SEMIDEFINITE PROGRAMMING FOR TDOA LOCALIZATION WITH LOCALLY SYNCHRONIZED ANCHOR NODES

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ABSTRACT

The most state-of-art time-difference-of-arrival (TDOA) localization algorithms are performed under the assumption that all the nodes are synchronized. However, for a widely distributed wireless sensor networks (WSNs), time synchronization between all the nodes is not a trival problem. In this paper, we study the problem of source localization using signal TDOA measurements in the system of nodes part synchronization. Starting from the maximum likelihood estimator (MLE), we develop a semidefinite programming (SDP) approach. Besides, we extend the SDP algorithm to the case of non-accurate sensor position. Simulation results validate the localization performance of the proposed SDP algorithms.

***Index Terms*—** Source localization, time-difference-of-arrival (TDOA), synchronization, semidefinite programming (SDP)

1. INTRODUCTION

Localization of an emitting source using multiple sensor nodes has many important applications, including wireless sensor networks (WSNs) [1], [2], wireless communication [3] and intelligent transport [4]. Some typical measurement techniques are used in the source localization, including direction-of-arrival (DOA), time-of-arrival (TOA), time-difference-of-arrival (TDOA) and received signal strength (RSS).

There has a lot of research concentrated on the estimation of source location based on TDOA measurements [5], [6], [7], [8]. It is known that the maximum likelihood estimator (MLE) is asymptotically efficient, but MLE is not easy to achieve in practice. Because it has to be realized by numerical iterative computation that requires sufficiently precise initial estimate for the global solution. Consequently, many researchers have been working on closed-form algebraic solution that can avoid the initialization problem which occurs in MLE. Nevertheless, the closed-form solution needs to square the nonlinear measurement equations, which result in the closed-form solution is only valid at sufficiently small noise conditions [5].

Due to the MLE has the optimal estimation performance, some semidefinite programming (SDP) based localization algorithms are proposed to directly relax the nonconvex MLE problem into convex problem [9], [10]. In [9], the authors use additional constraints, i.e., admissible physical region of the source, to improve the SDP algorithm performance. It shows that the SDP algorithm can attain Cramér-Rao lower bound (CRLB) at middle level noise, but it can not reach the CRLB at small level noise. Besides, the admissible source position information may not be available in some practical applications. In [10], Yang *et al.* develop an SDP algorithm with penalty term in the objective function, but the SDP algorithm without further local optimization can not reach the CRLB.

The above TDOA source localizaion algorithms are developed under the assumption that all the nodes (including reference node and the measurement nodes) are entirely synchronized. However, synchronization between reference node and the measurement nodes is not a trivial problem [11], [12] for a widely distributed WSN.

In this paper, we consider the problem of nodes partly synchronous TDOA source localization. We formulate an SDP algorithm for the problem, and then we consider the more complicated case: TDOA source localization in nodes partly synchronous system with the prensence of sensor position errors.

The rest of this paper is organized as follows. In Section II, the nodes partly synchronous TDOA measurements model is described, and the MLE problem is formulated. In Section III, we use SDP technique to relax the nonconvex MLE problem into convex problem. In Section IV, we propose a robust SDP localization algorithm for non-accurate sensor position. Simulation results are given in Section V to demonstrate the location estimation performance of the proposed estimators.

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In Section VI, we give conclusions.

2. PROBLEM STATEMENT

The following notations are used throughout this paper. Bold uppercase and bold lowercase letters denote matrices and vectors, respectively. $\mathbf{I}_m$ is the $m \times m$ identity matrix, $\mathbf{1}_m$ is the vector of m ones. $\|\cdot\|$ is the l_2 norm, and $tr(\cdot)$ is the trace operator. x_i is the ith element in vector $\mathbf{x}$, and $A_{i,j}$ is the ith row and jth column element in matrix $\mathbf{A}$. $\mathbf{A} \succeq \mathbf{B}$ means that $\mathbf{A} - \mathbf{B}$ is positive semidefinite.

We consider a nodes partly synchronous TDOA source localization system, which has N clusters, and each cluster has M_n sensors, where $n = 1, 2, \ldots, N$. The total number of sensors is $\sum_{n=1}^{N} M_n = f$. Let $\mathbf{s}_i^n \in \mathbb{R}^m$ and $\mathbf{u} \in \mathbb{R}^m$ be the known position of ith sensor in nth cluster and the unknown source position, respectively, where m is equal to 2 or 3. $f \geq max(N + m, 2N)$ is a required condition for a feasible localization system. In Fig. 1, we illustrate a nodes partly synchronous TDOA source localization system, which has 3 clusters. Let $d_i^n = \|\mathbf{u} - \mathbf{s}_i^n\|$ be the unknown distance between sensor $\mathbf{s}_i^n$ and source $\mathbf{u}$.

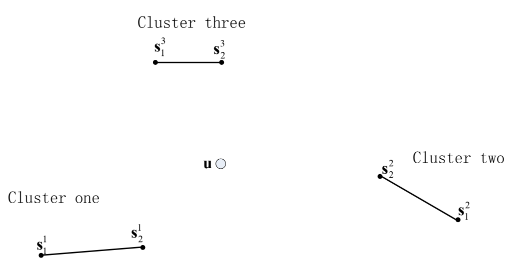

Fig. 1. Illustration of nodes partly synchronous TDOA source localization system, where the nodes connected by lines means they are synchronized.

In nth cluster, without loss of generality, let $\mathbf{s}_1^n$ be the reference node, and then the TDOA measurements can be denoted as

$$r_{i1}^n = d_i^n - d_1^n + e_{i1}^n, \quad n = 1, 2, \ldots, N, i = 2, 3, \ldots, M_n. \tag{1}$$

where r_{i1}^n is equal to the ith TDOA measurement t_{i1}^n multiplying by the signal propagation speed c, and e_{i1}^n is the distance difference measurement noise at ith node. For ease of analysis, we assume that $\mathbf{e}^n = [e_{21}^n, e_{31}^n, \ldots, e_{M_n 1}^n]^T$ is a zero-mean Gaussian vector with known covariance matrix $\mathbf{Q}_n$ [5].

Then the MLE can be written as the following optimization problem:

$$\min_{\mathbf{u}} \sum_{n=1}^{N} \sum_{i=2}^{M_n} \sum_{j=2}^{M_n} \left(r_{i1}^n - \|\mathbf{u} - \mathbf{s}_i^n\| + \|\mathbf{u} - \mathbf{s}_1^n\|\right) [Q_n^{-1}]_{(i-1),(j-1)} \cdot \left(r_{j1}^n - \left\|\mathbf{u} - \mathbf{s}_j^n\right\| + \|\mathbf{u} - \mathbf{s}_1^n\|\right) \tag{2}$$

The above problem is nonlinear and nonconvex, consequently the MLE is hard to achieve. Next, we will show how a nonconvex MLE problem be relaxed to a convex problem.

3. LOCALIZATION ALGORITHM

For easy of analysis, (2) can be written as the matrix-vector form

$$\min_{\mathbf{u}, \mathbf{d}^n} \sum_{n=1}^{N} (\mathbf{r}_d^n - \mathbf{A}_n \mathbf{d}^n)^T \mathbf{Q}_n^{-1} (\mathbf{r}_d^n - \mathbf{A}_n \mathbf{d}^n) \tag{3a}$$

$$s.t.\ d_i^n = \|\mathbf{u} - \mathbf{s}_i^n\|,\ n = 1, 2, \ldots, N, i = 1, 2, \ldots, M_n. \tag{3b}$$

where $\mathbf{r}_d^n = [r_{21}^n, r_{31}^n, \ldots, r_{M_n 1}^n]^T$, $\mathbf{d}^n = [d_1^n, d_2^n, \ldots, d_{M_n}^n]^T$, $\mathbf{A}_n = [-\mathbf{1}_{M_n - 1}, \mathbf{I}_{M_n - 1}]$. It can be seen that the objective function in (3a) is convex for $\mathbf{d}^n$. However, the constraints in (3b) are nonconvex for $\mathbf{d}^n$ and $\mathbf{u}$.

The objective function in (3a) can be rewritten as

$$\sum_{n=1}^{N} \left(tr(\mathbf{D}^n \mathbf{A}_n^T \mathbf{Q}_n^{-1} \mathbf{A}_n) - 2\mathbf{r}_d^{nT} \mathbf{Q}_n^{-1} \mathbf{A}_n \mathbf{d}^n + \mathbf{r}_d^{nT} \mathbf{Q}_n^{-1} \mathbf{r}_d^n\right) \tag{4}$$

where $\mathbf{D}^n = \mathbf{d}^n \mathbf{d}^{nT}$.

The constraints in (3b) can be expressed as

$$D_{i,i}^n = \|\mathbf{u} - \mathbf{s}_i^n\|^2 = y_s - 2\mathbf{u}^T \mathbf{s}_i^n + \mathbf{s}_i^{nT} \mathbf{s}_i^n, i = 1, 2, \ldots, M_n. \tag{5}$$

where $y_s = \mathbf{u}^T \mathbf{u}$.

Using the Cauchy-Schwartz inequality [13], we can obtain

$$D_{i,j}^n \geq |y_s - \mathbf{u}^T (\mathbf{s}_i^n + \mathbf{s}_j^n) + \mathbf{s}_i^{nT} \mathbf{s}_j^n|,\ 1 \leq i < j \leq M_n. \tag{6}$$

Note that $\mathbf{A}_n^T \mathbf{Q}_n^{-1} \mathbf{A}_n$ in (4) is singular, i.e., it is not a full rank matrix. To improve the accuracy, as in [14], we also introduce a penalty term $\sum_{n=1}^{N} tr(\mathbf{D}^n)$ into the objective function and add the second-order-cone (SOC) constraints

$$\|\mathbf{u} - \mathbf{s}_i^n\| \leq d_i^n,\ n = 1, 2, \ldots, N, i = 1, 2, \ldots, M_n. \tag{7}$$

By using the semidefinite relaxation (SDR) method [15], we can relax (3) into the following convex problem

$$\min_{\mathbf{d}^n, \mathbf{D}^n, \mathbf{u}, y_s} \sum_{n=1}^{N} \left(tr(\mathbf{D}^n \mathbf{A}_n^T \mathbf{Q}_n^{-1} \mathbf{A}_n) - 2\mathbf{r}_d^{nT} \mathbf{Q}_n^{-1} \mathbf{A}_n \mathbf{d}^n + \eta tr(\mathbf{D}^n)\right) \tag{8a}$$

$$s.t.\ D_{i,i}^n = y_s - 2\mathbf{u}^T \mathbf{s}_i^n + \mathbf{s}_i^{nT} \mathbf{s}_i^n, \quad n = 1, 2, \ldots, N, i = 1, 2, \ldots, M_n. \tag{8b}$$

$$\|\mathbf{u}-\mathbf{s}_i^n\| \le d_i^n,\ n=1,2,\ldots,N, i=1,2,\ldots,M_n. \tag{8c}$$

$$D_{i,j}^n \ge |y_s - \mathbf{u}^T(\mathbf{s}_i^n+\mathbf{s}_j^n) + \mathbf{s}_i^{nT}\mathbf{s}_j^n|,$$
$$n=1,2,\ldots,N, 1\le i<j\le M_n. \tag{8d}$$

$$\begin{bmatrix} 1 & \mathbf{d}^{nT} \\ \mathbf{d}^n & \mathbf{D}^n \end{bmatrix} \succeq \mathbf{0}, n=1,2,\ldots,N, \tag{8e}$$

$$\begin{bmatrix} \mathbf{I}_m & \mathbf{u} \\ \mathbf{u}^T & y_s \end{bmatrix} \succeq \mathbf{0}. \tag{8f}$$

where η is the regularization parameter which is difficult to determine. To alleviate this problem, first, we need to choose K different η, $\{\eta_k\}_{k=1}^K$, and then solve (8) with the K different choice of η, $\{\eta_k\}_{k=1}^K$, finally, from the K estimates $\{\hat{\mathbf{u}}_k\}_{k=1}^K$ to select $\hat{\mathbf{u}}$ that gives the minimum cost function J_k

$$J_k = \sum_{n=1}^{N} (\mathbf{r}_d^n - \mathbf{A}_n\hat{\mathbf{d}}_k^n)^T\mathbf{Q}_n^{-1}(\mathbf{r}_d^n - \mathbf{A}_n\hat{\mathbf{d}}_k^n),\ k=1,2,\ldots,K. \tag{9}$$

where $\hat{\mathbf{d}}_k^n = [\hat{d}_{k1}^n, \hat{d}_{k2}^n, \ldots, \hat{d}_{kM_n}^n]^T$, and $\hat{d}_{ki}^n = \|\hat{\mathbf{u}}_k - \mathbf{s}_i^n\|$, $n = 1,2,\ldots,N, i=1,2,\ldots,M_n$.

The method of determining a fitness penalty parameter is heuristic, and we need to solve K times of SDP problems to get a refined solution, which greatly increase the computational complexity. Nevertheless, we can solve the K times of SDP problems in parallel.

4. ROBUST LOCALIZATION ALGORITHM

In the previous discussion, the sensor positions are accurate. However, in practical, there exist the sensor position errors [16]. The obtained but erroneous sensor position can be expressed as

$$\mathbf{b}_i^n = \mathbf{s}_i^n + \boldsymbol{\beta}_i^n \tag{10}$$

where $\boldsymbol{\beta}_i^n$ is the sensor position error, which is modeled as Gaussian white noise with covariance matrix $\delta_i^{n2}\mathbf{I}_m$ [17].

Under the condition of independent noises $\boldsymbol{\beta}_i^n$ and e_{i1}^n, the MLE problem can be written as

$$\min_{\mathbf{u},\mathbf{s}_i^n} \sum_{n=1}^{N}\sum_{i=2}^{M_n}\sum_{j=2}^{M_n} \big(r_{i1}^n - \|\mathbf{u}-\mathbf{s}_i^n\| + \|\mathbf{u}-\mathbf{s}_1^n\|\big)[Q_n^{-1}]_{(i-1),(j-1)}\cdot$$
$$\big(r_{j1}^n - \big\|\mathbf{u}-\mathbf{s}_j^n\big\| + \|\mathbf{u}-\mathbf{s}_1^n\|\big) + \sum_{n=1}^{N}\sum_{i=1}^{M_n} \frac{\|\mathbf{b}_i^n - \mathbf{s}_i^n\|^2}{\delta_i^{n2}} \tag{11}$$

The above formulation can be reshaped as (constant terms are discarded)

$$\min_{\mathbf{X},\mathbf{d}^n} \sum_{n=1}^{N} (\mathbf{r}_d^n - \mathbf{A}_n\mathbf{d}^n)^T\mathbf{Q}_n^{-1}(\mathbf{r}_d^n - \mathbf{A}_n\mathbf{d}^n) +$$
$$\Big\|(\mathbf{X}(:,2:f+1) - \mathbf{B})\mathbf{W}^{\frac{1}{2}}\Big\|_F^2 \tag{12a}$$

$$s.t.\ d_i^n = \left\|\mathbf{X}(:,1) - \mathbf{X}(:,1+i+\sum_{q=0}^{n-1}M_q)\right\|,$$
$$n=1,2,\ldots,N, i=1,2,\ldots,M_n. \tag{12b}$$

where $M_0 = 0$, and

$$\mathbf{X} = [\mathbf{u}, \mathbf{s}_1^1, \ldots, \mathbf{s}_{M_1}^1, \ldots, \mathbf{s}_1^N, \ldots, \mathbf{s}_{M_N}^N], \tag{13}$$

$$\mathbf{B} = [\mathbf{b}_1^1, \ldots, \mathbf{b}_{M_1}^1, \ldots, \mathbf{b}_1^N, \ldots, \mathbf{b}_{M_N}^N], \tag{14}$$

$$\mathbf{W} = diag\big([\delta_1^{1\,-2}, \ldots, \delta_{M_1}^{1\ -2}, \ldots, \delta_1^{N\,-2}, \ldots, \delta_{M_N}^{N\ -2}]\big). \tag{15}$$

Let $\mathbf{Y} = \mathbf{X}^T\mathbf{X}$, the above objective function can be recast as

$$\sum_{n=1}^{N} \big(tr(\mathbf{D}^n\mathbf{A}_n^T\mathbf{Q}_n^{-1}\mathbf{A}_n) - 2\mathbf{r}_d^{nT}\mathbf{Q}_n^{-1}\mathbf{A}_n\mathbf{d}^n\big) +$$
$$tr\big(\mathbf{W}\mathbf{Y}(2:f+1, 2:f+1)\big) - 2tr\big(\mathbf{W}\mathbf{X}(:,2:f+1)^T\mathbf{B}\big) \tag{16}$$

Similar to the deviation of (8), we give the robust SDP localization algorithm

$$\min_{\mathbf{d}^n,\mathbf{D}^n,\mathbf{X},\mathbf{Y}} \sum_{n=1}^{N} \big(tr(\mathbf{D}^n\mathbf{A}_n^T\mathbf{Q}_n^{-1}\mathbf{A}_n) - 2\mathbf{r}_d^{nT}\mathbf{Q}_n^{-1}\mathbf{A}_n\mathbf{d}^n +$$
$$\eta tr(\mathbf{D}^n)\big) + tr\big(\mathbf{W}\mathbf{Y}(2:f+1,2:f+1)\big) -$$
$$2tr\big(\mathbf{W}\mathbf{X}(:,2:f+1)^T\mathbf{B}\big) \tag{17a}$$

$$s.t.\ D_{i,i}^n = Y(1,1) - 2Y(1,1+i+\sum_{q=0}^{n-1}M_q) +$$
$$Y(1+i+\sum_{q=0}^{n-1}M_q, 1+i+\sum_{q=0}^{n-1}M_q),$$
$$n=1,2,\ldots,N, i=1,2,\ldots,M_n. \tag{17b}$$

$$\left\|\mathbf{X}(:,1) - \mathbf{X}(:,1+i+\sum_{q=0}^{n-1}M_q)\right\| \le d_i^n,$$
$$n=1,2,\ldots,N, i=1,2,\ldots,M_n. \tag{17c}$$

$$D_{i,j}^n \ge |Y(1,1) - Y(1,1+i+\sum_{q=0}^{n-1}M_q) -$$
$$Y(1,1+j+\sum_{q=0}^{n-1}M_q) +$$
$$Y(1+i+\sum_{q=0}^{n-1}M_q, 1+j+\sum_{q=0}^{n-1}M_q)|,$$
$$n=1,2,\ldots,N, 1\le i<j\le M_n. \tag{17d}$$

$$\begin{bmatrix} 1 & \mathbf{d}^{nT} \\ \mathbf{d}^n & \mathbf{D}^n \end{bmatrix} \succeq \mathbf{0}, n=1,2,\ldots,N, \tag{17e}$$

$$\begin{bmatrix} \mathbf{I}_m & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Y} \end{bmatrix} \succeq \mathbf{0}. \tag{17f}$$

Table 1. Sensor positions in the unit of meters, $N = 4, M_1 = 2, M_2 = 2, M_3 = 2, M_4 = 2, f = 8$

Sensor no.	x	y	Ref.
1	0	0	Yes
2	10	0	No
3	90	0	Yes
4	100	0	No
5	0	90	Yes
6	0	100	No
7	90	90	Yes
8	90	100	No

The K times computation of (17) is similar to (8), and the selection of $\hat{\mathbf{u}}$ is also from (9).

5. SIMULATION RESULTS

In this section, we conduct several numerical simulations to demonstrate the performance of the proposed SDP algorithm. The proposed SDP algorithm is implemented by CVX toolbox [18], using SeDuMi as a solver [19], and the precision is set to *best*.

500 Monte Carlo realizations were done in the following simulations. The TDOA measurement noise covariance matrices is $\mathbf{Q}_n = \sigma^2\mathbf{R}$, where the diagonal elements in $\mathbf{R}$ equals to 1 and all other elements equals to 0.5 [16]. The sensor position errors variance is ${\delta_i^n}^2 = \delta^2$.

We consider the 2-D localization case. There are four clusters TDOA measurements, i.e., $N = 4$. The positions of the sensor nodes are listed in Table 1. Sensor 1 to sensor 2 are belong to cluster one, sensor 3 to sensor 4 are belong to cluster two, sensor 5 to sensor 6 are belong to cluster three, and sensor 7 to sensor 8 are belong to cluster four. In Table 1, 'Yes' means reference node. We set $K = 5$, $\eta_1 = 10^{-4}$, $\eta_2 = 10^{-3}$, $\eta_3 = 10^{-2}$, $\eta_4 = 10^{-1}$, $\eta_5 = 10^0$ for the computation of (8) and (17).

In Fig. 2, we show the estimation performance of the proposed algorithm. The 2SWLS and SDP-TDOA algorithms can not be applied in this case. However, from Fig. 2, it can been seen that the proposed algorithm performs well, and it is very close to the CRLB. Besides, we can see that the performance gap between partly synchronous and entirely synchronous is 13 dBm.

In Fig. 3, we show the estimation performance of the proposed robust algorithm in the presence of sensor position errors. From the figure, we can see that the proposed algorithm still perform well when consider the sensor position errors.

6. CONCLUSIONS

In this paper, we have investigated the problem of TDOA source localization in nodes partly synchronous system. First, we consider the situation of accurate sensor position, and we formulated the MLE problem, then we use the SDP techniques to relax the nonconvex MLE problem into convex problem. Besides, we develop a robust SDP algorithm for the case of non-accurate sensor position. The simulation results demonstrated that the proposed algorithms have good performance under the condition of accurate or non-accurate sensor position.

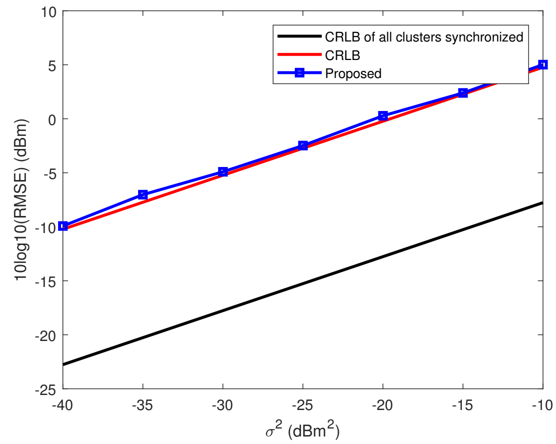

Fig. 2. RMSE vs σ^2, $\mathbf{u} = [74, 60]^T m$.

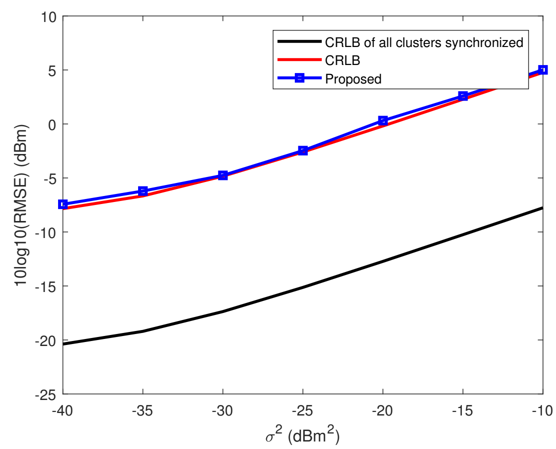

Fig. 3. RMSE vs σ^2, $\mathbf{u} = [74, 60]^T m$, $\delta = 0.01m$.

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