AN EFFICIENT TARGET LOCALIZATION ESTIMATOR FROM BISTATIC RANGE AND TDOA MEASUREMENTS IN MULTISTATIC RADAR

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ABSTRACT

This paper considers the target localization problem using the hybrid bistatic range and time difference of arrival (TDOA) measurements in multistatic radar. An algebraic closed-form solution to this nonlinear estimation problem is developed through two-stage processing, where the nuisance variables are introduced in the first stage and the localization error of first stage solution is estimated to improve the final target position estimate in the second stage. Theoretical analysis shows that the performance of the proposed method can reach the Cramer-Rao lower bound (CRLB) for Gaussian measurement noise over the small error region. Simulations are included to corroborate the performance of the proposed estimator.

Index Terms— Cramer-Rao lower bound (CRLB), bistatic range, time difference of arrival (TDOA), target localization, multistatic radar

1. INTRODUCTION

Target localization is a fundamental problem in multistatic radar systems, which has drawn considerable attentions in recent years [1-4]. Multistatic radar consists of multiple transmitter–receiver pairs, each of which can determine one bistatic range (BR) measurement. Each BR induces an ellipsoid where the target lies on, with the associated transmitter and receiver as its foci. Another common technique is to measure the time differences of arrival (TDOA) measurement of the source reflecting signal between the reference receiver and another receiver. Each TDOA defines a hyperboloid in which the source must lie. The intersection of these conicoid gives the target location estimate.

The source localization problem is potentially challenging due to the highly nonlinear relationship between the measurements and the unknown parameters. A lot of hyperbolic localization methods [5-11] have been developed using TDOA measurements over the last few decades. Recently, there is a rapidly growing literature concerned with the BR-based localization [12-17], which can be divided into iterative and closed-form algorithms. The closed-form solutions [13-17] are computationally attractive without requiring initial guesses and having divergence problem as compared to the iterative techniques [12]. In [13], elliptic equations were converted to hyperbolic ones, from which the target location was obtained using the least squares approach. The work [3, 18] conducted a preliminary study on the comparison between elliptic and hyperbolic positionings. Especially in [19], Rui and Ho performed a further investigation on the performance of elliptic localization with respect to the hyperbolic localization through CRLB analysis. More recently, the work by Liu [20] considered the problem of moving target localization using both BR and Doppler shift measurements and identified jointly the target position and velocity. Furthermore, [15] and [21] presented a hybrid BR and bearing measurements location algorithm that gives better accuracy than using BR alone. We shall consider in this paper the problem of target localization using both BR and TDOA measurements from multistatic radar.

This paper develops an efficient closed-form method for target localization in multistatic radar using the hybrid BR and TDOA measurements. The proposed estimator is based on two-stage processing [10]. In the first stage, a set of pseudolinear BR and TDOA equations are established by introducing nuisance parameters. In the second stage, the error term of the first stage solution is estimated using the relationship between the unknowns and nuisance variables. The obtained estimator gives a global minimum solution and is shown analytically and confirmed by simulations to be able to attain the CRLB performance under samll Gaussian measurement noise conditions.

The rest of this paper is organized as follows. Section 2 presents the target localization problem and the CRLB is derived in Section 3. The closed-form solution is proposed in Section 4. Section 5 provides the theoretical analysis and Section 6 examines the performance by simulations. Section 7 is the conclusion.

2. PRELIMINARY

We shall consider the target localization problem in three-dimensional (3D) space using M transmitters and N receivers, whose positions are denoted by $\boldsymbol{t}_i = \left[x_i^t, y_i^t, z_i^t\right]^T$,

$i = 1, 2, \cdots, M$ and $\boldsymbol{s}_j = \left[x_j^s, y_j^s, z_j^s\right]^T$, $j = 1, 2, \cdots, N$, where superscript T stands for the transpose. The true position of the target is represented by $\boldsymbol{u} = [x, y, z]^T$.

Each transmitter radiates a signal, and all receivers observe the signal from direct propagation and from indirect reflection of the target. The range between a transmitter-receiver pair is known as BR. The reflecting signal can be observed by all receivers to produce multiple TDOAs.

The range difference r_{ij}^o between the direct and the indirect signal from transmitter i at receiver j is given by

$$r_{ij}^o = r_i^t + r_j^s - d_{ij} \tag{1}$$

where $r_i^t = \|\boldsymbol{u} - \boldsymbol{t}_i\|$, $r_j^s = \|\boldsymbol{u} - \boldsymbol{s}_j\|$, $d_{ij} = \|\boldsymbol{t}_i - \boldsymbol{s}_j\|$, and $\|\cdot\|$ represents the Euclidean norm. It is obvious that BR is obtained through r_{ij}^o plus the range d_{ij}. We shall use BR and range difference interchangeably in this paper.

The noisy version of r_{ij}^o can be written by $r_{ij} = r_{ij}^o + \Delta r_{ij}$, where Δr_{ij} is the additive noise. Collecting the MN range measurements gives

$$\boldsymbol{r} = \left[\boldsymbol{r}_1^T, \boldsymbol{r}_2^T, \cdots, \boldsymbol{r}_N^T\right]^T = \boldsymbol{r}^o + \Delta \boldsymbol{r} \tag{2}$$

where $\boldsymbol{r}_j = \left[r_{1j}, r_{2j}, \cdots, r_{Mj}\right]^T = \boldsymbol{r}_j^o + \Delta \boldsymbol{r}_j$ contains the range difference measurements from the receiver j.

Without loss of generality, let us choose receiver $\boldsymbol{s}_1$ as the reference sensor. After multiplying TDOA by signal propagation speed, the range difference of arrival (RDOA) $\bar{r}_{j1}^o$ can be expressed as

$$\bar{r}_{j1}^o = r_j^s - r_1^s \tag{3}$$

where $j = 2, 3, \cdots, N$. TDOA and RDOA will be interchangeably used in this paper. The noisy RDOA measurement is modeled as $\bar{r}_{j1} = \bar{r}_{j1}^o + \Delta \bar{r}_{j1}$, where $\Delta \bar{r}_{j1}$ is the additive noise. We can define the $N-1$ RDOA measurements in vector form as

$$\bar{\boldsymbol{r}} = \left[\bar{r}_{21}, \bar{r}_{31}, \cdots, \bar{r}_{N1}\right]^T = \bar{\boldsymbol{r}}^o + \Delta \bar{\boldsymbol{r}} \tag{4}$$

where $\bar{\boldsymbol{r}}^o$ is the actual counterpart and $\Delta \bar{\boldsymbol{r}}$ is the corresponding error vector.

For notation simplicity, we stack $\boldsymbol{r}$, $\bar{\boldsymbol{r}}$ and represent them together by $\boldsymbol{m} = \left[\boldsymbol{r}^T, \bar{\boldsymbol{r}}^T\right]^T = \boldsymbol{m}^o + \Delta \boldsymbol{m}$. To simplify the development, we assume $\Delta \boldsymbol{r}$ and $\Delta \bar{\boldsymbol{r}}$ are independent of each other. The error vector $\Delta \boldsymbol{m}$ is assumed to be zero-mean Gaussian with a prior known covariance matrix $\boldsymbol{Q}_{\boldsymbol{m}} = \mathrm{diag}(\boldsymbol{Q}_{\boldsymbol{r}}, \boldsymbol{Q}_{\bar{\boldsymbol{r}}})$, where $\boldsymbol{Q}_{\boldsymbol{r}}$ and $\boldsymbol{Q}_{\bar{\boldsymbol{r}}}$ are covariance matrices of $\boldsymbol{r}$ and $\bar{\boldsymbol{r}}$, respectively.

We would like to accurately estimate the target position from the observed hybrid BR and TDOA measurements.

3. CRLB

We shall establish the performance bound through CRLB analysis for the target localization problem. The CRLB is the lowest possible variance that any unbiased estimator can achieve [22]. The density function of the composite Gaussian measurement is

$$f(\boldsymbol{m} \mid \boldsymbol{u}) = K \exp\left(-\frac{1}{2}(\boldsymbol{m} - \boldsymbol{m}^o)^T \boldsymbol{Q}_{\boldsymbol{m}}^{-1} (\boldsymbol{m} - \boldsymbol{m}^o)\right) \tag{5}$$

where K is a constant. Under this model, the CRLB of $\boldsymbol{u}$ is given by $\mathrm{CRLB} = \boldsymbol{J}(\boldsymbol{u})^{-1}$, where $\boldsymbol{J}(\boldsymbol{u})$ is the Fisher information matrix (FIM) equal to

$$\boldsymbol{J}(\boldsymbol{u}) = \nabla_{\boldsymbol{u}}^{\boldsymbol{m}^o T} \boldsymbol{Q}_{\boldsymbol{m}}^{-1} \nabla_{\boldsymbol{u}}^{\boldsymbol{m}^o} \tag{6}$$

where $\nabla_{\boldsymbol{a}}^{\boldsymbol{b}} = \partial \boldsymbol{b} / \partial \boldsymbol{a}$. $\nabla_{\boldsymbol{u}}^{\boldsymbol{m}^o} = \left[\nabla_{\boldsymbol{u}}^{\boldsymbol{r}^o T}, \nabla_{\boldsymbol{u}}^{\bar{\boldsymbol{r}}^o T}\right]^T$ are the partial derivatives of BR and TDOA measurement functions with respect to $\boldsymbol{u}$ evaluated at its true value.

Actually, $\nabla_{\boldsymbol{u}}^{\boldsymbol{r}^o}$ is a $MN \times 3$ matrix and the row is

$$\nabla_{\boldsymbol{u}}^{r_{ij}^o} = \boldsymbol{\rho}_{\boldsymbol{u}, \boldsymbol{t}_i}^T + \boldsymbol{\rho}_{\boldsymbol{u}, \boldsymbol{s}_j}^T \tag{7}$$

where $\boldsymbol{\rho}_{\boldsymbol{a}, \boldsymbol{b}} = (\boldsymbol{a} - \boldsymbol{b}) / \|\boldsymbol{a} - \boldsymbol{b}\|$ denotes a unit vector from $\boldsymbol{b}$ to $\boldsymbol{a}$.

Similar to $\nabla_{\boldsymbol{u}}^{\boldsymbol{r}^o}$, $\nabla_{\boldsymbol{u}}^{\bar{\boldsymbol{r}}^o}$ is a $(N-1) \times 3$ matrix, whose j th row can be written as

$$\nabla_{\boldsymbol{u}}^{\bar{r}_{j1}^o} = \boldsymbol{\rho}_{\boldsymbol{u}, \boldsymbol{s}_j}^T - \boldsymbol{\rho}_{\boldsymbol{u}, \boldsymbol{s}_1}^T \tag{8}$$

Given (7), (8) and (6), the CRLB of $\boldsymbol{u}$ can be obtained.

4. CLOSED-FORM SOLUTION

We shall develop a closed-form solution for the target localization problem using the hybrid BR and TDOA measurements. The proposed algorithm is comprised of two stages. The first stage establishes a set of pseudolinear equations by introducing nuisance parameters and estimates the target position. The second stage estimates the error term of first stage solution through exploring the relationship between target position and nuisance variables [10].

First Stage: We first express (1) as $r_{ij}^o - r_j^s + d_{ij} = r_i^t$. Substituting $r_{ij}^o = r_{ij} - \Delta r_{ij}$, rearranging and squaring both sides yields the BR measurement equations

$$\begin{aligned} 2 r_i^t \Delta r_{ij} \simeq\; & r_{ij}^2 + 2 r_{ij} d_{ij} + 2(\boldsymbol{s}_j - \boldsymbol{t}_i)^T \boldsymbol{s}_j \\ & - 2(\boldsymbol{s}_j - \boldsymbol{t}_i)^T \boldsymbol{u} - 2(r_{ij} + d_{ij}) r_j^s \end{aligned} \tag{9}$$

for $i = 1, 2, \cdots, M$ and $j = 1, 2, \cdots, N$.

Upon rewriting (3) as $\bar{r}_{j1}^o + r_1^s = r_j^s$, substituting $\bar{r}_{j1}^o = \bar{r}_{j1} - \Delta \bar{r}_{j1}$ and squaring both sides, we arrive at the TDOA measurement equations

$$2 r_j^s \Delta \bar{r}_{j1} \simeq \bar{r}_{j1}^2 + \boldsymbol{s}_1^T \boldsymbol{s}_1 - \boldsymbol{s}_j^T \boldsymbol{s}_j - 2(\boldsymbol{s}_1 - \boldsymbol{s}_j)^T \boldsymbol{u} + 2 \bar{r}_{j1} r_1^s \tag{10}$$

for $j = 2, \cdots, N$.

In both (9) and (10), the second-order noise terms have been ignored. Besides, they are nonlinear with respect to the unknown target position $\boldsymbol{u}$ since r_j^s is related to $\boldsymbol{u}$ through $r_j^s = \left\|\boldsymbol{u} - \boldsymbol{s}_j\right\|$. The solution derivation begins by defining an unknown vector $\boldsymbol{\varphi}^o = \left[\boldsymbol{u}^T, r_1^s, r_2^s, \cdots, r_N^s\right]^T$, which contains the unknown target position and N nuisance variables.

Stacking (9) and (10), and putting them together yields the matrix form equation

$$\boldsymbol{B}_1 \Delta \boldsymbol{m} = \boldsymbol{h}_1 - \boldsymbol{G}_1 \boldsymbol{\varphi}^o \tag{11}$$

In (11), the vector $\boldsymbol{h}_1 = \left[\boldsymbol{h}_{1,r}^T, \boldsymbol{h}_{1,\bar{r}}^T\right]^T$. The entries of $\boldsymbol{h}_{1,r}$ and $\boldsymbol{h}_{1,\bar{r}}$ are $r_{ij}^2 + 2r_{ij}d_{ij} + 2(\boldsymbol{s}_j - \boldsymbol{t}_i)^T \boldsymbol{s}_j$ and $\bar{r}_{j1}^2 + \boldsymbol{s}_1^T \boldsymbol{s}_1 - \boldsymbol{s}_j^T \boldsymbol{s}_j$, respectively. The matrix $\boldsymbol{G}_1$ can be represented as $\left[\boldsymbol{G}_{1,r}^T, \boldsymbol{G}_{1,\bar{r}}^T\right]^T$. The rows of $\boldsymbol{G}_{1,r}$ and $\boldsymbol{G}_{1,\bar{r}}$ are $2\left[(\boldsymbol{s}_j - \boldsymbol{t}_i)^T, \boldsymbol{0}_{j-1}^T, (r_{ij} + d_{ij}), \boldsymbol{0}_{N-j}^T\right]$ and $2\left[(\boldsymbol{s}_1 - \boldsymbol{s}_j)^T, -\bar{r}_{j1}, \boldsymbol{0}_{N-1}^T\right]$, where $\boldsymbol{0}_k$ denotes a k-dimensional zero column vector.

The matrix $\boldsymbol{B}_1 = blkdiag\left(\boldsymbol{B}_{1,r}, \boldsymbol{B}_{1,\bar{r}}\right)$. The submatrix $\boldsymbol{B}_{1,r}$ is equal to $2\boldsymbol{I}_N \otimes diag(r_1^t, r_2^t, \cdots, r_M^t)$, where $\boldsymbol{I}_N$ denotes an N-dimensional identity matrix and $\otimes$ stands for the Kronecker product. Submatrix $\boldsymbol{B}_{1,\bar{r}}$ is $2diag(r_2^s, r_3^s, \cdots, r_N^s)$.

The weighted least squares (WLS) solution to (11) is

$$\boldsymbol{\varphi} = (\boldsymbol{G}_1^T \boldsymbol{W}_1 \boldsymbol{G}_1)^{-1} \boldsymbol{G}_1^T \boldsymbol{W}_1 \boldsymbol{h}_1 \tag{12}$$

where $\boldsymbol{W}_1 = \boldsymbol{B}_1^{-T} \boldsymbol{Q}_m^{-1} \boldsymbol{B}_1^{-1}$ is the weighting matrix. The acquisition of $\boldsymbol{W}_1$ relys on the true target position through $\boldsymbol{B}_1$. So we can first set $\boldsymbol{W}_1$ as $\boldsymbol{Q}_m^{-1}$ to produce an initial solution. The initial position is used to form the desired $\boldsymbol{W}_1$. Then, the first stage solution is obtained from the new $\boldsymbol{W}_1$. According to the previous studies [7, 15-17], when the measurement noise is small, the first stage solution has negligible bias, and the covariance matrix of $\boldsymbol{\varphi}$ can be approximated by $\mathrm{cov}(\boldsymbol{\varphi}) \simeq (\boldsymbol{G}_1^T \boldsymbol{W}_1 \boldsymbol{G}_1)^{-1}$.

The first stage processing, however, cannot reach CRLB accuracy. In the next stage, we shall explore the dependency in the elements of $\boldsymbol{\varphi}$ to improve the localization accuracy.

Second Stage: The first stage solution can be represented as $\boldsymbol{\varphi} = \left[\hat{\boldsymbol{u}}^T, \hat{r}_1^s, \hat{r}_2^s, \cdots, \hat{r}_N^s\right]^T = \boldsymbol{\varphi}^o + \Delta\boldsymbol{\varphi}$, where $\Delta\boldsymbol{\varphi} = \left[\Delta\boldsymbol{u}^T, \Delta r_1^s, \Delta r_2^s, \cdots, \Delta r_N^s\right]^T$ is the related estimation error. Substituting $\boldsymbol{u} = \hat{\boldsymbol{u}} - \Delta\boldsymbol{u}$ into $r_j^s = \left\|\boldsymbol{u} - \boldsymbol{s}_j\right\|$ and applying the Taylor-series expansion up to the first-order term obtains

$$r_j^s = \left\|\boldsymbol{u} - \boldsymbol{s}_j\right\| \simeq \left\|\hat{\boldsymbol{u}} - \boldsymbol{s}_j\right\| - \boldsymbol{\rho}_{\hat{\boldsymbol{u}},\boldsymbol{s}_j}^T \Delta\boldsymbol{u} \tag{13}$$

Expressing r_j^s in terms of $r_j^s = \hat{r}_j^s - \Delta r_j^s$ and putting it into (13) yields the solution equation that relates the estimated source position $\hat{\boldsymbol{u}}$ and the nuisance variable $\hat{r}_j^s$

$$\Delta r_j^s \simeq \hat{r}_j^s - \left\|\hat{\boldsymbol{u}} - \boldsymbol{s}_j\right\| + \boldsymbol{\rho}_{\hat{\boldsymbol{u}},\boldsymbol{s}_j}^T \Delta\boldsymbol{u} \tag{14}$$

which is linear with respect to $\Delta\boldsymbol{u}$. In order to establish another cost function, we have an auxiliary equation

$$\Delta\boldsymbol{u} = \boldsymbol{0}_3 + \Delta\boldsymbol{u} \tag{15}$$

in which the left $\Delta\boldsymbol{u}$ denotes random error.

Combining (14) and (15) gives the matrix form equation

$$\boldsymbol{B}_2 \Delta\boldsymbol{\varphi} = \boldsymbol{h}_2 - \boldsymbol{G}_2 \Delta\boldsymbol{u} \tag{16}$$

where $\boldsymbol{h}_2 = \left[\boldsymbol{0}_3^T, \boldsymbol{h}_{2,r}^T\right]^T$ and $\boldsymbol{G}_2 = \left[-\boldsymbol{I}_3^T, \boldsymbol{G}_{2,r}^T\right]^T$. The j th element of $\boldsymbol{h}_{2,r}$ is $\hat{r}_j^s - \left\|\hat{\boldsymbol{u}} - \boldsymbol{s}_j\right\|$. The j th row of $\boldsymbol{G}_{2,r}$ is $-\boldsymbol{\rho}_{\hat{\boldsymbol{u}},\boldsymbol{s}_j}^T$. The matrix $\boldsymbol{B}_2$ is given by $\boldsymbol{B}_2 = \boldsymbol{I}_{3+N}$.

The WLS solution to (16) is

$$\Delta\hat{\boldsymbol{u}} = (\boldsymbol{G}_2^T \boldsymbol{W}_2 \boldsymbol{G}_2)^{-1} \boldsymbol{G}_2^T \boldsymbol{W}_2 \boldsymbol{h}_2 \tag{17}$$

where $\boldsymbol{W}_2 = \boldsymbol{B}_2^{-T} \mathrm{cov}(\boldsymbol{\varphi})^{-1} \boldsymbol{B}_2^{-1}$ is the weighting matrix. If we assume that the BR and TDOA measurements noise are sufficiently small so that the error in $\boldsymbol{G}_2$ can be ignored, and the covariance matrix of $\Delta\hat{\boldsymbol{u}}$ can be approximately equal to

$$\mathrm{cov}(\Delta\hat{\boldsymbol{u}}) \simeq (\boldsymbol{G}_2^T \boldsymbol{W}_2 \boldsymbol{G}_2)^{-1} \tag{18}$$

The final source position estimate is obtained from subtracting $\Delta\hat{\boldsymbol{u}}$ from the first stage localization result

$$\tilde{\boldsymbol{u}} = \hat{\boldsymbol{u}} - \Delta\hat{\boldsymbol{u}} \tag{19}$$

A point to note is that when $M < N$, we should use the $r_i^t, i = 1, 2, \cdots, M$ and r_1^s as the nuisance variables. Under this condition, we had $M + 4$ unknowns rather than $N + 3$. We will carry on the research in the future work.

5. PERFORMANCE ANALYSIS

We shall evaluate the performance of the proposed method by comparing its covariance matrix with the CRLB under small noise conditions. By substituting $\hat{\boldsymbol{u}} = \boldsymbol{u} + \Delta\boldsymbol{u}$ into (19), we have the final localization error $\tilde{\boldsymbol{u}} - \boldsymbol{u} = -(\Delta\hat{\boldsymbol{u}} - \Delta\boldsymbol{u})$. Hence, we realize that $\mathrm{cov}(\tilde{\boldsymbol{u}})$ would be equal to $\mathrm{cov}(\Delta\hat{\boldsymbol{u}})$.

After substituting sequentially $\boldsymbol{W}_2$, $\mathrm{cov}(\boldsymbol{\varphi})$ and $\boldsymbol{W}_1$, we can express the inverse of $\mathrm{cov}(\tilde{\boldsymbol{u}})$ as

$$\mathrm{cov}(\tilde{\boldsymbol{u}})^{-1} = \boldsymbol{G}_3^T \boldsymbol{Q}_m^{-1} \boldsymbol{G}_3 \tag{20}$$

where $\boldsymbol{G}_3 = \boldsymbol{B}_1^{-1} \boldsymbol{G}_1 \boldsymbol{B}_2^{-1} \boldsymbol{G}_2$.

Comparing (20) and (6) reveals that they have the same expression structure. The subsequent analysis require several small noise conditions where $\Delta r_{ij} \ll r_{ij}^o$, $\Delta r_{ij} \ll r_i^t$, $\Delta\bar{r}_{j1} \ll \bar{r}_{j1}^o$, and $\Delta\bar{r}_{j1} \ll r_j^s$. After some straightforward mathematical manipulations, under the above conditions, it can be verified that

$$\boldsymbol{G}_3 \simeq \nabla_{\boldsymbol{u}}^{\boldsymbol{m}^o} \tag{21}$$

Putting (21) into (20) and comparing with (6), it can be immediately concluded that

$$\mathrm{cov}(\tilde{\boldsymbol{u}}) \simeq \mathrm{CRLB}(\boldsymbol{u}) \tag{22}$$

Consequently, the proposed solver is able to reach the CRLB approximately when above conditions are satisfied.

6. SIMULATIONS

Simulations are performed to assess the performance of the proposed method. The number of available transmitters and receivers are $M=5$ and $N=5$, and their positions are tabulated in Table 1. The target is at $\boldsymbol{u}=[600,650,550]^T$ m. The covariance matrices of the BR and TDOA measurements are set as $\boldsymbol{Q}_{\boldsymbol{r}}=\delta_d^2\boldsymbol{I}_{MN}$ and $\boldsymbol{Q}_{\bar{\boldsymbol{r}}}=\delta_d^2\boldsymbol{I}_{N-1}$. We use the CRLB as a benchmark for performance evaluation. The number of ensemble runs is 5000.

Table 1. Positions of transmitters and receivers (m)

Tx	x_i^t	y_i^t	z_i^t	**Rx**	x_j^s	y_j^s	z_j^s
1	350	200	100	**1**	0	500	200
2	450	300	250	**2**	500	0	100
3	300	100	150	**3**	-500	0	150
4	400	150	100	**4**	0	-500	100
5	300	500	200	**5**	0	0	100

Fig. 1 illustrates the root mean square error (RMSE) of the proposed estimator as the range measurement noise power increases when $M=2$ and $N=5$. In this scenario, the target can be positioned using TDOA measurement only. As can be seen from the figure, the proposed estimator is shown to outperform the BR-based and TDOA-based methods. The performance improvement by adding TDOA measurements becomes apparent as noise power increases. In particular, the CRLB value of joint location is lowered by nearly 1.4 dB than that of elliptic location. The proposed algorithm works well and attains the CRLB accuracy until the noise power about 25 dB. Nevertheless, the BR-based and TDOA-based algorithms begin to deviate from each bound much earlier than that of the proposed method.

In addition, the accuracy improvement of the joint localization method with respect to the BR-based method is affected by the number of transmitters. Table 2 summarizes the simulation results, which indicate that the accuracy improvement decreases as the number of transmitters increases. Note that when $M=1$, the BR-based method is not able to locate the target, and so that of joint localization.

Table 2. The average improvement of localization accuracy (N=5)

Improvment	M=2	M=3	M=4	M=5
RMSE (dB)	3.80	1.37	0.96	0.56
CRLB (dB)	1.41	0.97	0.81	0.49

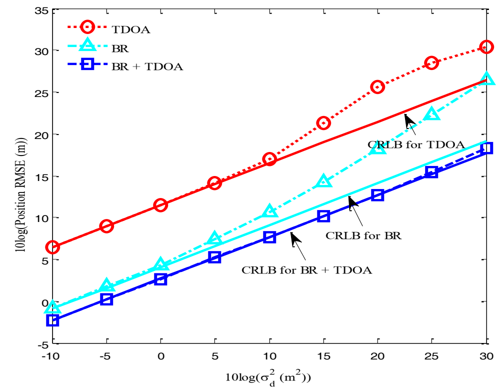

Fig. 1. The comparision of localization accuracy (M=2, N=5)

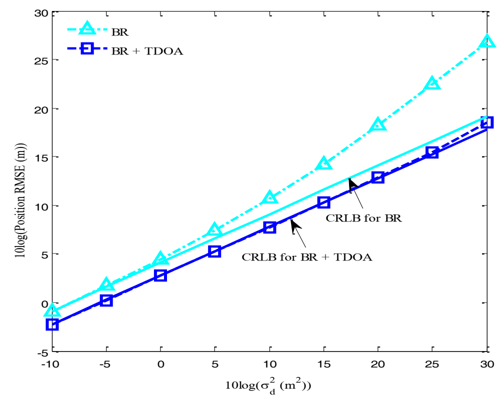

Fig. 2. The comparision of localization accuracy (M=2, N=4)

Fig. 2 shows the results at different levels of range measurement noise power when $M=2$ and $N=4$. In this scenario, the TDOA-based localization method fails to work due to the number of receivers is less than 5. The improvement of estimation accuracy owing to the contribution of TDOA measurements is obvious especially when the measurement noise is large, even though the target is not able to be located using TDOA measurements only. The other observations are similar to those from Fig. 1.

7. CONCLUSION

In this paper, we proposed a closed-form estimator to locate a single target using the hybrid BR and TDOA measurements in multistatic radar. The obtained two-stage algorithm was derived through introducing nuisance variables in the first stage and refining the estimate in the second stage. The use of TDOA measurements can improve the target localization accuracy. The proposed method was shown to be able to attain the CRLB accuracy under small Gaussian measurement noise, which is supported by the theoretical analysis and simulation results.

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