IMPROVED WEIGHTED INSTRUMENTAL VARIABLE ESTIMATOR FOR DOPPLER-BEARING SOURCE LOCALIZATION IN HEAVY NOISE

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ABSTRACT

In Doppler-bearing source localization, pseudolinear estimators are appealing alternatives to the divergence-prone and computationally-demanding iterative maximum likelihood estimator. Among the existing pseudolinear estimators, the weighted instrumental variable estimator (WIVE) is the most attractive option as it is asymptotically unbiased and efficient. However, the asymptotic unbiasedness of the WIVE relies on the approximation that the second-order noise term in the Doppler pseudolinear noise is zero, which is only valid for sufficiently small noise. In large noise, the second-order noise term can no longer be neglected, thereby leading to biased estimates. In this paper, we analyze the WIVE bias and propose a new improved version of the WIVE, called the I-WIVE, that overcomes the WIVE bias problems at large noise levels. The superior performance of the I-WIVE over the WIVE and other pseudolinear estimators is demonstrated by way of simulations. Specifically, we observe that the I-WIVE exhibits a negligible bias and produces a mean-squared error closest to the Cramér-Rao lower bound among the simulated estimators.

Index Terms— Source localization, bearing angle, Doppler shift, instrumental variables, bias compensation

1. INTRODUCTION

Source localization has been an active research area for several decades, playing a fundamental role in wide-ranging applications such as wireless sensor networks, radar and sonar, satellite geolocation, and search and rescue [1–29]. Source localization makes use of sensor data and measurements with direct functional dependence on the source location such as bearing angle, time of arrival, time difference of arrival, Doppler shift, and received signal strength. In this paper, we focus our attention on the problem of source localization in the 2D-plane using bearing angle and Doppler shift measurements collected by a single moving sensor platform.

Doppler-bearing source localization is a challenging estimation problem due to the highly nonlinear nature of the bearing angle and Doppler shift equations with respect to the unknown source location to be estimated. The maximum likelihood estimator (MLE) is a widely-used technique for solving nonlinear estimation problems [12]. The use of the MLE for Doppler-bearing source localization was presented in [13]. In spite of the fact that the MLE enjoys the desirable properties of asymptotic unbiasedness and efficiency, it does not admit a closed-form solution and must be implemented via iterative numerical search algorithms. The high computational complexity and vulnerability to divergence in the absence of good initialization makes the MLE less attractive in practice.

An appealing alternative to the MLE is the pseudolinear estimation approach which offers closed-form solutions and thus alleviates the complexity and divergence problems of the MLE. The main idea behind the pseudolinear estimation approach is to re-arrange the nonlinear measurement equations so as to make them linear in the unknowns, followed by linear least-squares estimation. This approach has been widely used in the literature for various localization and tracking problems (see e.g., [5–8, 14–28]).

The application of pseudolinear estimation to Doppler-bearing source localization was proposed in [14]. In particular, the work in [14] developed three pseudolinear estimators including the pseudolinear least-squares estimator (PLE), the bias-compensated PLE (BCPLE) and the weighted instrumental variable estimator (WIVE). The PLE, a simple least-squares solution of the linearized measurement equation system, was realized in [14] to suffer from a severe bias problem due to the correlation between the measurement matrix and the pseudolinear noise vector. To overcome the bias problem of the PLE, the BCPLE aims to estimate and remove the instantaneous bias of the PLE estimate, while the WIVE exploits the use of instrumental variables to eliminate the correlation between the measurement matrix and the pseudolinear noise vector. Among these three pseudolinear estimators, the WIVE was empirically shown to provide the best estimation performance [14]. In addition, the WIVE was also analytically proved in [14] to be asymptotically unbiased and efficient for sufficiently small measurement noise.

The asymptotic unbiasedness of the WIVE relies on the second-order noise term in the Doppler pseudolinear noise being negligible (i.e., the Doppler pseudolinear noise is approximately zero-mean), which is only valid for sufficiently small noise. In the presence of large noise, the Doppler pseudolinear noise becomes non-zero mean, causing a significant bias in the WIVE estimate. The main contribution of this paper is to analyze the bias of the WIVE and propose a new improved version of the WIVE, namely the I-WIVE, to resolve the bias problem of the WIVE in large noise. Specifically, the mean of the Doppler pseudolinear noise is estimated and used to calculate the WIVE bias which is subsequently subtracted from the WIVE estimate. The effectiveness of the proposed I-WIVE in removing the bias of the WIVE in large noise conditions is demonstrated via numerical Monte Carlo simulations. The I-WIVE is observed to significantly outperform the PLE, BCPLE and WIVE not only in terms of estimation bias but also in terms of mean-squared error.

2. PROBLEM FORMULATION

Fig. 1 depicts the problem of 2D source localization using bearing angle and Doppler frequency shift measurements collected by a moving sensor platform, where $\boldsymbol{p} = [p_x, p_y]^T$ is the unknown source position to be estimated, and $\boldsymbol{r}_k = [r_{x,k}, r_{y,k}]^T$ and $\boldsymbol{v}_k = [v_{x,k}, v_{y,k}]^T$ are the sensor position and velocity at time instant $k \in 1, \ldots, N$. Here the symbol ^{T} denotes the matrix transpose operator. The bearing angle and Doppler shift measurements obtained by the

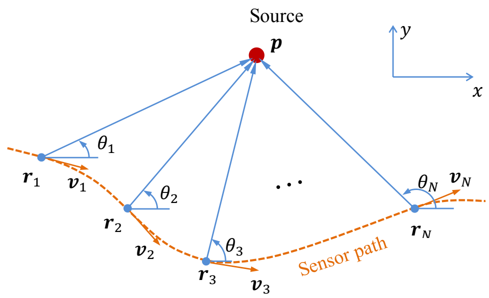

Fig. 1. Doppler-bearing source localization geometry.

sensor at time instant k are given by

$$\tilde{\theta}_k = \theta_k + n_{\theta,k}, \quad \theta_k = \tan^{-1}\frac{p_y - r_{y,k}}{p_x - r_{x,k}} \tag{1}$$

$$\tilde{\zeta}_k = \zeta_k + n_{\zeta,k}, \quad \zeta_k = \frac{(\boldsymbol{r}_k - \boldsymbol{p})^T\boldsymbol{v}_k}{\|\boldsymbol{r}_k - \boldsymbol{p}\|} \tag{2}$$

where $\tan^{-1}$ is the 4-quadrant arctangent, $\|\cdot\|$ denotes the Euclidean norm, and $n_{\theta,k}$ and $n_{\zeta,k}$ are measurement noise terms which are assumed to be zero-mean independent Gaussian random variables with variance $E\{n_{\theta,k}^2\} = \sigma_{\theta,k}^2$ and $E\{n_{\zeta,k}^2\} = \sigma_{\zeta,k}^2$. Note that the Doppler-shift ζ_k are already normalized by the factor $f_\circ/c$, where $f_\circ$ is the emitted signal frequency, which is assumed to be known *a priori*, and c is the speed of signal propagation. In practice, the emitted signal frequency $f_\circ$ is available in active sensors, e.g., sonar or airborne radar systems, or can be estimated by passive sensors, such as unmanned underwater vehicles or rotary-wing unmanned aerial vehicles, during the loiter mode (in which the sensor position remains unchanged). In this paper, the sensor position and velocity, $\boldsymbol{p}_k$ and $\boldsymbol{v}_k$, are assumed to be known with negligible error.

3. OVERVIEW OF PSEUDOLINEAR ESTIMATORS

After some algebraic manipulations, the bearing angle and Doppler shift measurement equations in (1) and (2) can be rearranged into a pseudolinear form as [14]

$$\boldsymbol{A}_{\theta,k}\boldsymbol{p} = b_{\theta,k} + \eta_{\theta,k} \tag{3}$$

$$\boldsymbol{A}_{\zeta,k}\boldsymbol{p} = b_{\zeta,k} + \eta_{\zeta,k} \tag{4}$$

where

$$\boldsymbol{A}_{\theta,k} = [\sin\tilde{\theta}_k, -\cos\tilde{\theta}_k] \tag{5a}$$

$$b_{\theta,k} = [\sin\tilde{\theta}_k, -\cos\tilde{\theta}_k]\boldsymbol{r}_k \tag{5b}$$

$$\eta_{\theta,k} = \|\boldsymbol{p} - \boldsymbol{r}_k\| \sin n_{\theta,k} \tag{5c}$$

$$\boldsymbol{A}_{\zeta,k} = \tilde{\zeta}_k\boldsymbol{u}_1^T + (\sin\tilde{\theta}_k + \cos\tilde{\theta}_k)\boldsymbol{v}_k^T \tag{5d}$$

$$b_{\zeta,k} = \tilde{\zeta}_k\boldsymbol{u}_1^T\boldsymbol{r}_k + (\sin\tilde{\theta}_k + \cos\tilde{\theta}_k)\boldsymbol{v}_k^T\boldsymbol{r}_k \tag{5e}$$

$$\eta_{\zeta,k} = \boldsymbol{v}_k^T(\boldsymbol{p} - \boldsymbol{r}_k)\Big((\cos\theta_k - \sin\theta_k)\sin n_{\theta,k} - 2(\sin\theta_k + \cos\theta_k)\sin^2(n_{\theta,k}/2)\Big) + \boldsymbol{u}_1^T(\boldsymbol{p} - \boldsymbol{r}_k)n_{\zeta,k} \tag{5f}$$

with $\boldsymbol{u}_1 = [1, 1]^T$.

By stacking (3) and (4) for $k = 1, \ldots, N$, we obtain

$$\boldsymbol{A}\boldsymbol{p} = \boldsymbol{b} + \boldsymbol{\eta} \tag{6}$$

where

$$\boldsymbol{A} = [\boldsymbol{A}_1^T, \boldsymbol{A}_2^T, \ldots, \boldsymbol{A}_N^T]^T, \quad \boldsymbol{A}_k = [\boldsymbol{A}_{\theta,k}^T, \boldsymbol{A}_{\zeta,k}^T]^T, \tag{7a}$$

$$\boldsymbol{b} = [\boldsymbol{b}_1^T, \boldsymbol{b}_2^T, \ldots, \boldsymbol{b}_N^T]^T, \quad \boldsymbol{b}_k = [b_{\theta,k}, b_{\zeta,k}]^T, \tag{7b}$$

$$\boldsymbol{\eta} = [\boldsymbol{\eta}_1^T, \boldsymbol{\eta}_2^T, \ldots, \boldsymbol{\eta}_N^T]^T, \quad \boldsymbol{\eta}_k = [\eta_{\theta,k}, \eta_{\zeta,k}]^T. \tag{7c}$$

The PLE estimate of $\boldsymbol{p}$ is obtained by solving (6) in the least-squares sense [14]

$$\hat{\boldsymbol{p}}_{\text{PLE}} = \arg\min_{\boldsymbol{p}\in\mathbb{R}^2} \|\boldsymbol{A}\boldsymbol{p} - \boldsymbol{b}\|^2 = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{b}. \tag{8}$$

The estimation bias of $\hat{\boldsymbol{p}}_{\text{PLE}}$ is given by

$$\boldsymbol{\delta}_{\text{PLE}} = E\{\hat{\boldsymbol{p}}_{\text{PLE}}\} - \boldsymbol{p} = -E\{(\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{\eta}\}. \tag{9}$$

It has been proved in [14] that $E\{\boldsymbol{A}^T\boldsymbol{\eta}/N\} \neq 0$ even for $N \to \infty$ because of the correlation between $\boldsymbol{A}$ and $\boldsymbol{\eta}$, and thus

$$\boldsymbol{\delta}_{\text{PLE}} \to -E\left\{\frac{\boldsymbol{A}^T\boldsymbol{A}}{N}\right\}^{-1} E\left\{\frac{\boldsymbol{A}^T\boldsymbol{\eta}}{N}\right\} \neq \boldsymbol{0} \text{ as } N \to \infty \tag{10}$$

which implies that the PLE has an asymptotically nonvanishing bias.

To moderate the bias problem of the PLE, the BCPLE was developed in [14] based on an estimation of the instantaneous bias $\boldsymbol{\delta}_{ins} = -(\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{\eta}$, i.e.,

$$\hat{\boldsymbol{\delta}}_{ins} = -(\boldsymbol{A}^T\boldsymbol{A})^{-1}\hat{E}\{\boldsymbol{A}^T\boldsymbol{\eta}\} \tag{11}$$

with $\hat{E}\left\{\boldsymbol{A}^T\boldsymbol{\eta}\right\} = \sum_{k=1}^N \hat{E}\left\{\boldsymbol{A}_{\theta,k}^T\eta_{\theta,k}\right\} + \sum_{k=1}^N \hat{E}\left\{\boldsymbol{A}_{\zeta,k}^T\eta_{\zeta,k}\right\}$. The expressions for $\hat{E}\left\{\boldsymbol{A}_{\theta,k}^T\eta_{\theta,k}\right\}$ and $\sum_{k=1}^N \hat{E}\left\{\boldsymbol{A}_{\zeta,k}^T\eta_{\zeta,k}\right\}$ are given in (12) (at the top of next page), where $\hat{\theta}_k$ and $\hat{\zeta}_k$ are computed from $\hat{\boldsymbol{p}}$ with $\hat{\boldsymbol{p}} = \hat{\boldsymbol{p}}_{\text{PLE}}$. The BCPLE is obtained by subtracting (11) from (8) as

$$\hat{\boldsymbol{p}}_{\text{BCPLE}} = \hat{\boldsymbol{p}}_{\text{PLE}} - \hat{\boldsymbol{\delta}}_{ins} = \hat{\boldsymbol{p}}_{\text{PLE}} + (\boldsymbol{A}^T\boldsymbol{A})^{-1}\hat{E}\{\boldsymbol{A}^T\boldsymbol{\eta}\}. \tag{13}$$

Since the BCPLE still suffers from bias (although less so than the PLE for small noise) due to the error existing in the bias estimate $\hat{\boldsymbol{\delta}}_{ins}$, a more attractive solution for the bias problem of the PLE is to exploit the use of instrumental variables to eliminate the correlation between $\boldsymbol{A}$ and $\boldsymbol{\eta}$, thus resulting in the WIVE [14]. Specifically, the PLE normal equations $\boldsymbol{A}^T\boldsymbol{A}\hat{\boldsymbol{p}}_{\text{PLE}} = \boldsymbol{A}^T\boldsymbol{b}$ are modified to $\boldsymbol{G}^T\boldsymbol{A}\hat{\boldsymbol{p}}_{\text{IVE}} = \boldsymbol{G}^T\boldsymbol{b}$, where $\boldsymbol{G}$ is the instrumental variable (IV) matrix and is constructed so that $E\{\boldsymbol{G}^T\boldsymbol{A}/N\}$ is nonsingular and $E\{\boldsymbol{G}^T\boldsymbol{\eta}/N\} = \boldsymbol{0}$ as $N \to \infty$. Thus, the resulting IV estimator is given by

$$\hat{\boldsymbol{p}}_{\text{IVE}} = (\boldsymbol{G}^T\boldsymbol{A})^{-1}\boldsymbol{G}^T\boldsymbol{b}. \tag{14}$$

Introducing a weighting matrix $\boldsymbol{W} = E\{\boldsymbol{\eta}\boldsymbol{\eta}^T\}$ leads to the WIVE:

$$\hat{\boldsymbol{p}}_{\text{WIVE}} = (\boldsymbol{G}^T\boldsymbol{W}^{-1}\boldsymbol{A})^{-1}\boldsymbol{G}^T\boldsymbol{W}^{-1}\boldsymbol{b}. \tag{15}$$

The optimal choice for $\boldsymbol{G}$ is the noise-free version of $\boldsymbol{A}$, denoted as $\boldsymbol{A}_\circ$. Unfortunately, $\boldsymbol{A}_\circ$ is a function of the unknown bearing angle θ_k and Doppler shift ζ_k, and thus is not available. A suboptimal IV matrix can be constructed based on an estimate of $\boldsymbol{A}_\circ$ obtained using $\hat{\theta}_k$ and $\hat{\zeta}_k$ calculated from the BCPLE estimate $\hat{\boldsymbol{p}}_{\text{BCPLE}}$:

$$\boldsymbol{G} = [\boldsymbol{G}_1^T, \boldsymbol{G}_2^T, \ldots, \boldsymbol{G}_N^T]^T, \quad \boldsymbol{G}_k = [\boldsymbol{G}_{\theta,k}^T, \boldsymbol{G}_{\zeta,k}^T]^T, \tag{16}$$

where

$$\boldsymbol{G}_{\theta,k} = [\sin\hat{\theta}_k, -\cos\hat{\theta}_k] \tag{17a}$$

$$\boldsymbol{G}_{\zeta,k} = \hat{\zeta}_k\boldsymbol{u}_1^T + (\sin\hat{\theta}_k + \cos\hat{\theta}_k)\boldsymbol{v}_k^T. \tag{17b}$$

$$\sum_{k=1}^{N} \hat{E}\left\{\boldsymbol{A}_{\theta,k}^T \eta_{\theta,k}\right\} = \sum_{k=1}^{N} E\{\sin^2 n_{\theta,k}\}(\hat{\boldsymbol{p}} - \boldsymbol{r}_k) \tag{12a}$$

$$\sum_{k=1}^{N} \hat{E}\left\{\boldsymbol{A}_{\zeta,k}^T \eta_{\zeta,k}\right\} = \sum_{k=1}^{N} E\{n_{\zeta,k}^2\}\boldsymbol{u}_1^T(\hat{\boldsymbol{p}} - \boldsymbol{r}_k)\boldsymbol{u}_1 - 2\sum_{k=1}^{N} \hat{\zeta}_k(\sin\hat{\theta}_k + \cos\hat{\theta}_k)E\left\{\sin^2\left(\frac{n_{\theta,k}}{2}\right)\right\}\boldsymbol{v}_k^T(\hat{\boldsymbol{p}} - \boldsymbol{r}_k)\boldsymbol{u}_1$$
$$+ \sum_{k=1}^{N}\left\{\left(-2(\sin\hat{\theta}_k + \cos\hat{\theta}_k)^2\left(E\left\{\sin^2\left(\frac{n_{\theta,k}}{2}\right)\right\} - 2E\left\{\sin^4\left(\frac{n_{\theta,k}}{2}\right)\right\}\right) + (\cos\hat{\theta}_k - \sin\hat{\theta}_k)^2 E\{\sin^2 n_{\theta,k}\}\right)\boldsymbol{v}_k^T(\hat{\boldsymbol{p}} - \boldsymbol{r}_k)\boldsymbol{v}_k\right\}. \tag{12b}$$

The expression for the weighting matrix $\boldsymbol{W}$ is

$$\boldsymbol{W} = E\{\boldsymbol{\eta}\boldsymbol{\eta}^T\} = \operatorname{diag}(E\{\boldsymbol{\eta}_1\boldsymbol{\eta}_1^T\}, E\{\boldsymbol{\eta}_2\boldsymbol{\eta}_2^T\}, \ldots, E\{\boldsymbol{\eta}_N\boldsymbol{\eta}_N^T\}) \tag{18}$$

where

$$E\{\boldsymbol{\eta}_k\boldsymbol{\eta}_k^T\} = \begin{bmatrix} E\{\eta_{\theta,k}^2\} & E\{\eta_{\theta,k}\eta_{\zeta,k}\} \\ E\{\eta_{\zeta,k}\eta_{\theta,k}\} & E\{\eta_{\zeta,k}^2\} \end{bmatrix} \tag{19}$$

and

$$E\{\eta_{\theta,k}^2\} = \|\boldsymbol{p} - \boldsymbol{r}_k\|^2 E\{\sin^2 n_{\theta,k}\} \tag{20a}$$

$$E\{\eta_{\theta,k}\eta_{\zeta,k}\} = E\{\eta_{\zeta,k}\eta_{\theta,k}\}$$
$$= \|\boldsymbol{p} - \boldsymbol{r}_k\|\boldsymbol{v}_k^T(\boldsymbol{p} - \boldsymbol{r}_k)(\cos\theta_k - \sin\theta_k)E\{\sin^2 n_{\theta,k}\} \tag{20b}$$

$$E\{\eta_{\zeta,k}^2\} = \left(\boldsymbol{u}_1^T(\boldsymbol{p} - \boldsymbol{r}_k)\right)^2 E\{n_{\zeta,k}^2\}$$
$$+ \left(\boldsymbol{v}_k^T(\boldsymbol{p} - \boldsymbol{r}_k)\right)^2\Big(4(\sin\theta_k + \cos\theta_k)^2 E\left\{\sin^4(n_{\theta,k}/2)\right\}$$
$$+ (\cos\theta_k - \sin\theta_k)^2 E\{\sin^2 n_{\theta,k}\}\Big). \tag{20c}$$

Note that, since the knowledge of $\boldsymbol{p}$, θ_k and ζ_k is not available, $\hat{\boldsymbol{p}}_{\mathrm{BCPLE}}$ as well as $\hat{\theta}_k$ and $\hat{\zeta}_k$ calculated from $\hat{\boldsymbol{p}}_{\mathrm{BCPLE}}$ are used instead to approximate $\boldsymbol{W}$.

4. BIAS ANALYSIS

The bias of the WIVE is given by

$$\boldsymbol{\delta}_{\mathrm{WIVE}} = E\{\hat{\boldsymbol{p}}_{\mathrm{WIVE}} - \boldsymbol{p}\} = -E\{(\boldsymbol{G}^T\boldsymbol{W}^{-1}\boldsymbol{A})^{-1}\boldsymbol{G}^T\boldsymbol{W}^{-1}\boldsymbol{\eta}\}. \tag{21}$$

By following the derivation in [14] based on the probability theory, it is straightforward to show that

$$\boldsymbol{\delta}_{\mathrm{WIVE}} \to -E\left\{\frac{\boldsymbol{G}^T\boldsymbol{W}^{-1}\boldsymbol{A}}{N}\right\}^{-1} E\left\{\frac{\boldsymbol{G}^T\boldsymbol{W}^{-1}\boldsymbol{\eta}}{N}\right\} \text{ as } N \to \infty. \tag{22}$$

Since the covariance of $\hat{\boldsymbol{p}}_{\mathrm{BCPLE}}$ tends to zero as $N \to \infty$, the correlation between $\hat{\boldsymbol{p}}_{\mathrm{BCPLE}}$ and $\boldsymbol{\eta}$ vanishes as $N \to \infty$. Consequently, given that $\boldsymbol{G}$ is constructed based on $\hat{\boldsymbol{p}}_{\mathrm{BCPLE}}$, we have

$$E\left\{\frac{\boldsymbol{G}^T\boldsymbol{W}^{-1}\boldsymbol{\eta}}{N}\right\} \to \frac{E\{\boldsymbol{G}\}^T\boldsymbol{W}^{-1}E\{\boldsymbol{\eta}\}}{N} \text{ as } N \to \infty, \tag{23}$$

and thus

$$\boldsymbol{\delta}_{\mathrm{WIVE}} \to -E\left\{\frac{\boldsymbol{G}^T\boldsymbol{W}^{-1}\boldsymbol{A}}{N}\right\}^{-1} \frac{E\{\boldsymbol{G}\}^T\boldsymbol{W}^{-1}E\{\boldsymbol{\eta}\}}{N} \quad \text{as } N \to \infty. \tag{24}$$

For sufficiently small measurement noise, the second-order noise term $\sin^2(n_{\theta,k}/2)$ in $\eta_{\zeta,k}$ (see (5f)) is approximately zero and thus $E\{\boldsymbol{\eta}\} \approx \boldsymbol{0}$. As a result, we have $\boldsymbol{\delta}_{\mathrm{WIVE}} \to \boldsymbol{0}$ as $N \to \infty$, which implies the asymptotic unbiasedness of the WIVE under the small noise assumption in agreement with [14].

In contrast, in large noise conditions, the second-order noise term $\sin^2(n_{\theta,k}/2)$ becomes significant and the zero-mean approximation of the pseudolinear noise $\boldsymbol{\eta}$ is no longer valid. Instead, we have $E\{\boldsymbol{\eta}\} \neq \boldsymbol{0}$. Specifically, the expression of $E\{\boldsymbol{\eta}\}$ is given by

$$E\{\boldsymbol{\eta}\} = [E\{\boldsymbol{\eta}_1\}^T, E\{\boldsymbol{\eta}_2\}^T, \ldots, E\{\boldsymbol{\eta}_N\}^T]^T \tag{25}$$

where

$$E\{\boldsymbol{\eta}_k\} = [E\{\eta_{\theta,k}\}, E\{\eta_{\zeta,k}\}]^T \tag{26a}$$

$$E\{\eta_{\theta,k}\} = 0 \tag{26b}$$

$$E\{\eta_{\zeta,k}\} = -2\boldsymbol{v}_k^T(\boldsymbol{p} - \boldsymbol{r}_k)(\sin\theta_k + \cos\theta_k)E\{\sin^2(n_{\theta,k}/2)\}. \tag{26c}$$

Since $E\{\boldsymbol{\eta}\} \neq \boldsymbol{0}$, we have

$$\boldsymbol{\delta}_{\mathrm{WIVE}} \to -E\left\{\frac{\boldsymbol{G}^T\boldsymbol{W}^{-1}\boldsymbol{A}}{N}\right\}^{-1} \frac{E\{\boldsymbol{G}\}^T\boldsymbol{W}^{-1}E\{\boldsymbol{\eta}\}}{N} \neq \boldsymbol{0} \quad \text{as } N \to \infty. \tag{27}$$

which implies an asymptotically nonvanishing bias for the WIVE under heavy noise.

5. PROPOSED ESTIMATOR

In this section, we propose a new variant of the WIVE, the I-WIVE, to overcome the bias problems of the WIVE in the case of large measurement noise.

Using (27), for sufficiently large N, the bias of the WIVE can be approximated by

$$\boldsymbol{\delta}_{\mathrm{WIVE}} \approx -E\left\{\frac{\boldsymbol{G}^T\boldsymbol{W}^{-1}\boldsymbol{A}}{N}\right\}^{-1} \frac{E\{\boldsymbol{G}\}^T\boldsymbol{W}^{-1}E\{\boldsymbol{\eta}\}}{N} \tag{28a}$$

$$\approx -(\boldsymbol{G}^T\boldsymbol{W}^{-1}\boldsymbol{A})^{-1}\boldsymbol{G}^T\boldsymbol{W}^{-1}E\{\boldsymbol{\eta}\}. \tag{28b}$$

Since $E\{\boldsymbol{\eta}\}$ is a function of the unknowns $\boldsymbol{p}$ and θ_k, an estimate of the approximate bias $\boldsymbol{\delta}_{\mathrm{WIVE}}$ in (28) can be obtained by replacing $E\{\boldsymbol{\eta}\}$ with $\hat{E}\{\boldsymbol{\eta}\}$

$$\hat{\boldsymbol{\delta}}_{\mathrm{WIVE}} = -(\boldsymbol{G}^T\boldsymbol{W}^{-1}\boldsymbol{A})^{-1}\boldsymbol{G}^T\boldsymbol{W}^{-1}\hat{E}\{\boldsymbol{\eta}\} \tag{29}$$

where $\hat{E}\{\boldsymbol{\eta}\}$ has the same expression as $E\{\boldsymbol{\eta}\}$ in (25) and (26), except that $\boldsymbol{p}$ is replaced by $\hat{\boldsymbol{p}}_{\mathrm{BCPLE}}$ and θ_k is replaced by $\hat{\theta}_k$ calculated from $\hat{\boldsymbol{p}}_{\mathrm{BCPLE}}$.

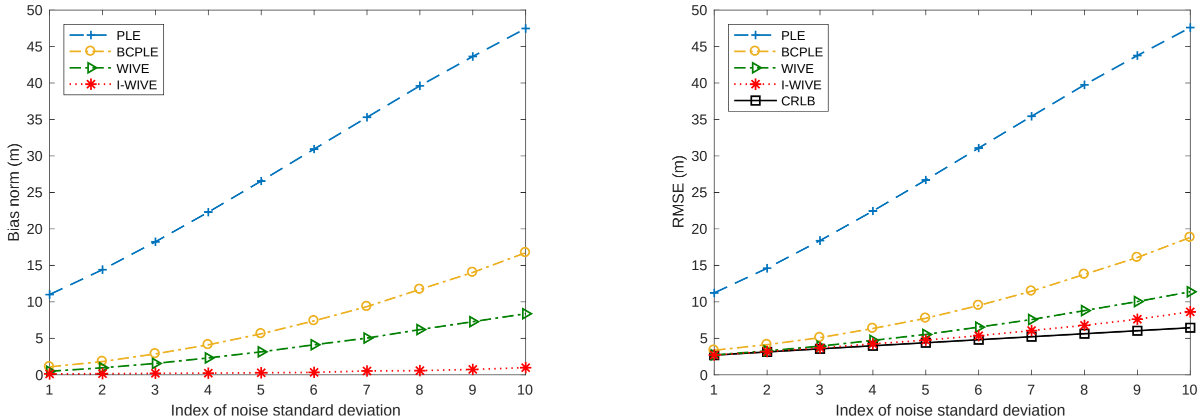

Fig. 2. Bias norm and RMSE versus noise standard derivation in Table 1 for the proposed I-WIVE in comparison with the existing PLE, BCPLE and WIVE.

The I-WIVE is now obtained by subtracting the bias estimate $\hat{\boldsymbol{\delta}}_{\text{WIVE}}$ from the WIVE estimate $\hat{\boldsymbol{p}}_{\text{WIVE}}$:

$$\hat{\boldsymbol{p}}_{\text{I-WIVE}} = \hat{\boldsymbol{p}}_{\text{WIVE}} - \hat{\boldsymbol{\delta}}_{\text{WIVE}} \tag{30a}$$

$$= (\boldsymbol{G}^T \boldsymbol{W}^{-1} \boldsymbol{A})^{-1} \boldsymbol{G}^T \boldsymbol{W}^{-1} (\boldsymbol{b} + \hat{E}\{\boldsymbol{\eta}\}). \tag{30b}$$

Note that $\boldsymbol{G}$, $\boldsymbol{W}$ and $\hat{E}\{\boldsymbol{\eta}\}$ in (30) are computed using the BCPLE estimate $\hat{\boldsymbol{p}}_{\text{BCPLE}}$.

6. SIMULATION STUDIES

We consider a simulated Doppler-bearing source localization geometry with a source located at $\boldsymbol{p} = [70, 60]^T$ m and a sensor traveling in a straight line $r_{y,k} = -0.2\, r_{x,k} + 10$ with a constant velocity of $\boldsymbol{v}_k = [-10, 2]^T$ m/s. The sensor collects $N = 100$ bearing angle and Doppler shift measurements at equally spaced points along the segment $5 \le r_{x,k} \le 35$. We consider a large noise scenario with measurement noise standard deviations listed in Table 1.

The bias norm and root mean-squared error (RMSE) measures are used for performance comparison. The bias norm is defined by $\|E\{\hat{\boldsymbol{p}}\} - \boldsymbol{p}\|$ while the RMSE is defined by $(\text{tr}\, E\{(\hat{\boldsymbol{p}} - \boldsymbol{p})(\hat{\boldsymbol{p}} - \boldsymbol{p})^T\})^{1/2}$, where $\hat{\boldsymbol{p}}$ is an estimate of $\boldsymbol{p}$. The bias and RMSE performance is estimated using 10,000 Monte Carlo simulation runs. In addition, the square root of the trace of the CRLB (referred to as RCRLB for simplicity) is also computed as the theoretical benchmark for the RMSE performance. The expression for the CRLB is

$$\text{CRLB} = (\boldsymbol{J}^T \boldsymbol{K}^{-1} \boldsymbol{J})^{-1} \tag{31}$$

where $\boldsymbol{K} = \text{diag}(\boldsymbol{K}_1, \boldsymbol{K}_2, \ldots, \boldsymbol{K}_N)$ is the noise covariance with $\boldsymbol{K}_k = \text{diag}(\sigma^2_{\theta,k}, \sigma^2_{\zeta,k})$, and $\boldsymbol{J} = [\boldsymbol{J}_1^T, \boldsymbol{J}_2^T, \ldots, \boldsymbol{J}_N^T]^T$ is the Jacobian matrix evaluated at the true source position $\boldsymbol{p}$ with $\boldsymbol{J}_k = [\boldsymbol{J}^T_{\theta,k}, \boldsymbol{J}^T_{\zeta,k}]^T$. Here,

$$\boldsymbol{J}_{\zeta,k} = \frac{[-v_{x,k} \sin^2\theta_k + \frac{1}{2} v_{y,k} \sin 2\theta_k, -v_{y,k} \cos^2\theta_k + \frac{1}{2} v_{x,k} \sin 2\theta_k]}{\|\boldsymbol{p} - \boldsymbol{r}_k\|} \tag{32}$$

Table 1. Measurement noise standard deviation

Index	1	2	3	4	5	6	7	8	9	10
σ_θ (deg.)	6	8	10	12	14	16	18	20	22	24
σ_ζ (m/s)	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8

and $\boldsymbol{J}_{\theta,k} = \frac{[-\sin\theta_k, \cos\theta_k]}{\|\boldsymbol{p} - \boldsymbol{r}_k\|}$. In this simulation, we use the following approximations: $E\{\sin^2 n_{\theta,k}\} = \sigma^2_{\theta,k} - \sigma^4_{\theta,k}$, $E\{\sin^2(n_{\theta,k}/2)\} = \sigma^2_{\theta,k}/4 - \sigma^4_{\theta,k}/16$, and $E\{\sin^4(n_{\theta,k}/2)\} = 3\sigma^4_{\theta,k}/16$.

Fig. 2 shows the performance of the proposed I-WIVE and the existing algorithms (the PLE, BCPLE and WIVE) against the measurement noise in Table 1. The PLE exhibits a severe bias as expected. The bias problem of the PLE is greatly moderated by the BCPLE and is even further reduced by the WIVE. However, in such a large noise scenario, the bias of the WIVE is still noticeably large. This observation agrees with the analytical findings presented in Section 4 that the WIVE is asymptotically biased under large noise due to the second-order noise term in the Doppler pseudolinear noise not being negligible. In contrast, by estimating and subtracting the WIVE bias from the WIVE estimate, the proposed I-WIVE produces a very small bias compared with that of the WIVE.

It is also observed in Fig. 2 that, although the WIVE exhibits the best RMSE performance among the existing pseudolinear estimators, its RMSE significantly deviates from the RCRLB as noise increases because of its bias problem. On the other hand, the RMSE of the proposed I-WIVE is appreciably smaller and much closer to the RCRLB than that of the WIVE thanks to the compensation of the WIVE bias in the I-WIVE. This observation demonstrates the superior performance of the I-WIVE over the existing pseudolinear estimators not only in terms of the estimation bias but also the RMSE.

7. CONCLUSION

The WIVE presented in [14] for single-platform Doppler-bearing source localization suffers from significant bias problems in the presence of heavy measurement noise due to the second-order noise term in the Doppler pseudolinear noise which can no longer be neglected. In this paper, we have analysed the asymptotic bias of the WIVE and proposed a new refinement of the WIVE, namely the I-WIVE, to compensate the bias of the WIVE in large noise conditions. Specifically, the I-WIVE incorporates the BCPLE to estimate the WIVE bias and removes it from the WIVE estimate. The performance advantages of the I-WIVE over the existing pseudolinear estimators under heavy noise were demonstrated by way of Monte Carlo simulations. The I-WIVE was observed to significantly outperform the PLE, BCPLE and WIVE, exhibiting a negligible bias and producing an RMSE closest to the RCRLB.

8. REFERENCES

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