REDUCED DIMENSION MINIMUM BER PSK PRECODING FOR CONSTRAINED TRANSMIT SIGNALS IN MASSIVE MIMO

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ABSTRACT

Recently a number of nonlinear precoding algorithms have been developed for designing a downlink transmit signal that is constrained by some nonlinearity, such as one-bit quantization, power-amplifier saturation or constant modulus. These methods use iterative search algorithms to directly design the signal that is transmitted from each antenna. Since the dimension of the search space equals the number of antennas, the computational complexity of these approaches can be high for massive MIMO scenarios. Thus, in this paper we pose the problem in a smaller dimensional space by constraining the signal prior to the nonlinearity to be the output of a linear precoder. The search is then over the vector of predistorted symbols at the input to the linear precoder, which is typically much smaller than the number of antennas. We focus on algorithms that minimize the bit error rate at the receivers, and show that performance can be obtained that is similar to algorithms that operate directly in the antenna domain.

Index Terms— one-bit quantization, constant modulus, massive MIMO, linear precoding, per-antenna power constraints, minimum probability of error precoding

1. INTRODUCTION

There are a number of situations in which the signal transmitted by a given antenna is constrained in some way. The most common type of this constraint is a per antenna power constraint, in which the amplitude of the signal cannot exceed a certain value, due for example to saturation in the power amplifier (PA). In other situations, to maximize the energy efficiency of the PA, one may wish to constrain the transmit signal to have a constant modulus. Another example that has recently gained attention is the use of a low-resolution (*e.g.,* one-bit) digital-to-analog converter (DAC), which limits the transmit signal to one of only a finite number of possibilities.

Linear precoding schemes are not directly compatible with such constraints, since the precoder output will in general not satisfy them. A simple approach to dealing with this situation is to use a linear precoder anyway, and then project its output onto the nearest constrained signal. This approach has recently been used for the case of one-bit DACs in [1–3]. Its performance was analyzed in [3] for the zero-forcing precoder, and was found to perform reasonably well as long as the ratio of the number of antennas to the number of receivers was large enough. However, direct nonlinear design methods that take the one-bit constraint into account when designing the transmit signal vector have been shown to perform considerably better. Some examples include [4–6] for one-bit DACs and [5, 7–13] for constant modulus signals. For per-antenna power constraints, most prior work has focused on linear precoders [14–20], although direct design of the transmit vector has recently been considered for this problem as well [21].

The primary drawback of these direct nonlinear design approaches is their computational complexity. First, they are symbol-level precoders, which means that an entirely new precoding must be designed for each set of transmit symbols; this is in contrast to linear precoders, which remain fixed during the coherence time of the channel. Second, they require iterative algorithms in a search space of dimension equal to the number of antennas, which can be very large in the case of massive MIMO systems.

In this paper, we focus on the massive MISO downlink in which a basestation (BS) with a large number of antennas transmits PSK symbols to a number of single antennas users. The method we will present also requires symbol-level precoding, but it operates in a space whose dimension is equal to the number of users, which is typically much smaller than the number of antennas. The algorithm operates like the methods above that simply project the output of a linear precoder onto the constraint space, but it attempts to predistort the signals in order to find a better linear precoder output before the constraints are applied. In particular, the algorithm adjusts the input to the linear precoder to minimize the worst-case bit error rate (BER) at the users, where the minimum BER criterion is achieved by maximizing the "saftey margin" described in [6]. The minimum BER criterion has also been used in "constructive interference" precoding [13, 22, 23].

In the next section we present our data modeling assumptions, and in Section 3 we define the safety margin metric for minimizing the users' BER. Section 4 then provides details of

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the algorithm, followed by the results of several simulations in Section 5 that illustrate its performance.

2. DATA MODEL AND ASSUMPTIONS

We assume a flat fading downlink scenario with an M-antenna BS and K single antenna user terminals. We will focus on a massive MIMO scenario in which $M \gg K$, although neither this nor the flat fading assumption is strictly necessary for the proposed methods. We make no assumptions regarding the $K \times M$ channel matrix $\mathbf{H}$ other than it is generically full rank. We let the M-element vector $\mathbf{x}$ represent the transmitted signal at the BS, so that the downlink data model can be represented as

$$\hat{\mathbf{r}} = \mathbf{H}\mathbf{x} + \mathbf{n} \ , \tag{1}$$

where the k-th element of $\hat{\mathbf{r}}$, denoted by $\hat{r}_k$, represents the signal received by user k, and $\mathbf{n}$ is a vector whose K elements are independent identically distributed (i.i.d.) Gaussian noise. We will denote $\mathbf{r} = \mathbf{H}\mathbf{x}$ as the noise-free version of $\hat{\mathbf{r}}$.

In standard linear precoding, $\mathbf{x}$ is given by the product of the $K \times 1$ vector of symbols $\mathbf{s}$ that the BS desires to send to the users times an $M \times K$ precoder $\mathbf{P}$; *i.e.,* $\mathbf{x} = \gamma\mathbf{P}\mathbf{s}$, where the scalar γ is chosen to meet an average power constraint. Common precoding approaches include maximum ratio transmission $\mathbf{P} = \mathbf{H}^H$ and zero-forcing (ZF) $\mathbf{P} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}$, where $(\cdot)^H$ denotes the conjugate transpose. In this paper, we will assume that the elements of $\mathbf{x}$ are constrained in some nonlinear way to lie in a certain set $\mathcal{X}$ due to hardware requirements that prevent the use of standard linear precoding. Some examples of such constraint sets $\mathcal{X}$ are listed below in terms of the constant ρ, which is chosen to satisfy power constraints:

- *Low-Resolution DACs:* In this case, the elements of $\mathbf{x}$ are the outputs of coarse quantizers, and thus they are restricted to lie at certain fixed constellation points. For example, in the case of a one-bit quantizer, we have $x_i = \sqrt{\frac{\rho}{2}}(\pm 1 \pm j)$.
- *Saturation Nonlinearity:* Here the amplitude of the elements of x_i cannot exceed a certain value due, for example, to PA saturation: $|x_i| \leq \sqrt{\rho}$.
- *Constant Modulus:* To maximize the energy efficiency of the PAs, in some cases the transmit signal may be constrained to be at the peak power level for all antennas: $|x_i| = \sqrt{\rho}$.

As explained above, several methods have recently been developed to design transmit vectors $\mathbf{x}$ that satisfy these constraints and optimize some performance criterion. These methods rely on iterative algorithms in an M-dimensional space, and they often make use of relaxation techniques. When M is large as with massive MIMO, the computational complexity of these algorithms may be prohibitive.

An alternative and very simple approach to designing $\mathbf{x}$ is to simply project the output of a standard linear precoder onto the constraint set: $\mathbf{x} = \mathcal{Q}(\mathbf{P}\mathbf{s})$, where the function $\mathcal{Q}(\cdot)$ represents the nonlinear constraint. The performance of this approach was studied in [3] for the case of one-bit DACs, where it was shown that reasonably good performance can be obtained when the ratio M/K is large enough. It was shown in [24] that improved performance can be obtained by predistorting the symbols in a certain way prior to the nonlinearity: $\mathcal{Q}(\mathbf{P}(\mathbf{s} + \boldsymbol{\epsilon}))$, although the predistortion in [24] was made in an *ad hoc* way. Here we study a more systematic method for perturbing the symbols that allows this method to achieve performance similar to the direct design methods that require an M-dimensional search. The advantage of the perturbation approach is that the search for the predistortion $\boldsymbol{\epsilon}$ is only of dimension K, which is normally much smaller than M.

3. MINIMUM BER METRIC FOR PSK

We will assume in this paper that the elements of $\mathbf{s}$ are drawn from a D-PSK alphabet, *i.e.,* $s_k = \exp(j\pi(2d+1)/D)$ for some $d \in \{0, \cdots, D-1\}$, although the methods we present can be generalized to other constellations. At the user terminals, the received signal $\hat{r}_k$ is compared with the D-PSK symbol boundaries, and the closest symbol in the constellation is taken to be the transmitted signal. Thus, r_k need not be "close" to s_k to be decoded correctly, it just must lie in the correct decision region. In fact, for PSK, a lower BER is obtained when $|r_k|$ can be made to increase well beyond $|s_k|$ in a direction that is farther removed from the decision boundaries. As mentioned above, this idea has been referred to as constructive interference precoding in other work [13, 22, 23], and used for symbol-level precoder design. The approach we take here however is different.

To describe the BER metric, we define $\mathbf{S} = \mathrm{diag}(\mathbf{s})$ and follow the approach of [6] by rotating the coordinate system of the received data:

$$\mathbf{z} = \mathbf{S}^H\mathbf{r} = \mathbf{S}^H\mathbf{H}\mathbf{x} = \tilde{\mathbf{H}}\mathbf{x} \ ,$$

where we have defined $\tilde{\mathbf{H}} = \mathbf{S}^H\mathbf{H}$. Since $\mathbf{S}$ is unitary, this will have no impact on the distribution of the Gaussian noise when it is present. If the design of $\mathbf{x}$ is successful, then the elements of the vector $\mathbf{z}$ will all lie in a $2\pi/D$ pie-shaped region centered on the horizontal axis, as depicted in Fig. 1. The quantity δ_k represents the "safety margin," or the distance of the received sample z_k from the decision boundary, and is defined as

$$\delta_k = z_{kR}\sin\theta - |z_{kI}|\cos\theta \ , \tag{2}$$

where $\theta = \pi/D$, and $z_k = z_{kR} + jz_{kI}$ is the decomposition of z_k into real and imaginary parts. The larger δ_k, the smaller the BER. Thus, our approach to designing $\mathbf{x}$ will be to maximize the worst case δ_k over all the users $k \in \mathcal{K} = \{1, \cdots, K\}$:

$$\delta = \arg\max_{\mathbf{x}\in\mathcal{X}} \min_{k\in\mathcal{K}} \delta_k \ . \tag{3}$$

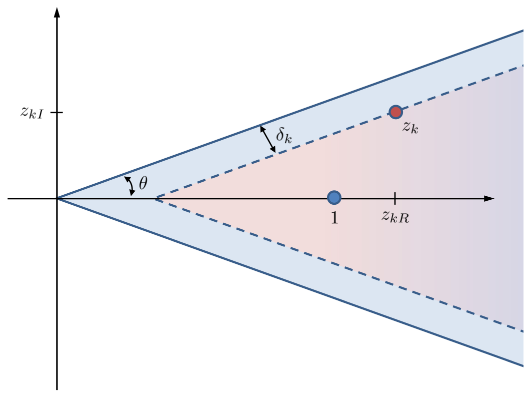

Fig. 1. Safety margin for received data at user k.

4. MINIMUM BER PREDISTORTION PRECODING

The algorithm of [6] maximizes (3) for one-bit quantization using a relaxed box constraint, which allows for a linear programming solution. The resulting solution is then mapped onto the nearest quantization points $\sqrt{\frac{\rho}{2}}(\pm 1 \pm j)$. The linear programming search is over $\mathbf{x}$, which is of dimension M. In the method we propose, we take a different approach by assuming that $\mathbf{x} = \mathcal{Q}(\mathbf{P}(\mathbf{s}+\boldsymbol{\epsilon}))$ for some fixed linear precoder $\mathbf{P}$ that depends only on the channel and not $\mathbf{s}$. The advantage of this approach is that the optimization of (3) will be over the predistortion vector $\boldsymbol{\epsilon}$, which is only of dimension $K \ll M$.

The proposed approach uses an approximate stochastic gradient descent method to update $\boldsymbol{\epsilon}$ and increase δ. Instead of computing the discontinuous gradient with respect to the nonlinearity $Q(\cdot)$, the function $Q(\cdot)$ is ignored and the gradient is computed as if it were not present. At iteration p, the approximate gradient, denoted here by $\tilde{\nabla}$, is given by

$$\begin{aligned}
\tilde{\nabla}_{\boldsymbol{\epsilon}}\delta &= \tilde{\nabla}_{\boldsymbol{\epsilon}}\left[z_{kR}\sin\theta - \mathrm{sign}(z_{kI})z_{kI}\cos\theta\right] \\
&= \tilde{\nabla}_{\boldsymbol{\epsilon}}\left[(\mathbf{e}_k^T\tilde{\mathbf{H}}\mathbf{x})_R\sin\theta - \mathrm{sign}(z_{kI})(\mathbf{e}_k^T\tilde{\mathbf{H}}\mathbf{x})_I\cos\theta\right] \\
&= \tilde{\nabla}_{\boldsymbol{\epsilon}}\left[(\mathbf{e}_k^T\tilde{\mathbf{H}}\mathcal{Q}(\mathbf{P}\boldsymbol{\epsilon}))_R\sin\theta - \mathrm{sign}(z_{kI})(\mathbf{e}_k^T\tilde{\mathbf{H}}\mathcal{Q}(\mathbf{P}\boldsymbol{\epsilon}))_I\cos\theta\right] \\
&\approx \tilde{\nabla}_{\boldsymbol{\epsilon}}\left[(\mathbf{e}_{k(p)}^T\tilde{\mathbf{H}}\mathbf{P}\boldsymbol{\epsilon})_R\sin\theta - \mathrm{sign}(z_{kI})(\mathbf{e}_k^T\tilde{\mathbf{H}}\mathbf{P}\boldsymbol{\epsilon})_I\cos\theta\right] \\
&\approx \mathbf{P}^H\tilde{\mathbf{H}}^H\mathbf{e}_k\left(\sin\theta + j\mathrm{sign}(z_{kI})\cos\theta\right) \ ,
\end{aligned} \tag{4}$$

where index k denotes the user with the worst case δ_k, $\mathbf{e}_k$ is a vector with a one in position k and zeros elsewhere, and the subscripts R and I again denote the real and imaginary parts, respectively. The first approximation ignores the nonlinearity $\mathcal{Q}(\cdot)$, and the second assumes we never evaluate the gradient at $z_{kI} = 0$. In addition to the errors caused by these approximations, taking a step in this direction may also cause the worst-case user index, and hence the approximate gradient, to change. These factors result in a relatively large misadjustment error, but also tend to allow the algorithm to escape from local minima and better explore the solution space. As such, the stopping criterion is not based on the norm of the gradient; the algorithm keeps track of the $\boldsymbol{\epsilon}$ iterate that produces the largest value of δ during the search, and then takes this as the solution after a fixed number of steps.

In the sequel, we will refer to the algorithm as Reduced Dimension Minimum BER (RedMinBER) precoding. A detailed description of the algorithm is given below.

1. Given $\mathbf{s}, \tilde{\mathbf{H}}, \mathbf{P}$, number of iterations N_p, and stepsize μ, set $p = 1$ and $\boldsymbol{\epsilon}(1) = 0$.
2. Calculate $\mathbf{z} = \tilde{\mathbf{H}}\mathcal{Q}(\mathbf{P}\mathbf{s})$ and $\delta(1)$ from (2)-(3).
3. Set $\mathbf{s}_{opt} = \mathbf{s}$ and $\delta_{opt} = \delta(1)$.
4. For $p = 1$ to N_p, do
   (a) Find $\boldsymbol{\epsilon}(p+1) = \boldsymbol{\epsilon}(p) + \mu\tilde{\nabla}^*_{\boldsymbol{\epsilon}}\delta(p)$.
   (b) Calculate $\mathbf{z} = \tilde{\mathbf{H}}\mathcal{Q}(\mathbf{P}(\mathbf{s}+\boldsymbol{\epsilon}(p+1)))$ and $\delta(p+1)$.
   (c) If $\delta(p+1) > \delta_{opt}$, set $\delta_{opt} = \delta(p+1)$ and $\mathbf{s}_{opt} = \mathbf{s} + \boldsymbol{\epsilon}(p+1)$.
5. Output solution $\mathbf{s}_{opt}$.

The bulk of the computational load for RedMinBER is the calculation of $\mathbf{z}$ at each step, but this requires a minimal amount of effort for the following two reasons: (i) since only one element of $\boldsymbol{\epsilon}(p)$ changes at each iteration, updating $\mathbf{P}(\mathbf{s}+\boldsymbol{\epsilon}(p))$ only takes $O(K)$ operations; (ii) the adjustments to $\boldsymbol{\epsilon}(p)$ at each iteration are relatively small, so in many cases (*e.g.*, for one-bit DACs) the number of terms, say $\tilde{M}$, in $\mathcal{Q}(\mathbf{P}(\mathbf{s}+\boldsymbol{\epsilon}(p)))$ that actually change their value from one iteration to the next will often be small. Computation of the gradient approximation is trivial since $\tilde{\mathbf{H}}\mathbf{P}$ only needs to be calculated once, and the gradient is just a scaled version of one of its rows. Thus, the overall computation is $O(\tilde{M}K)$ per iteration, where typically $\tilde{M} \ll M$. This compares favorably with the linear programming method of [6], which requires $O(M^3)$ operations per iteration. On the other hand, due to the discontinuous nature of the nonlinear transmit constraints and the resulting approximate gradient ascent method, RedMinBER will require a larger number of iterations. In the examples presented in the next section, about $2M$ iterations were required to get the best results. Still, even with this larger number of iterations, the overall computational load of RedMinBER is still significantly less than optimization methods that operate directly on $\mathbf{x}$.

5. SIMULATION EXAMPLES

For the simulation results presented here, we assume a base-station with $M = 128$ antennas serving $K = 16$ single-antenna users. We will consider two of the three types of transmit signals described in Section 2: one-bit quantized and constant modulus (CM) signals. In each case, the per antenna power is equal to ρ. The channel $\mathbf{H}$ is composed of circular complex independent identically distributed (c.c.i.i.d) Gaussian random variables with zero mean and unit variance. The additive noise at the receivers is also c.c.i.i.d Gaussian with zero mean and unit variance, and thus the SNR seen at the receivers is $10\log_{10}\rho M$ dB due to the array gain. We will let $\mathbf{s}$ be composed of QPSK symbols, and thus $\theta = \pi/4$. Our performance metric is the symbol error rate (SER) averaged over all trials and over all users. The ZF precoder will be used for $\mathbf{P}$ in both examples. The proposed algorithm was implemented with $\mu = 0.005$ and $N_p = 250$.

In the first example, we study the case of one-bit DACs. Figure 2 shows the SER as a function of the SNR at the receivers for standard (unquantized) ZF, quantized ZF (the method studied in [3]), the linear programming approach of [6], and the RedMinBER method proposed here using quantized ZF as the initialization. We see that RedMinBER achieves performance similar to, and at higher SNRs even slightly better than, the more complex linear programming method. Fig. 3 shows the value of δ averaged over 2000 channel realizations as a function of the number of RedMinBER iterations for an SNR of 4dB. We see that on the order of 250 iterations are needed on average for convergence, which results in an increase in δ from about 10 to 17. The resulting gain in dB is $20\log_{10}(17/10) = 4.6$dB, which matches the improvement achieved by RedMinBER over quantized ZF observed in Fig. 2 at 4dB SNR. The number of iterations can likely be reduced by replacing the fixed-step-size approach with some type of line search, albeit with an increase in the complexity per iteration.

Fig. 4 shows results for the case where $\mathbf{x}$ is constrained to be CM. In addition to RedMinBER, the SER is plotted for standard ZF, "ZF-CM" which refers to standard ZF with its output mapped onto the unit circle to enforce the CM constraint (similar to quantized ZF in the previous example), and the CVX-CIO method of [23], which minimizes the BER as in (3) but over the entire transmit vector $\mathbf{x}$ with the relaxed constraint $|x_i| \leq \rho$. Once the solution to the optimization is found, CVX-CIO maps the result onto the unit circle. We see that CVX-CIO and RedMinBER give essentially identical results, even though RedMinBER performs the search in a 16- rather than a 128-dimensional complex space.

6. CONCLUSIONS

This paper has presented a new method for downlink precoding in massive MIMO systems under general types of constraints on the transmit signals. Unlike methods that directly design the M-dimensional constrained transmit vector, the new algorithm searches for a predistortion of the signal applied to a standard linear precoder prior to application of the constraint. This search is only of dimension equal to the number of receivers, which is typically much smaller than M. Simulations show that the new algorithm achieves performance essentially identical to methods that operate in the larger-dimensional space, but at a fraction of the computational cost.

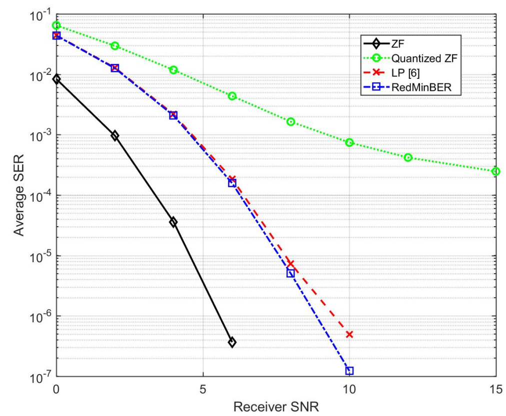

Fig. 2. SER vs. SNR at receivers for one-bit DACs.

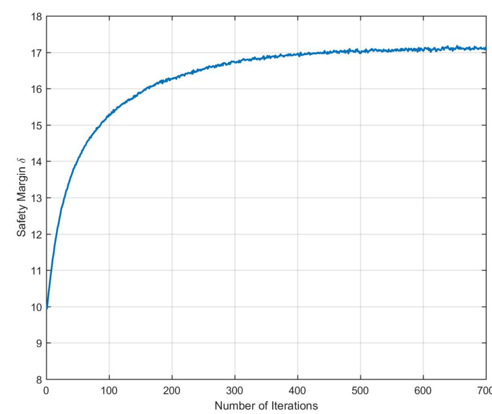

Fig. 3. Average change in δ versus number of iterations.

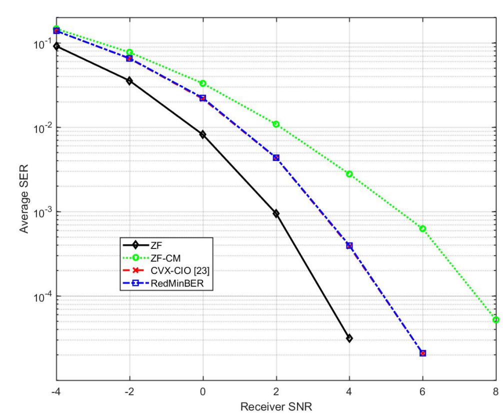

Fig. 4. SER vs. SNR at receivers for CM signals.

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