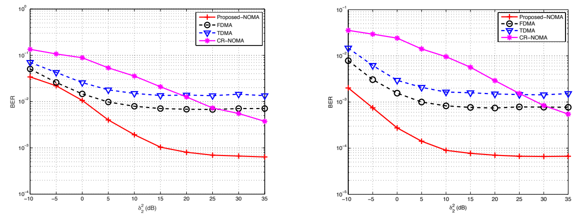
In this section, we conduct computer simulations to verify the effectiveness of our NOMA design in comparison to the constellation rotation (CR)-NOMA design proposed in [23] and the OMA methods including time-division multiple access (TDMA) and frequency-division multiple access (FDMA) schemes in various channel conditions and system configurations.

Without loss of generality, we assume that $P_1 = P_2 = 1$ and the system signal-to-noise ratio (SNR) is defined by $\rho \triangleq 1/2\sigma^2$. All channels are subject to Rayleigh distribution such that $h_k \sim \mathcal{CN}(0, 2\delta_k^2)$, $k = 1, 2$.

We first compare the average BER of all the schemes where the variances of the channel coefficients are the same, i.e., $(\delta_1^2, \delta_2^2) = (1, 1)$ in Fig. 1(a). In the simulation, without loss of generality, we assume that each user adopts 64-QAM for the proposed NOMA design and 64-PSK is used by each user in CR-NOMA. Meanwhile, for TDMA and FDMA methods, each user uses 4096-QAM. As can be observed from Fig. 1(a) that, the proposed NOMA design outperforms all the designs in moderate and high SNR regimes. In addition, the FDMA method has a better error performance than the TDMA scheme as expected. The CR-NOMA has the highest BER due to the fact that the PSK constellation has a smaller Euclidean distance under the same power constraint compared with QAM constellation.

In the following simulation, we take the near-far effect into consideration by letting $(\delta_1^2, \delta_2^2) = (1, 1/64)$ as shown in Fig. 1(b). Likewise, the proposed NOMA design has the lowest BER compared with all the benchmark schemes. Also, we can observe that the gap between the proposed NOMA and the FDMA as well as TDMA is larger than that in the case of equal channel gain. For example, at the BER 10^{-3}, the proposed NOMA has around 5dB SNR gain in Fig. 1(a), while the SNR gain is approximately 10dB in Fig. 1(b). Interestingly, we also observe that the error performance of CR-NOMA improves substantially compared to TDMA and FDMA in this case with near-far effect.

From both Figs. 1(a) and 1(b), we can observe that the performance gain of NOMA is highly related to the relative strength of the channel coefficients. To show this phenomenon clearly, we now study the BER against the relative strength of the channel coefficients under different SNRs. More specifically, in Fig.2(a), we set the variance of user S_1 as $\delta_1^2 = 1$, and we plot the BER against the variance of user S_2, i.e., δ_2^2, in dB. It can be observed from Fig. 2(a) that, for $\rho = 40$dB (i.e., the SNR is relatively low relative to the target transmission rate), our proposed NOMA scheme outperforms all the benchmark schemes. When δ_2^2 is less than 1 (i.e., less than 0dB), the error performance is mainly limited by user S_1 and even if δ_2^2 equals to 1, the BER gain of the proposed NOMA method is still marginal. However, with the increase of δ_2^2, the BER gain of the proposed NOMA method increases and finally gets saturated. Actually, when δ_2^2 is extremely large, the BER of the proposed NOMA is close to the system with one user transmitting with 64-QAM in both orthogonal blocks, while for the OMA method, it saturates as one user transmits using 4096-QAM in one block. This validates our observation that the proposed NOMA has a higher SNR gain when there is near-far effect. With the increase of δ_2^2, the performance of CR-NOMA improves dramatically and it eventually outperforms the OMA methods. However, the BER performance is poor when the channel gains of the two users are close. This is due to the fact that with the same spectral efficiency, a PSK constellation has a smaller minimum Euclidean distance than a QAM constellation. Moreover, the sum-constellation of two PSK constellations at the receiver does not have a good geometric structure. In Fig. 2(b), we can see that with the near-far effect, the BER gain of the proposed NOMA also become more significant. The BER gain of the proposed NOMA is evident even if $\delta_2^2 = 1$, which coincides well with the phenomenon observed in Fig. 1.



(a) (b)

Fig. 2. Comparison between the Proposed-NOMA with CR-NOMA, TDMA, and FDMA methods, 64-QAM are used for our case and 64-PSK are used for CR based method with (a) $\rho = 40$dB. (b) $\rho = 50$dB.

4. CONCLUSIONS

In this paper, we have presented a practical design framework for the non-orthogonal multiple access (NOMA) scheme in a classical two-user multiple access channel (MAC) with quadrature amplitude modulation (QAM) constellations at both users, the sizes of which are not necessarily the same. More specifically, by using a maximum likelihood (ML) detector, we aimed to maximize the minimum Euclidean distance of the sum-constellation at the receiver by adjusting the instantaneous transmit power of each user under an individual average power constraint. The design objective was formulated into a *mixed continuous-discrete* optimization problem. By introducing a new mathematical concept termed *punched Farey sequence* and investigating its fundamental properties, we managed to attain a compact closed-form solution for our original optimization problem. Computer simulations were conducted to verify our derivation under various channel configurations, and the simulation results demonstrated that our proposed NOMA scheme outperforms OMA and existing NOMA significantly and the performance gap can be further enlarged when there is a near-far effect between the users.

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