HIGH ACCURACY ACOUSTIC ESTIMATION OF MULTIPLE TARGETS

Mohammed H. AlSharif[1], *Mohamed Saad*[1], *Mohamed Siala*[2], *Hatem Boujemaa*[2], *Tarig Ballal*[1], *Tareq Y. Al-Naffouri*[1]

[1]King Abdullah University of Science & Technology (KAUST), Thuwal, Saudi Arabia
[2] Higher School of Communication of Tunis (SUP'COM), University of Carthage, Tunisia
{mohammed.alsharif, mohamed.saadeldin, tarig.ahmed, tareq.alnaffouri}@kaust.edu.sa
{mohamed.siala, boujemaa.hatem}@supcom.tn

ABSTRACT

This paper presents a new adaptation of a Gaussian echo model (GEM) to estimate the distances to multiple targets using acoustic signals. The proposed algorithm utilizes m-sequences and opens the door for applying other modulations and signal designs for acoustic estimation in a similar way. The proposed algorithm estimates the system impulse response and uses the GEM to limit the effect of noise before applying deconvolution to estimate the time of arrival (TOA) to multiple targets with high accuracy. The algorithm was experimentally evaluated for different scenarios with active (transmitters) and passive (reflectors) targets at proximity. In the case of closely spaced static passive targets, results show that 90% of the ranging errors are below 7 mm. When tracking two moving active targets approaching very close proximity, results show that 90% of the ranging errors are less than 10 mm.

***Index Terms*—** targets estimation, acoustic, tracking

1. INTRODUCTION

Several recent technologies and applications require precise estimation of the distances to multiple targets such as in location-aware networks, gesture controlled devices, navigation systems, activity detection and many other applications. Various ranging approaches utilize many types of signals to estimate the distances. Infrared and lasers are two of the most accurate technologies used in this field [1, 2, 3, 4]. However, they are expensive and complicated. Another ranging method uses radio signals to estimate the distance to a target based on two main approaches; time of arrival (TOA) and received signal strength (RSS). TOA-based target estimation requires very accurate synchronization because small errors in time estimation result in very high errors in estimating the distance to the target. The RSS approach usually utilizes a Wi-Fi or a Bluetooth signal, which requires pre-calibration and has low accuracy [5]. Another range estimation approach uses ultra-wideband signals. Recently, ultra-wideband round trip estimation reported 10-20 cm accuracy [6]. This accuracy degrades significantly in non-line of sight scenarios. Acoustic signals have the advantage of low propagation speed in the air allowing accurate TOA-based estimation using low-cost hardware [7]. However, the presence of multi-paths significantly degrades the estimation accuracy. In many applications, it is required to estimate the distances to multiple targets (active/passive) that are separated by tiny distances. Estimation of these targets with high accuracy becomes challenging as they get closer to each other.

Time of arrival (TOA) estimation using cross-correlation is one of the most common and straightforward ways where the received (Rx) signal is correlated with the reference transmitted (Tx) signal generating a peak associated with the TOA of the signal. The resolution of the TOA depends on the width of this peak which is a function of the available bandwidth. The resolution of the cross-correlation is given by $r = \frac{2v_s}{BW} = D$, where v_s is the speed of sound in air and BW is the hardware bandwidth. If two or more targets are separated by a distance less than D, their cross-correlation peaks will overlap, which makes it challenging to estimate TOA of each target. In [8, 9], R. Demirli and J. Saniie proposed a model and an algorithm to resolve overlapping acoustic echoes of a transmitted Gaussian pulse. They introduced a model-based deconvolution algorithm to determine the overlapping echoes with high accuracy. This algorithm requires finding the inverse of the convolution matrix or diagonalizing it using SVD, which is computationally demanding. The model-based estimation in [8, 9] was proposed for nondestructive evaluation applications which require pulse-echoes estimation for short distances. However, for long distances, using a pulse signal results in echoes with low signal to noise ratio (SNR) which significantly degrades the accuracy. Another concern with the long distance applications is the large size of the convolution matrix, which makes the system implementation computationally expensive.

This paper proposes the use of the Gaussian echo model (GEM) in [8] to estimate the impulse response of the system utilizing m-sequences. This GEM estimation is performed as a de-noising of the impulse response of the system. Then the deconvolution method is applied to the de-noised system impulse response to estimate the targets. Compared to the previous work on multiple targets estimation, the proposed algorithm contributes to the following:

- Implementation of GEM estimation on the cross-correlation of m-sequences instead of pulse-echoes wavelets. This opens the door for the use of the GEM on other signal designs.
- Using m-sequences which improves the SNR and allows for a multi-user system by utilizing the orthogonality of m-sequences.
- Implementation of a novel and less complex criteria to determine the number of targets and their initial parameters for fast convergence of the GEM estimator.
- Experimental evaluation of multiple targets estimation in an indoor environment using low-cost hardware.

2. MULTIPLE TARGETS ESTIMATION

The deconvolution method provides high TOA resolution in multiple targets estimation, but its performance degrades significantly in low SNR scenarios. Using a pulse signal for long distance estimation results in a low SNR. Two pre deconvolution steps are applied to improve the SNR. The first step is to use a pseudorandom sequence instead of a pulse signal. Then, instead of applying the algorithm to the Rx signal, apply it to the cross-correlation between Tx and Rx which represents the system impulse response. This approach improves the SNR and allows for a multi-user system. The system impulse response represents the global filter of the system. The GEM

This work is supported by the KAUST-MIT-TUD consortium under grant OSR-2015-Sensors-2700.

is a good model for the global filter due to the similarity between the shape of the filter and the Gaussian pulse. The second step is to de-noise the impulse response by modeling it using the GEM [8]. The proposed pseudorandom sequence is a maximal length sequence (m-sequence) which has good correlation properties.

2.1. M-Sequences

M-sequences are pseudorandom binary sequences that are generated using maximal linear feedback shift registers. The periodic autocorrelation function is given by [10]:

$$R(n) = \frac{1}{N}\sum_{m=1}^{N} a(m)a^*(m+n) = \begin{cases} 1, & \text{if } n = 0, \\ \frac{-1}{N}, & \text{if } 0 < n < N. \end{cases} \quad (1)$$

where $N = 2^m - 1$ is the sequence length, $a(m)$ takes one of two values in $\{-1, 1\}$, and $*$ denotes the complex conjugation operation.

2.2. System Impulse Response Estimation

Estimating the impulse response of the system is required to apply the deconvolution method. In the noiseless case and with no delay, the received signal can be modeled as:

$$y(t(nT_s)) = h(t(nT_s)) * x(t(nT_s)), \quad (2)$$

where $h(t(nT_s))$ is the impulse response of the system which is a combination of analog filtering at both the Tx and Rx sides, $x(t(nT_s))$ is the Tx signal, $t(nT_s)$ are the discrete samples of the time variable t, n is an integer, T_s is the sampling interval, and $*$ denotes the convolution operation. Cross-correlating the Rx signal with the Tx signal gives $r_{yx}(l) = h(l) * r_{xx}(l)$, where $r_{xx}(l)$ is the autocorrelation of the Tx signal. The cross-correlation between the Rx signal and the Tx signal is the convolution between the impulse response of the system and the autocorrelation of the Tx signal. If the Tx signal is a pseudorandom sequence, such as an m-sequence, which has an autocorrelation function approximately equals to an impulse function for large values of m, then we have $r_{yx}(l) = h(l)$. This means that the cross-correlation between the Rx signal $y(t(nT_s))$ and the Tx signal $x(t(nT_s))$ is the impulse response of the system. A pseudorandom signal with a bandwidth that is large enough to encompass the bandwidth of the system can be used to sound the channel. The experimental setup has a bandwidth of 7 KHz and a center frequency f_c of 20 KHz. An m-sequence of length of $2^{17} - 1$ was transmitted in a reverberation free environment to estimate the impulse response of the system. The cross-correlation between the Tx m-sequence and the Rx signal (the system impulse response) and its spectrum are shown in Figure 1.

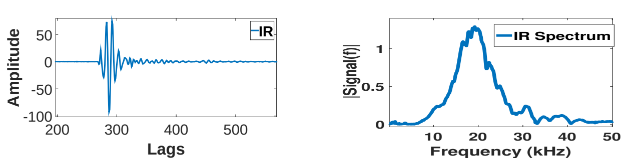


Fig. 1: System Impulse Response and its spectrum

Applying the deconvolution algorithm to the sum of the time shifted versions of the system impulse response enables the estimation of the TOAs of the targets. However, this estimation will be noisy and inaccurate due to the presence of noise. A precise knowledge of the impulse response of the system is a must for the de-noising of the received signal. The Gaussian echo model (GEM) is a good model for the global impulse response which will be adopted next as a de-noising step before applying the deconvolution method.

2.3. Gaussian Echo Modeling

The cross-correlation vector is composed of corrupted replicas of the system impulse response with different time shifts and different amplitudes. The system impulse response $h(t(nT_s))$ in (2) can be modeled using a Gaussian echo model (GEM) which is given by [8]:

$$s(\theta; t(nT_s)) = \alpha e^{-\beta(t(nT_s)-\tau)^2} \cos(2\pi f_c(t(nT_s) - \tau) + \phi)$$

where $\theta = [\beta, \tau, f_c, \phi, \alpha]$, β is a bandwidth factor which determines the bandwidth of the target signal, τ is the time of arrival, f_c is the center frequency, ϕ is the phase and α is the amplitude of the target signal. For a fixed parameter vector θ, the system impulse response can be modeled as $h(t(nT_s)) = s(\theta; t(nT_s)) + n(t(nT_s))$, where $n(t(nT_s))$ is the noise coming from the discrepancy between the system impulse response and the GEM. This model can be extended to multiple targets as:

$$q(t(nT_s)) = \sum_{m=1}^{M} h(t(nT_s))_m = \sum_{m=1}^{M} s(\theta_m; t(nT_s)) + n(t(nT_s)), \quad (3)$$

where each parameter vector θ_m completely defines each target and M is the number of targets. Since the transformation from the signal space to the parameter space is nonlinear, we need to solve the following nonlinear optimization problem for a single target:

$$\min_{\theta} \|h(t(nT_s)) - s(\theta, t(nT_s))\|^2, \quad (4)$$

where $h(t(nT_s))$ is the system impulse response for a single target. This optimization problem is an unconstrained nonlinear least squares problem. It can be solved using the Levenberg-Marquardt algorithm [11]. This algorithm is an iterative search algorithm. At each iteration we must solve $\min_{p_k} \frac{1}{2}\|H_k p_k + r_k\|^2$ subject to $\|p_k\| \leq \Delta_k$, where H_k is the gradient vector of the model with respect to the parameter vector θ, r_k is the residual function, and Δ_k is the trust region radius. A model function m_k, whose behavior near a point θ_k is similar to r_k, is given by [11]:

$$m_k(p) = \frac{1}{2}\|r_k\|^2 + p_k^T H_k^T r_k + \frac{1}{2} p_k^T H_k^T H_k p_k \quad (5)$$

Minimizing the residual function m_k (5) gives the step vector: $p_k^{GN} = -(H_k^T H_k)^{-1} H_k^T r_k$. Given this step p_k^{GN}, we define [11]:

$$\rho_k = \frac{r_k(\theta) - r_k(\theta + p_k^{GN})}{m_k(0) - m_k(p_k^{GN})} \quad (6)$$

If $\rho_k < 0$, then the step must be rejected and Δ_k must be shrunk. If $\rho_k \approx 1$, then it is safe to expand Δ_k. If $\rho_k > 0$ but not close to 1 then Δ_k is kept the same. If $\|p_k^{GN}\| > \Delta_k$, then there must be $\lambda > 0$ such that [11]

$$\begin{aligned} (H_k^T H_k + \lambda \text{diag}(H_k^T H_k)) p_k^{LM} &= -H_k^T r_k \\ \|p_k^{LM}\| &= \Delta_k \end{aligned} \quad (7)$$

where λ is called the damping factor and it is updated at each iteration according to the strategy in [12]. The step vector is given by:

$$p_k = \begin{cases} -(H_k^T H_k)^{-1} H_k^T r_k, & \text{if } \|p_k^{GN}\| \leq \Delta_k, \\ -(H_k^T H_k + \lambda \text{diag}(H_k^T H_k))^{-1} H_k^T r_k, & \text{if } \|p_k^{GN}\| > \Delta_k. \end{cases}$$

The Levenberg-Marquardt iteration formula for estimating the parameter vector is: $\theta^{(k+1)} = \theta^{(k)} + p_k$. Figure 2 shows the estimated system impulse response using GEM.

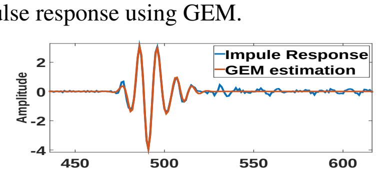


Fig. 2: Estimated system impulse response using GEM

The previous steps are for a single target estimation. Considering the case for multiple targets, the Expectation-Maximization (EM) algorithm is used [8] to solve the following optimization problem:

$$\min_{\theta_m} \|q - \sum_{m=1}^{M} s(\theta_m)\|^2 \tag{8}$$

where q is the observed data vector which represents the cross-correlation vector, $s(\theta_m)$ is the GEM and θ_m is the parameter vector of the m^{th} target and M is the number of targets. The EM algorithm defines M unobserved data vectors as $h_m = s(\theta_m) + n_m$, where n_m is an AWGN sequence. The relation between the observed data and the unobserved data is given by: $q = \sum_{m=1}^{M} h_m$. The expectation of the unobserved data h_m can be computed as [8]

$$\hat{h}_m^{(k)} = s(\theta_m^{(k)}) + \frac{1}{M}(q - \sum_{l=1}^{M} s(\theta_l^{(k)})) \tag{9}$$

This is called the expectation step of the EM algorithm. The maximization step iterates the parameter vector $\theta_m^{(k)}$ by minimizing:

$$\theta_m^{(k+1)} = \arg_{\theta_m} \min \|\hat{h}_m^{(k)} - s(\theta_m)\|^2 \tag{10}$$

This maximization step can be solved using Levenberg-Marquardt algorithm as was shown previously.

2.3.1. *Initialization of model estimation algorithm*

The initialization of the model estimation algorithm is a crucial step that affects the algorithm convergence. If the initial guess is far from the optimal solution, the Levenberg-Marquardt algorithm might not converge. The model estimation algorithm needs an initial guess of the number of targets M, and the initial parameter vector $\theta_m^{(0)}$ of each target. It also needs to determine overlapping targets. The number of targets M is determined initially as the number of cross-correlation peaks that are higher than a certain threshold. The location and amplitude of these peaks determine the initial guess of the TOA $\tau_m^{(0)}$, and the target amplitude $\alpha_m^{(0)}$, respectively. The initial guess of the bandwidth factor $\beta_m^{(0)}$ and the center frequency $f_{c_m}^{(0)}$ is the same for all the targets and depends on the available hardware bandwidth and center frequency. The initial phase $\phi_m^{(0)}$ is set to zero for all targets. Two criteria are used to determine the presence of overlapping targets. The first criterion is based on the width of the cross-correlation peak which is determined by the available bandwidth as was shown previously. The width of the cross-correlation peak increases based on the number of overlapping targets and how close they are to each other. The second criterion is based on the inverse-square law which states that the signal energy is inversely proportional to the square of the distance from the source (amplitude $\propto \frac{1}{\text{distance}^2}$). The factor of proportionality is set as the threshold for determining the overlapping targets. Assume that the m^{th} peak was determined by any of the two criteria to have multiple overlapping targets. Then, the initial guess for the overlapping targets is the same as the initial guess of the m^{th} target except that the TOAs are perturbed with small perturbations.

2.4. Deconvolution

To improve TOA estimation for multiple targets, the deconvolution method is applied to the estimated GEM. The Fast Fourier Transform (FFT) of the received signal can be written as:

$$Q(f) = \text{FFT}(\sum_{k=1}^{M} s(\theta_k)) = S(f) * \text{FFT}(\sum_{k=1}^{M} c_k \delta(t - \tau_k)), \tag{11}$$

where c_k is the amplitude of the k^{th} target, $\delta(t)$ is the Dirac delta function, M is the number of targets, $s(\theta_k)$ is the estimated system impulse response using the GEM and $S(f) = \text{FFT}(s(\theta_k))$. Dividing by $S(f)$ then taking the IFFT:

$$Z(f) = \frac{Q(f)}{S(f)} \implies z(t) = \text{IFFT}(Z(f)) = \sum_{k=1}^{M} c_k \delta(t - \tau_k) \tag{12}$$

The deconvolution method provides a high-resolution estimation of the targets. The TOA of these targets is estimated by taking the peaks of $z(t)$ that are higher than a certain threshold value. In this method, $s(\theta_k)$ is chosen as the estimated impulse response of one of the targets.

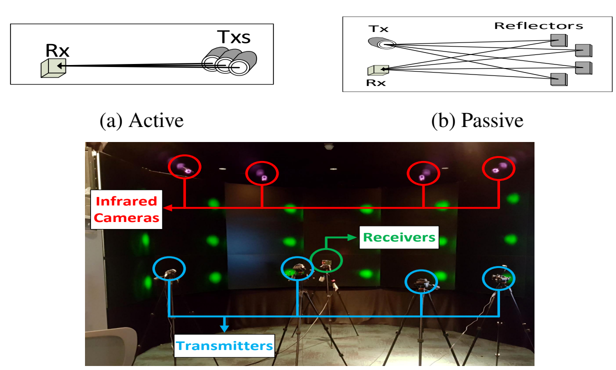
2.5. Algorithm Summary

In summary, the proposed algorithm can be implemented as follows:

- Step 1: GEM estimation algorithm initialization:
  - Cross-correlate the received signal with the transmitted signal and determine the initial values of M and $\Theta^{(0)} = [\theta_1^{(0)}; \theta_2^{(0)}; ...; \theta_M^{(0)}]$.
  - Determine the overlapping targets using the two criteria.
  - Set the initial guess of the overlapping target at the m^{th} peak as $\theta_m^{(0)} + [0, \Delta_t, 0, 0, 0]$ where Δ_t is half the width of the cross-correlation peak, and set $M = M +$ overlapping targets.
  - Set $k = 0$ (iteration index), and $m = 1$ (number of targets index).
- Step 2: GEM estimation algorithm:
  - Compute the expected signal for the m^{th} target given by (9)
  - Use Levenberg-Marquardt algorithm to solve (10) for the m^{th} parameter vector and set $\theta_m^{(k)} = \theta_m^{(k+1)}$.
  - Set $m \to m+1$ and move to the start of Step 2 unless $m > M$.
  - If $\|\Theta^{(k+1)} - \Theta^{(k)}\| \leq$ tolerance, then move to step 3. Otherwise set $k \to k + 1$ and $m = 1$ and go to start of Step 2.
- Step 3: Apply the deconvolution algorithm given by (12).

3. EXPERIMENTAL SETUP

This section presents the experimental setups used to test the proposed algorithm. The transmitters (Tx) used are Clarion SRG213H tweeters, which have a bandwidth of 7 kHz centered at 20 kHz. Microphones on a customized board, are used as receivers (Rx). Both Tx and Rx are connected to a PC through an E44 Express soundcard providing sampling rate up to 192 kHz. Recorded data is saved to be processed off-line using MATLAB. The algorithm was tested and evaluated for estimating the targets in two approaches; active and passive. In the active approach, Txs were closely positioned to evaluate resolving overlapping targets. In the passive approach, reflectors were placed very close to each other to evaluate resolving reflected signals from the objects. To provide a benchmark for evaluating the performance of the proposed algorithm, Tacklife Advanced Laser Measure 131 Ft [13] is used to give the ground truth with ±1.5 mm accuracy.



(a) Active

(b) Passive

(c) Tracking setup

Fig. 3: Experimental setup for static multiple targets estimation

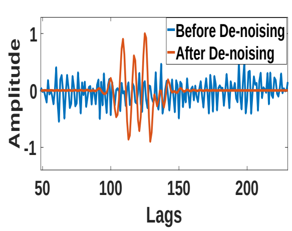
The second setup is for tracking two moving transmitters that are using the same m-sequence. The experimental setup is installed in an indoor environment with dimensions 300 cm $\times$ 300 cm $\times$ 400 cm. ARTTRACK infrared-based tracking system [14] is used to provide a benchmark for evaluating the performance of the algorithm with 0.1 mm resolution. Figure 3 shows the experimental setups.

4. RESULTS

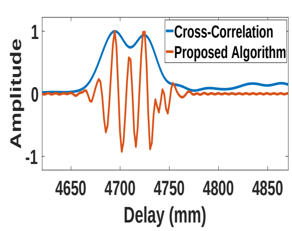
In this section, the proposed algorithm is evaluated under different scenarios by presenting the experimental results. For each setup, a signal of 100 repetitions of an m-sequence was transmitted. In the first scenario, a single receiver (Rx) was fixed at a certain location, and three transmitters were widely spaced (919 mm, 958 mm, and 1031 mm away from the Rx). Then, the three transmitters were placed close to each other (919 mm, 929 mm, and 940 mm away from the Rx). In the second scenario, we are interested in estimating the reflections from objects that are positioned close to each other. Three reflectors were positioned 876 mm, 897 mm, and 923 mm away from Rx in the third experiment. In the fourth experiment, the reflectors were positioned closer to each other (1108 mm, 1114 mm, and 1125 mm away from the Rx). Another reflector was added in the fifth experiment (885 mm, 900 mm, 922 mm, and 940 mm away from Rx). In the sixth experiment, three transmitters were positioned 4666 mm, 4679 mm and 4657 mm away from the receiver to evaluate the performance of the system at longer distances. Figure 4a, shows the deconvolution results before and after de-noising the cross-correlation. Table 1 shows the root mean square error (RMSE), the standard deviation (σ) and the number of the estimated targets M using the proposed method, GEM estimation, and cross-correlation. Figure 5 shows the percentage of estimates with an error less than x mm using the proposed algorithm and GEM estimation in four different scenarios. Figure 4b shows that the overlapping targets cannot be distinguished using cross-correlation while the proposed algorithm resolves these overlapping targets even at far distances.

Exps	Proposed Algorithm,			GEM estimation			CC
	RMSE (mm)	σ (mm)	M	**RMSE** (mm)	σ (mm)	M	M
Exp 1	2.11	2.10	3	2.86	2.26	3	3
Exp 2	5.89	5.81	3	10.78	5.91	3	2
Exp 3	3.52	3.48	3	7.32	4.54	3	2
Exp 4	5.11	4.13	3	8.67	3.15	3	1
Exp 5	3.39	2.43	4	3.46	3.34	4	2
Exp 6	3.99	3.08	3	8.52	5.61	3	2

Table 1: Estimated multiple targets RMSE, σ and M



(a) Deconvolution



(b) Three targets at far distance

Fig. 4

In the third scenario, four receivers were placed at certain locations, and two transmitters were moving. Both transmitters were transmitting the same m-sequence which cause targets to overlap. The results of tracking the two transmitters are summarized in table 2 for all the four receivers. Figure 6 shows tracking results using the proposed method, the GEM estimation and cross-correlation for one of the receivers. The proposed algorithm achieves high accuracy even when the two transmitters are overlapping with more than 90 % estimates with an error less than 10 mm. This percentage degrades to 78 % when using GEM estimation method only. Cross-correlation fails to estimate the two transmitters when they are overlapping.

Receivers	Proposed Algorithm			GEM estimation		
	RMSE (mm)	σ (mm)	% error < 5 mm	**RMSE** (mm)	σ (mm)	% error< 5 mm
Rx1-Tx1	2.30	1.48	99.40	1.07	0.97	100
Rx2-Tx1	3.11	3.10	88.80	3.60	3.01	84.43
Rx3-Tx1	2.86	2.38	96.80	2.26	2.20	95.41
Rx4-Tx1	2.35	2.35	97.40	3.28	2.97	90.82
Rx1-Tx2	3.98	2.53	76.65	3.02	2.51	93.41
Rx2-Tx2	8.30	6.35	27.54	9.03	6.15	27.74
Rx3-Tx2	7.21	5.70	50.9	6.21	5.75	42.32
Rx4-Tx2	6.02	4.56	66.67	11.13	10.78	57.68

Table 2: Tx1 and Tx2 tracking results

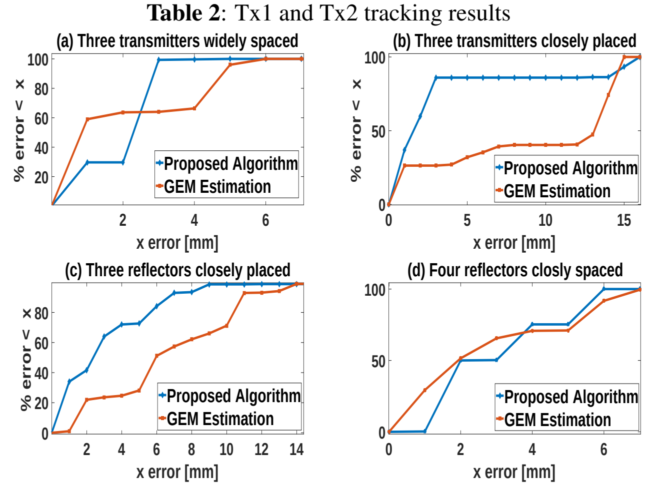


Fig. 5: Percentage of estimates with error less than x mm

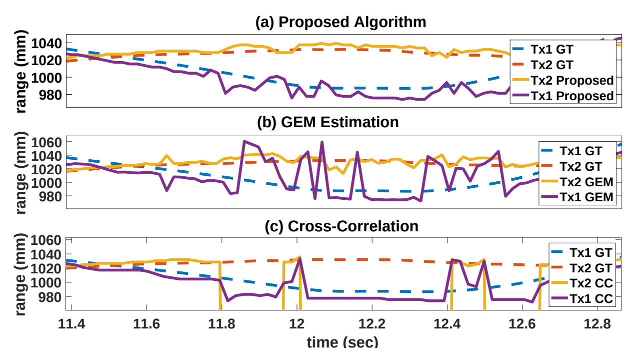


Fig. 6: Tracking of two transmitters with the ground truth (GT)

5. CONCLUSION

A new high accuracy multiple targets estimation algorithm using an m-sequence coded acoustic signal has been implemented. This algorithm estimates the impulse response of the system by utilizing the cross-correlation of the m-sequence and de-noise this impulse response by modeling it using Gaussian echo model. To provide high accuracy estimation, the deconvolution method is applied to the estimated model and TOAs for the targets are determined. The performance of the system was evaluated experimentally using a low-cost hardware in an indoor environment. The results show the high resolution and accuracy of the proposed algorithm in estimating multiple targets at proximity. In most of the evaluated scenarios, 90% of errors is less than 10 mm.

6. REFERENCES

[1] Jeffrey Hightower and Gaetano Borriello, "A survey and taxonomy of location systems for ubiquitous computing," *IEEE computer*, vol. 34, no. 8, pp. 57–66, 2001.

[2] Kamin Whitehouse, Chris Karlof, and David Culler, "A practical evaluation of radio signal strength for ranging-based localization," *ACM SIGMOBILE Mobile Computing and Communications Review*, vol. 11, no. 1, pp. 41–52, 2007.

[3] Çağri Yüzbaşioğlu and Billur Barshan, "Improved range estimation using simple infrared sensors without prior knowledge of surface characteristics," *Measurement Science and Technology*, vol. 16, no. 7, pp. 1395, 2005.

[4] Markus-Christian Amann, Thierry Bosch, Marc Lescure, Risto Myllyla, and Marc Rioux, "Laser ranging: a critical review of usual techniques for distance measurement," *Optical engineering*, vol. 40, no. 1, pp. 10–19, 2001.

[5] Matteo Cypriani, Frédéric Lassabe, Philippe Canalda, and François Spies, "Open wireless positioning system: A wi-fi-based indoor positioning system," in *Vehicular Technology Conference Fall (VTC 2009-Fall), 2009 IEEE 70th*. IEEE, 2009, pp. 1–5.

[6] Ram M Narayanan and Muhammad Dawood, "Doppler estimation using a coherent ultrawide-band random noise radar," *IEEE Transactions on Antennas and Propagation*, vol. 48, no. 6, pp. 868–878, 2000.

[7] Mohammed H AlSharif, Mohamed Saad, Mohamed Siala, Tarig Ballal, Hatem Boujemaa, and Tareq Y Al-Naffouri, "Zadoff-chu coded ultrasonic signal for accurate range estimation," .

[8] Ramazan Demirli and Jafar Saniie, "Model-based estimation of ultrasonic echoes. part i: Analysis and algorithms," *IEEE transactions on ultrasonics, ferroelectrics, and frequency control*, vol. 48, no. 3, pp. 787–802, 2001.

[9] Ramazan Demirli and Jafar Saniie, "Model-based estimation of ultrasonic echoes. part ii: Nondestructive evaluation applications," *IEEE transactions on ultrasonics, ferroelectrics, and frequency control*, vol. 48, no. 3, pp. 803–811, 2001.

[10] Dilip V Sarwate and Michael B Pursley, "Crosscorrelation properties of pseudorandom and related sequences," *Proceedings of the IEEE*, vol. 68, no. 5, pp. 593–619, 1980.

[11] Stephen J Wright and Jorge Nocedal, "Numerical optimization," *Springer Science*, vol. 35, no. 67-68, pp. 7, 1999.

[12] Donald W Marquardt, "An algorithm for least-squares estimation of nonlinear parameters," *Journal of the society for Industrial and Applied Mathematics*, vol. 11, no. 2, pp. 431–441, 1963.

[13] https://www.tacklifetools.com/product/product/index/id/26/, "Tacklife advanced laser measure 131 ft," 2017.

[14] http://www.ar-tracking.com/products/tracking-systems/systems overview/, "Arttrack system overview," 2017.