ALTITUDE MEASUREMENT OF LOW-ANGLE TARGET UNDER COMPLEX TERRAIN ENVIRONMENT FOR METER-WAVE RADAR

Yuan Liu, Hongwei Liu, Bo Jiu and Lei Zhang

National Laboratory of Radar Signal Processing, Xidian University, Xi'an, China
Collaborative Innovation Center of Information Sensing and Understanding at Xidian University
Email: ly000212@126.com
{hwliu, bojiu, leizhang}@xidian.edu.cn

ABSTRACT

For modern meter-wave radar, the performance of low-angle target altitude measurement is limited by multipath phenomenon, especially in the complex terrain environment where the multipath signal is perturbed by irregular surface. To address this problem, a practical signal model for meter-wave radar in practical terrain is first presented, where the influence of the perturbed multipath caused by irregular reflecting surface is taken into consideration. A novel compressive sensing (CS) based altitude measurement algorithm, combined with alternative optimization and dictionary updating techniques, is then proposed, in which the perturbation caused by the complex terrain can be iteratively compensated to estimate the target altitude more precisely. Numerical results based on both simulated data and real data demonstrate the effectiveness of the proposed algorithm under complex terrain environment.

***Index Terms*—** Compressed sensing, perturbational multipath, meter-wave radar, online dictionary updating.

1. INTRODUCTION

In modern warfare, the recognition of meter-wave radar has been increasing gradually due to its potential advantage of addressing the critical threats from stealth aircraft and anti-radiation missile (ARM). However, the localization accuracy of meter-wave radar for low-angle target is affected by the highly correlated multipath caused by ground surface reflection [1]-[3]. It is difficult to deal with this problem mainly because of the fact that the direct and multipath signals lie within a half-power beamwidth and they usually have fairly small differences in both time delay and radar radial velocity. Therefore, the real target can hardly be distinguished from its multipath image in the spatial, Doppler, and/or time domains.

To address the problem mentioned above, considerable research work on the array signal processing techniques has been carried out. These commonly used methods can be mainly classified into two categories: subspace methods [4-6] and parametric methods [7-11]. The first group algorithms, including multiple signal classification (MUSIC) algorithm [4], suffer from significant performance degradation due to the high correlation of the real target and its multipath signals. Although the forward/backward spatial smoothing (FBSS) technique [5, 12] could be introduced to achieve decorrelation preprocessing, it reduces the effective aperture and results in performance degradation. Alternatively, parametric methods contain a variety of maximum likelihood (ML) based estimator. Ballance has demonstrated that the solutions of these methods are equivalent [7]. In order to reduce the computation complexity of the ML algorithm, a simplified highly deterministic multipath signal model is studied by the refined maximum likelihood (RML) algorithm [8] with the assumption that the Earth's surface is perfectly smooth. However, the steering vector of the multipath signal may be perturbed by an irregular terrain reflection, which leads to the simplified model produces mismatch. This mismatch may invalidate the existing altitude measurement algorithms.

In this paper, a practical multipath signal model for meter-wave radar is first presented, where the perturbation caused by irregular reflecting surface is considered. A novel altitude measurement algorithm based on CS framework [13-14], combined with alternative optimization and dictionary updating techniques, is then proposed. Inspired by the sufficient sparsity of the direct and multipath signals in the spatial domain, the CS technique is introduced to achieve the target direction by constructing a parameterized dictionary. In addition, the alternative optimization and dictionary updating techniques are utilized to mitigate the influence of the perturbed multipath, in which the dictionary atoms can be updated step-by-step to compensate the perturbation caused by irregular terrain and to locate the target more precisely.

The remainder of this paper is organized as follows. Section 2 formulates the perturbational multipath signal model. In Section 3, the proposed low-angle target altitude measurement

This work is supported by the National Science Fund for Distinguished Young Scholars (61525105).

algorithm is detailed. Experimental results based on computer simulation and real data are presented in Section 4. Finally, we conclude the paper in Section 5.

2. SIGNAL MODEL

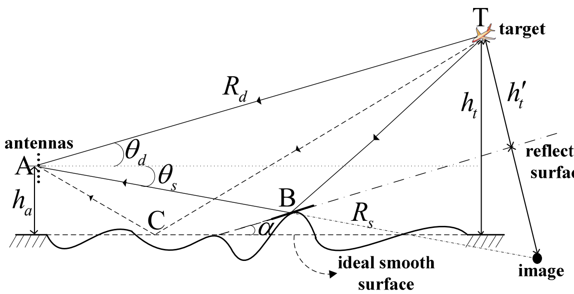

Fig.1. Geometry of perturbational multipath model for VHF radar.

The illustration geometry of perturbational multipath signal model is depicted in Fig. 1. We consider a uniform linear array (ULA) with *M*-elements. The center height of the array and the point target height are h_a and h_t, respectively. R_d and R_s are the lengths of the direct and the multipath paths, respectively. The dotted line (ACT) shows the conventional multipath model that reflected by a perfectly smooth surface, whereas the solid line (ABT) represents the introduced perturbational multipath model that reflected by a complex terrain, which is the main interest in this paper. The two separate paths of target and its image have the directions of θ_d and θ_s, respectively. The vertical distance from the target to the reflected surface and the included angle between the reflected surface and the ideal smooth surface are h_t' and α, respectively.

The signal observed by the antennas can be represented as

$$\begin{aligned}\mathbf{s}(t) &= \left[s_1(t), s_2(t), \cdots, s_M(t)\right]^{\mathrm{T}} \\ &= \left[\mathbf{a}(\theta_d) + \rho\mathbf{\Gamma}\mathbf{a}(\theta_s)\right]g(t) + \mathbf{n}(t)\end{aligned} \tag{1}$$

where $(\cdot)^{\mathrm{T}}$ stands for the transpose. $\mathbf{a}(\theta_d)$ and $\mathbf{a}(\theta_s)$ denote the steering vectors in the directions of the target and its multipath, which can be respectively expressed as

$$\mathbf{a}(\theta_d) = \left[1, e^{-j\varphi\sin(\theta_d)}, \cdots, e^{-j(M-1)\varphi\sin(\theta_d)}\right]^{\mathrm{T}} \tag{2}$$

$$\mathbf{a}(\theta_s) = \left[1, e^{-j\varphi\sin(\theta_s)}, \cdots, e^{-j(M-1)\varphi\sin(\theta_s)}\right]^{\mathrm{T}} \tag{3}$$

where $\varphi = 2\pi d/\lambda$, d and λ are the inter-element distance and the wavelength, respectively. $\rho = \rho_0 \exp(-j2\pi\Delta R/\lambda)$ is the attenuation coefficient with ρ_0 and $\Delta R = R_s - R_d$ represent the specular reflection coefficient and the path difference, respectively. $\mathbf{\Gamma} = \mathrm{diag}\left[\beta_1, \beta_2, \cdots, \beta_M\right]$ is the perturbation matrix, which is produced by the complex terrain reflection. $g(t)$ and $\mathbf{n}(t) = \left[n_1(t), n_2(t), \cdots, n_M(t)\right]^{\mathrm{T}}$ are the complex envelope of the signal and the additive Gaussian white noise vector, respectively.

Considering a more general multi-snapshot condition, (1) becomes the following representation

$$\mathbf{S} = \left[\mathbf{a}(\theta_d) + \rho\mathbf{\Gamma}\mathbf{a}(\theta_s)\right]\mathbf{g} + \mathbf{N} \tag{4}$$

where $\mathbf{S} = \left[\mathbf{s}(t_1), \mathbf{s}(t_2), \cdots, \mathbf{s}(t_L)\right] \in \mathbb{C}^{M\times L}$, $\mathbf{g} \in \mathbb{C}^{1\times L}$ and $\mathbf{N} \in \mathbb{C}^{M\times L}$ are defined similarly. L is the number of snapshots, $\mathbb{C}^{M\times L}$ denotes an $M \times L$ complex matrix.

3. THE PROPOSED METHOD

In this section, a novel target altitude measurement method based on CS framework with alternative optimization and dictionary updating techniques is proposed, which performs effectively in a complex terrain condition. The detail of this algorithm is shown in the following three subsections.

3.1. Overcomplete representation based on the sparse representation framework

We start to formulate the target localization problem as a sparse representation problem [15]. Let $\left\{\theta_d^1, \cdots, \theta_d^p, \cdots, \theta_d^P\right\}$ and $\left\{\theta_s^1, \cdots, \theta_s^q, \cdots, \theta_s^Q\right\}$ be two sampling grids of target and its multipath locations of interest, respectively. Generally, both the number of potential target directions *P* and the number of potential image directions *Q* are assumed much greater than the number of sensors *M*, that is, $P \gg M$ and $Q \gg M$. An overcomplete dictionary $\mathbf{B}(\mathbf{\Gamma}) = \left[\mathbf{A}_d, \mathbf{\Gamma}\mathbf{A}_s\right]$ in terms of all possible directions is introduced, where $\mathbf{A}_d$ and $\mathbf{A}_s$ can be respectively given as

$$\mathbf{A}_d = \left[\mathbf{a}(\theta_d^1), \cdots, \mathbf{a}(\theta_d^p), \cdots, \mathbf{a}(\theta_d^P)\right] \tag{5}$$

$$\mathbf{A}_s = \left[\mathbf{a}(\theta_s^1), \cdots, \mathbf{a}(\theta_s^q), \cdots, \mathbf{a}(\theta_s^Q)\right] \tag{6}$$

Thus, the signal model (4) is reduced to

$$\mathbf{S} = \mathbf{B}(\mathbf{\Gamma})\mathbf{W} + \mathbf{N} \tag{7}$$

where $\mathbf{W} = \left[\boldsymbol{\omega}_1, \cdots, \boldsymbol{\omega}_l, \cdots, \boldsymbol{\omega}_L\right] \in \mathbb{C}^{(P+Q)\times L}$ represents the sparse coefficient matrix, $\boldsymbol{\omega}_l = \left[\boldsymbol{\omega}_{dl}, \boldsymbol{\omega}_{sl}\right]^{\mathrm{T}} \in \mathbb{C}^{(P+Q)\times 1}$, $\boldsymbol{\omega}_{dl} = \left[\omega_{dl}^1, \cdots, \omega_{dl}^P\right]$ and $\boldsymbol{\omega}_{sl} = \left[\omega_{sl}^1, \cdots, \omega_{sl}^Q\right]$ are the associated coefficient vectors for the target and its multipath image, respectively.

In fact, this overcomplete representation allows us to convert the problem of direction estimation into the problem of sparse spectrum estimation, which can be solved by utilizing the following l_1 methodology

$$\left\{\hat{\mathbf{\Gamma}}, \hat{\mathbf{W}}\right\} = \arg\min_{\mathbf{\Gamma}, \mathbf{W}} \left\|\mathbf{S} - \mathbf{B}(\mathbf{\Gamma})\mathbf{W}\right\|_2^2 + \mu\left\|\mathbf{W}\right\|_1 \tag{8}$$

where $\|\cdot\|_1$ and $\|\cdot\|_2$ are the l_1-norm and l_2-norm, respectively. μ is a parameter controls the tradeoff between the residual norm and the sparsity of the spectrum. The main drawback of this method is the required computational complexity increases rapidly with the increasing L. Thus, when L is large, it is difficult to satisfy the real-time application.

3.2. Direction estimation with multiple snapshots based on CS and dictionary updating

To reduce the computational complexity and enhance the robustness to noise, the singular value decomposition (SVD) of the data matrix $\mathbf{S}$ is firstly implemented. The key idea is to decompose $\mathbf{S}$ into the signal and noise subspaces and just keep the signal subspace. Due to the highly correlation of direct and multipath signals, the set of vectors $\{\mathbf{s}(t_l)\}_{l=1}^{L}$ would lie in a one-dimensional subspace (without noise). Thus, a reduced-dimension signal subspace basis $\mathbf{y}$, which contains most of the signal energy, can be obtained by

$$\mathbf{y}=\Theta_{\delta}(\mathbf{S})\mathbf{V1} \tag{9}$$

where Θ_{δ} means the singular value thresholding

$$\Theta_{\delta}(\mathbf{S})=\mathbf{U}\boldsymbol{\Sigma}_{\delta}\mathbf{V}^{\mathrm{H}} \tag{10}$$

Here, $\boldsymbol{\Sigma}_{\delta}=diag\left[(\eta_1-\delta)_+,\cdots,(\eta_M-\delta)_+\right]$, $\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{H}}$ is the SVD operation of $\mathbf{S}$, $\boldsymbol{\Sigma}=diag[\eta_1,\cdots,\eta_M]$, and $t_+=\max(t,0)$. $(\cdot)^{\mathrm{H}}$ denotes the conjugate transpose, $\mathbf{1}$ is an $L\times 1$ ones vector. In our problem, δ is set as the second largest singular value of $\mathbf{S}$ due to the highly coherent nature of the sources.

After the above operation is applied, the cost function in (8) can be reduce as

$$\left\{\hat{\boldsymbol{\Gamma}},\hat{\boldsymbol{\gamma}}\right\}=\arg\min_{\boldsymbol{\Gamma},\boldsymbol{\gamma}}\left\|\mathbf{y}-\mathbf{B}(\boldsymbol{\Gamma})\boldsymbol{\gamma}\right\|_2^2+\mu\|\boldsymbol{\gamma}\|_1 \tag{11}$$

where $\boldsymbol{\gamma}=[\boldsymbol{\gamma}_d,\boldsymbol{\gamma}_s]^{\mathrm{T}}$, $\boldsymbol{\gamma}_d=\left[\gamma_d^1,\cdots,\gamma_d^P\right]$ and $\boldsymbol{\gamma}_s=\left[\gamma_s^1,\cdots,\gamma_s^Q\right]$ are the associated coefficient vectors for the target and its multipath, respectively.

For objective function (11), the alternative optimization method is utilized, which produces to the following subproblems:

1) Sparse coding:

$$\begin{aligned}\hat{\boldsymbol{\gamma}}_k=\arg\min_{\boldsymbol{\gamma}}&\left\|\mathbf{y}-\mathbf{B}(\boldsymbol{\Gamma})\boldsymbol{\gamma}\right\|_2^2+\mu\|\boldsymbol{\gamma}\|_1\\ \text{s.t.}\quad &\boldsymbol{\Gamma}=\boldsymbol{\Gamma}_k\end{aligned} \tag{12}$$

where $\hat{\boldsymbol{\gamma}}_k$ is the estimation of $\boldsymbol{\gamma}$ in the kth iteration. $k=0,1,\cdots,K$, K is the maximum number of iterations. The ideal smooth surface is firstly considered, $\boldsymbol{\Gamma}_0$ is a diagonal matrix that all the elements are all one. Problem (12) is a convex optimization problem and the unique solution can be achieved by using the convex programming [16].

2) Dictionary updating:

$$\begin{aligned}\hat{\boldsymbol{\Gamma}}_k=\arg\min_{\boldsymbol{\Gamma}}&\left\|\mathbf{y}-\mathbf{B}(\boldsymbol{\Gamma})\boldsymbol{\gamma}\right\|_2^2+\mu\|\boldsymbol{\gamma}\|_1\\ \text{s.t.}\quad &\boldsymbol{\gamma}=\hat{\boldsymbol{\gamma}}_k\end{aligned} \tag{13}$$

Here, $\hat{\boldsymbol{\Gamma}}_k$ is the estimation of $\boldsymbol{\Gamma}$ in the kth iteration. A convenient CVX solver [17] can be used to solve the problem in (13). The choice of μ and the convergence property are analyzed in [18-21]. Repeat the aforementioned two steps until the iteration stopping condition is satisfied. In our experiment, the iteration is stopped when the iterative index reaches a given maximum number K.

3.3. Target DOA estimation and altitude calculation

After the alternative optimization terminates, the real target direction can be easily obtained by one-dimensional searching, which can be represented as follows

$$\hat{\theta}_d=\arg\max_{\theta}P\left(\hat{\boldsymbol{\gamma}}_k(\theta)\right) \tag{14}$$

where $P\left(\hat{\boldsymbol{\gamma}}_k(\theta)\right)$ denotes the angular spectrum of $\hat{\boldsymbol{\gamma}}_k$ with $\theta\in\left\{\theta_d^p\right\}_{p=1}^{P}$. The target altitude h_t can also be calculated with the final estimation $\hat{\theta}_d$, which can be expressed as

$$\hat{h}_t=R_d\sin\hat{\theta}_d+R_d/2R_e+h_a \tag{15}$$

where R_d and h_a are the target range and the antenna height, respectively. R_e is the equivalent effective radius of the earth. Generally, $R_e=4R_0/3$, where $R_0=6370$ km denotes the real radius of the Earth.

The proposed low-angle target altitude finding algorithm can be summarized in Algorithm 1.

Algorithm 1: Pseudocode for the proposed low-angle target altitude finding algorithm

Input: Array measurement $\mathbf{S}$, initial perturbation matrix $\boldsymbol{\Gamma}_0$, maximum number of iterations K, k=0.

Steps:

1. Obtain the signal subspace vector $\mathbf{y}$:
   $$\mathbf{y}=\Theta_{\delta}(\mathbf{S})\mathbf{V1}$$
2. **while** $k\le K$ **do**
3. sparse coding:
   $$\begin{aligned}\hat{\boldsymbol{\gamma}}_k=\arg\min_{\boldsymbol{\gamma}}&\left\|\mathbf{y}-\mathbf{B}(\boldsymbol{\Gamma})\boldsymbol{\gamma}\right\|_2^2+\mu\|\boldsymbol{\gamma}\|_1\\ \text{s.t.}\quad &\boldsymbol{\Gamma}=\boldsymbol{\Gamma}_k\end{aligned}$$
4. dictionary updating:
   $$\begin{aligned}\hat{\boldsymbol{\Gamma}}_k=\arg\min_{\boldsymbol{\Gamma}}&\left\|\mathbf{y}-\mathbf{B}(\boldsymbol{\Gamma})\boldsymbol{\gamma}\right\|_2^2+\mu\|\boldsymbol{\gamma}\|_1\\ \text{s.t.}\quad &\boldsymbol{\gamma}=\hat{\boldsymbol{\gamma}}_k\end{aligned}$$
5. k=k+1
6. **end while**
7. Calculate the target altitude by (14) and (15).

Output: Estimated target altitude $\hat{h}_t$.

4. SIMULATION RESULTS AND MEASURED DATA VALIDATION

4.1. Simulation results

In this section, an ULA consisting of M=16 isotropic elements spaced half a wavelength apart is utilized, the wavelength is $\lambda=1$m. Some comparisons are made with the SSMUSIC algor-

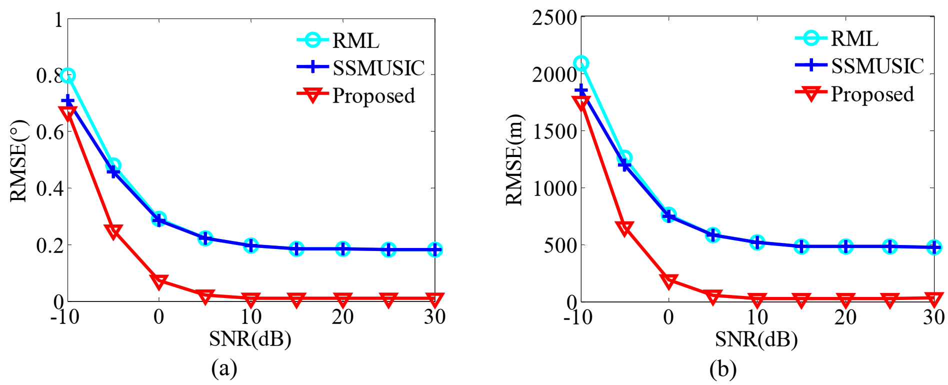

Fig.2. RMSE estimation against SNR for h_t=5km. (a) target angle, (b) target altitude.

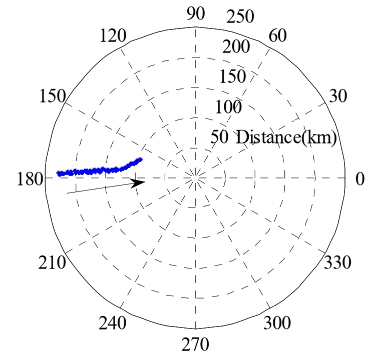

Fig.3. Navigational trace map.

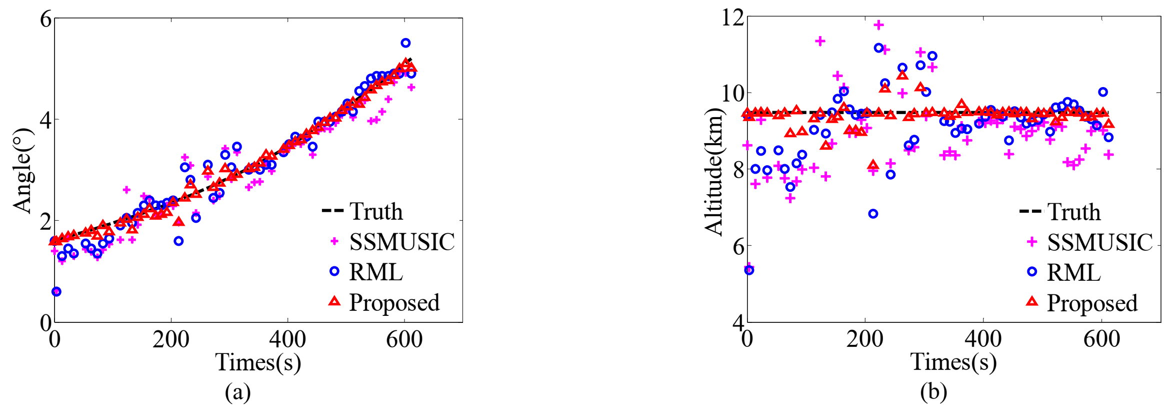

Fig.4. Results for measured data. (a) Comparison of target angle, (b) Comparison of target altitude.

ithm [5] and the general RML algorithm [8]. Assume the real perturbational coefficients $\{\beta_m\}_{m=1}^{M}$ is defined as $\beta_m = \Delta\beta_{mg}\exp\left(j\pi\Delta\beta_{mp}/180^\circ\right)$ with $\Delta\beta_{mg} \sim U(-0.2, 0.2)$ and $\Delta\beta_{mp} \sim U(-10, 10)$, $U(\cdot)$ denotes the uniform distribution. The radar height is $h_a = 100$ m, the target is located at $R_d = 150$ km with an altitude of $h_t = 5000$ m. The vertical distance and the included angle are $h_t' = 4500$ m and $\alpha = 3°$, respectively. Fig. 2 shows the RMSEs of θ_d and h_t via 500 independent Monte Carlo trials for each SNR.

It is clear from Fig. 2 that the performances of the SSMUSIC algorithm and the RML algorithm are heavily decreased. This is mainly because that the assumed highly deterministic signal model produces mismatch in the complex terrain. Particularly, the proposed algorithm is able to distinguish the real target from its multipath under this condition.

4.2. Measured data validation

We use a real data that is measured by an experimental meter-wave array radar located at a hilly terrain environment to verify the performance of the proposed algorithm in this subsection. The number of sensors of the radar is eighteen. The flight path of the target relative to the radar is shown in Fig. 3. Fig. 4 shows the results of the measured data. The dashed line is the real target angle or height recorded by the global positioning system (GPS) located on the target. It can be observed that the results of the SSMUSIC algorithm and the RML algorithm have large estimation error, especially in the low-angle condition, while the proposed method can provide relatively accurate estimations of the target angle and the target altitude. These results indicate that our algorithm performs better for low-angle target altitude measurement in the complex terrain environment.

5. CONCLUSIONS

In this paper, we have addressed a novel low-angle target altitude measurement algorithm for meter-wave radar under complex terrain. The compressive sensing framework, combined with the alternative optimization and dictionary updating techniques, is adopted in our algorithm to mitigate the influence of the perturbed multipath. With a perturbation matrix being introduced, the dictionary atoms can be updated step-by-step by dictionary updating technique to estimate the target altitude more precisely. Numerical results based on both simulated data and real data demonstrate the efficiency of our proposed algorithm under complex multipath environment.

6. REFERENCES

[1] A. V. Mrstik and P. G. Smith, "Multipath limitations on low-angle radar tracking," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 14, No.1, pp. 85–102, Jan. 1978.

[2] W. P. Balance and A. G. Jaffer, "Direction finding in the presence of fully correlated specular multipath," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.* (*ICASSP*), New York, NY, USA, Apr. 1988, pp. 2849–2852.

[3] M. H. S. Sajjadiehy and A. Asif, "Joint time reversal and compressive sensing based localization algorithms for multiple-input multiple-output radars," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.* (*ICASSP*), Brisbane, QLD, Australia, Apr. 2015, pp. 2354–2358.

[4] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propagat.*, vol. 34, no. 3, pp. 276–280, Mar. 1986.

[5] S. U. Pillai and B. H. Kwon, "Forward/backward spatial smoothing techniques for coherent signal identification," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 37, no. 1, pp. 8–15, Jan. 1989.

[6] G. H. Zhao and G. M. Shi, "Altitude measurement of low elevation target based on iterative subspace projection," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 47, no. 2, pp. 1256–1264, Apr. 2011.

[7] W Balance and A. G. Jaffer, "Low-angle direction finding based on maximum likelihood: a unification," in *Proc. Asilomar Conf. Signals, Syst., Comput.*, Pacific Grove, CA, USA, Nov. 1987, pp. 2–4.

[8] E. Bosse, R. M. Turner, and M. Lecours, "Tracking swerling fluctuating targets at low altitude over the sea," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 27, no. 5, pp. 806–822, Sep. 1991.

[9] M. Djeddou, A. Belouchrani, and S. Aouada, "Maximum likelihood angle-frequency estimation in partially known correlated noise for low-elevation targets," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 3057–3064, Aug. 2005.

[10] L. S. Biradar and V. U. Reddy, "A newton-based ziskind-wax alternating projection algorithm applied to spectral estimation," *IEEE Trans. Signal Process.*, vol. 41, no. 3, pp. 1435–1438, Sep. 1993.

[11] T. Lo and J. Litva, "Use of a highly deterministic multipath signal model in low-angle tracking," *IEE Proc. F-Radar Signal Process.*, vol.138, no. 2, pp. 163–171, Apr. 1991.

[12] J. Li, "Improved angular resolution for spatial smoothing techniques," *IEEE Trans. Signal Process.*, vol. 40, no. 12, pp. 3078–3081, Dec. 1992.

[13] R. G. Baraniuk, "Compressive sensing," *IEEE Signal Process. Mag.*, vol. 24, no. 4, pp. 118–121, Jul. 2007.

[14] D. L. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289–1306, Apr. 2006.

[15] M. Cetin, D. M. Malioutov and A. S. Willsky, "Avariational technique for source localization based on a sparse signal reconstruction perspective," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.* (*ICASSP*), Orlando, FL, USA, May. 2002, pp. 2965–2968.

[16] E. Candes and T. Tao, "Decoding by linear programming," *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4203-4215, 2005.

[17] M. Grant and S. Boyd, CVX: Matlab software for disciplined convex programming. ver. 2.1, Mar. 2014.

[18] Y. Wang, J. Yang, W. Yin and Y. Zhang, "A new alternating minimization algorithm for total variation image reconstruction," *SIAM J. Imaging. Sci.*, vol. 1, no. 3, pp. 248–272, Jul. 2008.

[19] A. Maleki, L. Anitori, Z. Yang and R. G. Baraniuk, "Asymptotic analysis of complex LASSO via complex approximate message passing (CAMP)," *IEEE Trans. Inf. Theory*, vol. 59, no. 7, pp. 4290–4308, Jul. 2013.

[20] X. X. Zhu and R. Bamler, "Tomographic SAR inversion by L_1 norm regularization – the compressive sensing approach," *IEEE Trans. Geosci. Remote Sens.*, vol. 48, no. 10, pp. 3839–3846, Oct. 2010.

[21] D. Malioutov, M. Cetin and A. S. Willsky "A sparse signal reconstruction perspective for source localization with sensor arrays," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 3010–3022, Aug. 2005.