A CAPITALIST SCHEME FOR ENERGY MANAGEMENT IN INFERENTIAL SENSOR NETWORKS

Stefano Marano

DIEM
University of Salerno, Fisciano (SA), Italy
marano@unisa.it

*Peter Willett**

ECE Department
University of Connecticut, Storrs, CT, USA
peter.willett@uconn.edu

ABSTRACT

Suppose that the energy made available to a sensor network at the beginning of a time slot is proportional to the success of the network inferential task during the previous slot. And further, assume that such energy is to be apportioned to charge the individual sensors, such that the more energy one sensor receives, the better it does its job. Then, the information gathered by the network in the long run consequently obeys a multiplicative rule, which enables us to adapt some results from portfolio theory to design the optimal apportionment. Two regimes emerge, one in which the expected value of the long-run information is key and all the energy is assigned to the "best" sensor, and another – more tricky – where the expected *logarithm* of the long-run information matters, and the solution is given by Cover's log-optimal apportionment.

Index Terms— Wireless sensor networks, energy harvesting sensors, Cover's log-optimal portfolio.

1. INTRODUCTION

One of the main bottlenecks of inferential sensor networks is the energy constraint [1–4], which motivates a mounting interest in networks with energy-harvesting sensors [5–8]. Sensor networks with *mobile agents* provide several advantages with respect to classical architectures where the fusion center is static [9–11]. The idea that the mobile agent, aside from collecting data, could also serve as energy charger for the sensors has been recently proposed, see [12, 13] and references therein. Hierarchical structures in which the system is composed of several sub-networks are also very popular for several advantageous characteristics [14].

Accordingly, we consider an inference-making system consisting of several sub-networks to which the energy is supplied externally by a supervisor at regular intervals of time. Using this energy, in each time slot each sensor network performs its inferential task and communicates the result of the inference to the supervisor, in the form of some *information*; we assume that the amount of information gathered by the sensor network can be precisely measured[1]. Then it makes perfect sense that, at the beginning of the $(n+1)$-th time slot, the supervisor rewards the sensor network with an amount of energy E_{n+1} proportional to the amount of information I_n gathered by the sensor network in the previous slot. This way, the more the sensor network performs efficiently its inference task (i.e., the larger is I_n), the more energy will be made available to it in the next time slot (the larger will be E_{n+1}), which provides a virtuous mechanism for a parsimonious resource allocation, allowing competition[2] among different sensor networks.

Each sub-network is a sensor network with a mobile agent, made of remote sensors tasked with inference whose output is a measurable quantity. The task requires energy that must be supplied (maybe wirelessly) by the agent, which is in turn charged with E_n unit of energy by the supervisor at the beginning of the n-th time slot. Thus, for the sensor network there is a total energy E_n to be spent in the n-th time slot, which is apportioned among the S sensors of the sensor network by the mobile agent. The more energy the agent apportions to sensor s, the more efficient is the sensor inferential task; but the amount of information that the sensor gathers is also related to the actual operational characteristic (e.g. noise/disturbance levels), modeled as a random quantity, that it experiences during the time slot. Therefore, the amount $i_n(s)$ of information gathered by sensor s during the n-th time slot is proportional to the energy $e_n(s)$ provided to it, multiplied by a random variable $X_n(s)$, which characterizes statistically the sensor.

Information is additive: at time slot n the sensor network made of S sensors gathers $I_n = \sum_{s=1}^{S} i_n(s)$ units of information and therefore $E_{n+1} \propto I_n$ is the energy at disposal of the sensor network for the time slot $n+1$. We ask: How the agent should allocate the energy E_{n+1} among the S sensors of the network? What fraction of the energy should be supplied to what sensor?

Winner-take-all is one possibility: The best sensor, – say, the one for which $\mathbb{E}[X_n(s)]$ is maximum – is recharged with E_{n+1} units of energy and nothing is left for the others. This is an *aggressive* energy apportionment strategy, but is this extremely meritocratic approach giving no chances to all but one sensor the best? One can argue that if the runner-up sensor is, on the average, only infinitesimally less good than the best, some fraction of E_{n+1} should be granted to it: statistical fluctuations of $X_n(s)$ may make the runner-up better than the best "in-expectation". At the other extreme, the fully *performance-unaware* approach is to

*P. Willett was supported by NPS via ONR contract N00244-16-1-0017.

[1] . . . as per Shannon.

[2] Rather brutal capitalism.

allocate $1/S$-th of E_{n+1} to each of the S sensors. Should one ignore at all the differences (if any) among sensors? It makes little sense. And, of course, between these extremes, plenty of other approaches could be conceived.

The theme of this work is to provide rigorous answers to the described energy allocation problem, which also reveals analogies with Cover's portfolio optimization [15, Chap. 16].

2. FORMALIZATION AND MAIN RESULTS

Let S be the number of sensors in the considered sensor network. Let $I_n = \sum_{s=1}^{S} i_n(s)$ be the total information gathered by the sensor network, and $i_n(s)$ the information gathered by sensor s thanks to the $e_n(s)$ units of energy that have been provided to it at the beginning of time slot n. Suppose that $i_n(s) = a\, e_n(s) X_n(s)$, where a is a positive dimensional constant (bits/Joule, assuming that information is measured in bits, and energy in Joule), and where $X_n(s)$ is a real-valued dimensionless non-negative random variable. Let $\mathbf{X}_n \triangleq [X_n(1), \ldots, X_n(S)]$. The vectors $\mathbf{X}_1, \mathbf{X}_2, \ldots$ are IID (independent and identically distributed) with common CDF (cumulative distribution function) $F(\mathbf{x})$. This CDF characterizes the S sensors (note that we are not assuming statistical independence among these sensors), and is assumed known and constant. The case of time-dependent distributions is not covered by our analysis.

To execute its n-th inferential task, sensor s is charged with an amount of energy given by $e_n(s) = p_n(s)\, E_n$, where $p_n(s)$ is the fraction of the total energy E_n transferred from the agent to node s, at the beginning of the n-th time slot. The vector $\mathbf{p}_n \triangleq [p_n(1), \ldots, p_n(S)]$ verifies

$$p_n(s) \geq 0, \qquad \sum_{s=1}^{S} p_n(s) = 1. \tag{1}$$

The class of all such vectors will be denoted by $\mathcal{P}$. Finally, $E_n = b\, I_{n-1}$ – the total energy – is proportional to the information I_{n-1} gathered by the sensor network during stage $n-1$. Here b is a positive (say, in Joule/bits) constant.

With no loss of generality, let $ab = 1$ and $I_0 = 1$. We have

$$\begin{aligned} I_n &= a \sum_{s=1}^{S} e_n(s) X_n(s) \\ &= E_n\, a \sum_{s=1}^{S} p_n(s) X_n(s) = I_{n-1} \sum_{s=1}^{S} p_n(s) X_n(s) \\ &= I_{n-1}\, R_n(\mathbf{p}_n) = \prod_{j=1}^{n} R_j(\mathbf{p}_j), \end{aligned} \tag{2}$$

where $R_n(\mathbf{p}_n) \triangleq \sum_{s=1}^{S} p_n(s) X_n(s) = \mathbf{p}_n \cdot \mathbf{X}_n$. Our goal is to find the best sequence $\mathbf{p}_1, \ldots, \mathbf{p}_n$, maximizing the long-run ($N \to \infty$) total information

$$J_N(\mathbf{p}_{1:N}) \triangleq \sum_{n=1}^{N} \prod_{j=1}^{n} R_j(\mathbf{p}_j) \tag{3}$$

gathered by the system. Since $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n$ are IID, we limit the analysis to sequences of constant energy apportionments, i.e., $\mathbf{p}_1 = \mathbf{p}_2 \cdots = \mathbf{p}_n$, which entails no loss of generality, see [16].

Accordingly, for notational convenience, we will often omit the subscripts n, and simply write, e.g., $R(\mathbf{p}) = \mathbf{p} \cdot \mathbf{X}$. Let us introduce the following definitions

$$\eta(\mathbf{p}) \triangleq \mathbb{E}\left[\frac{1}{R(\mathbf{p})}\right], \tag{4}$$

$$\mu_{\max} \triangleq \max_{s=1,\ldots,S} \mathbb{E}[X(s)], \tag{5}$$

and two apportionment strategies that deserve special attention:

$$\mathbf{p}^* = \arg\max_{\mathbf{p}\in\mathcal{P}} \mathbb{E}[\log R(\mathbf{p})], \tag{6}$$

$$\bar{\mathbf{p}} = \arg\max_{\mathbf{p}\in\mathcal{P}} \mathbb{E}[R(\mathbf{p})]. \tag{7}$$

Note that $\mathbf{p}^*$ is Cover's log-optimal portfolio for the multiplicative wealth model [15]. The main result is now stated.

THEOREM
(a) Suppose $\mathbb{E}[\log R(\mathbf{p})] > 0$. *Then:*

$$\lim_{N\to\infty} \frac{1}{N} \log J_N(\mathbf{p}) = \mathbb{E}[\log R(\mathbf{p})], \tag{8}$$

and the limit is maximized by $\mathbf{p}^*$. *If* $\eta(\mathbf{p}) < 1$ *the convergence in (8) is a.e.; if* $\eta(\mathbf{p}) \geq 1$ *(and* $\mathbb{V}[\log R(\mathbf{p})]$ *is finite and nonzero) the convergence is in probability.*
(b) Suppose $\mathbb{E}[\log R(\mathbf{p})] \leq 0$. *Then:*

$$\lim_{N\to\infty} J_N(\mathbf{p}) = J_\infty(\mathbf{p}), \qquad \text{a.e.} \tag{9}$$

If $\mu_{\max} < 1$, *the random variable* $J_\infty(\mathbf{p})$ *is finite a.e., and its expectation is maximized by* $\bar{\mathbf{p}}$:

$$\mathbb{E}[J_\infty(\mathbf{p})] \leq \mathbb{E}[J_\infty(\bar{\mathbf{p}})] = \frac{\mu_{\max}}{1-\mu_{\max}}.$$

If $\mu_{\max} \geq 1$, *the random variable* $J_\infty(\mathbf{p})$ *is not necessarily finite a.e.* □

PROOF Space limitations prevent us from providing the detailed proof. The basic idea behind part (a) with $\eta(\mathbf{p}) < 1$ is that each summand of $\sum_{n=1}^{N} \prod_{j=1}^{n} R_j(\mathbf{p}_j)$ grows exponentially and therefore the sum is dominated by the last term $\prod_{j=1}^{N} R_j(\mathbf{p}_j)$. Then, the result follows straightforwardly by the strong law of large numbers. The case $\eta(\mathbf{p}) \geq 1$ exploits the fact that $J_\infty = \sum_{n=1}^{\infty} \prod_{j=1}^{n} R_j(\mathbf{p}_j)$ has the same distribution of $R'(J_\infty + 1)$, where R' is an independent copy of the random variable R_j. This allows us to exploit known results on the theory of *perpetuities* [17, 18] about the convergence in distribution of a normalized version of J_∞, which is then manipulated by probability tools to obtain the final claim. Similar tools (results on perpetuity random process combined with asymptotic probability tools) apply to part (b). The detailed complete proof is given in [16]. □

The above theorem states that if $\mathbb{E}[\log R(\mathbf{p}^*)] > 0$, then there exist energy allocation vectors $\mathbf{p} \in \mathcal{P}$ such that

$$J_N(\mathbf{p}) \approx \exp\{N\, \mathbb{E}[\log R(\mathbf{p})]\}, \tag{10}$$

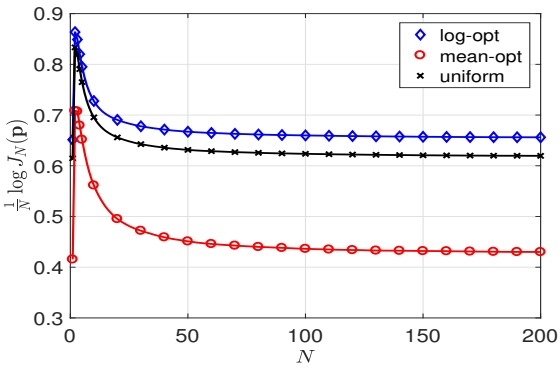

Fig. 1. Exponential rate of information versus the number N of time slots, for the first experiment. Cover's log-optimal portfolio gives the best energy apportionment. The winner-take-all policy (mean-opt) is much worse than the log-optimal, and even worse than the uniform apportionment.

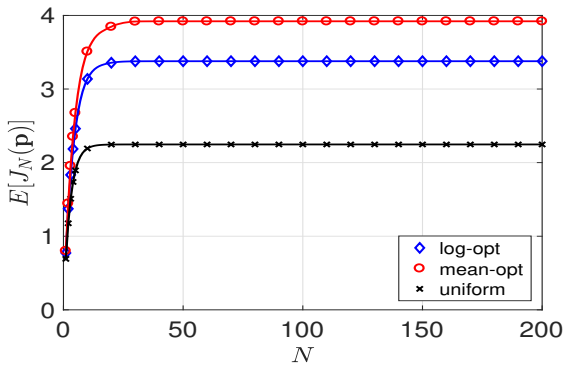

Fig. 2. Expected value of the information $J_N(\mathbf{p})$ versus the number N of time slots, for the second experiment. In this case the winner-take-all energy apportionment (mean-opt) achieves the best performance.

namely, $J_N(\mathbf{p})$ grows exponentially with rate $\mathbb{E}[\log R(\mathbf{p})]$, which is maximized by $\mathbf{p}^*$. Conversely, if $\mathbb{E}[\log R(\mathbf{p}^*)] \leq 0$, then there exists no energy allocation vector $\mathbf{p} \in \mathcal{P}$ such that $J_N(\mathbf{p})$ grows exponentially. Still, $J_N(\mathbf{p})$ converges a.e. to a limiting random variable $J_\infty(\mathbf{p})$, which, if $\mu_{\max} < 1$, has finite expectation

$$\mathbb{E}[J_\infty(\mathbf{p})] \leq \frac{\mu_{\max}}{1-\mu_{\max}} \tag{11}$$

The energy apportionment $\bar{\mathbf{p}}$ achieves the maximum in this case.

3. COMPUTER EXPERIMENTS

The applications of the theorem presented in the previous section requires the computation of the vectors defined in (6) and (7). Computing the latter is straightforward. The former in general cannot be found in closed form, but the following algorithm provides a simple and accurate numerical solution.

Algorithm 1: Pseudocode to compute $\mathbf{p}^*$

$k \leftarrow 0$; $\quad p^{(0)}(s) \leftarrow 1/S, \Delta^{(k)}(s) = \infty, \quad s = 1, \ldots, S$;
while $\max_s \log \Delta^{(k)}(s) \geq \epsilon$ **do**
  $\Delta^{(k)}(s) \leftarrow \left\langle \frac{X(s)}{R(\mathbf{p}^{(k)})} \right\rangle$, where $\langle\cdot\rangle$ denotes a numerical approximation of the expectation (e.g., via Monte Carlo);
  $p^{(k+1)}(s) \longleftarrow p^{(k)}(s)\,\Delta^{(k)}(s), \quad s = 1, \ldots, S$;
  $k \longleftarrow k+1$;

The rationale of the algorithm is explained in [19], where it is shown that, given $\epsilon > 0$ as input, if the empirical average $\langle\cdot\rangle$ would be replaced by the true expectation $\mathbb{E}[\cdot]$, then the apportionment vector in output, say $\mathbf{p}^{(\underline{k})}$, has the property that $\mathbb{E}[\log R(\mathbf{p}^{(\underline{k})})]$ is ϵ-close to the true $\mathbb{E}[\log R(\mathbf{p}^*)]$.

We now illustrate the power of the previous theorem by computer experiments, in which the long-run behavior of the information gathered by the sensor network is simulated. For comparison purposes, aside from the best energy apportionment vector predicted by the theorem, we also show the case of the other allocation shown in (6)-(7), and the case in which energy is "blindly" distributed uniformly to the sensors. In the latter case the allocation vector is denoted by $\mathbf{u} \triangleq [1/S, \ldots, 1/S]$. In the experiments we consider a sensor network made of $S = 6$ sensors and assume that $X_n(s)$ are independent (but not identically-distributed) random variables each having the Gamma distribution:

$$f_X(x) = \frac{1}{\theta_S^{\xi_s}\Gamma(\xi_s)} x^{\xi_s - 1} e^{-x/\theta_s},$$

for $x \geq 0$. Different sensors are characterized by different parameters (ξ_s, θ_s).

The parameters used in the first computer experiment are shown in Table 1, where are also shown the corresponding energy allocation vectors $\mathbf{p}^*$ (computed by the aforementioned algorithm) and $\bar{\mathbf{p}}$. Note that $\bar{\mathbf{p}}$ is a *degenerate* apportionment vector, assigning all the energy to the first sensor. This is due to the fact that the expected value $\mathbb{E}[X_n(s)] = \xi_s\theta_s$ of the Gamma random variable attains its maximum only at the first sensor.

s	ξ_s	θ_s	$p^*(s)$	$\bar{p}(s)$
1	2	1	.137	1
2	3.9	.5	.172	0
3	4	.498	.260	0
4	4	.495	.234	0
5	4	.49	.197	0
6	5	.3	0	0

Table 1. Parameters of the first computer experiment.

We find $\mathbb{E}[\log R(\mathbf{p}^*)] \approx 0.653$ and $\eta(\mathbf{p}^*) \approx 0.537$. According to the results of the theorem [part (a), $\eta(\mathbf{p}) < 1$], we know that the best option is the energy apportionment $\mathbf{p}^*$, and that all the realizations of the random process $\frac{1}{N}\log J_N(\mathbf{p}^*)$ approach the *deterministic* limit $\mathbb{E}[\log R(\mathbf{p}^*)]$. The results of computer experiments shown in Fig. 1 corroborate the theoretical results. The figure depicts $\frac{1}{N}\log J_N(\mathbf{p})$, with $\mathbf{p} = \mathbf{p}^*$ (referred to as the

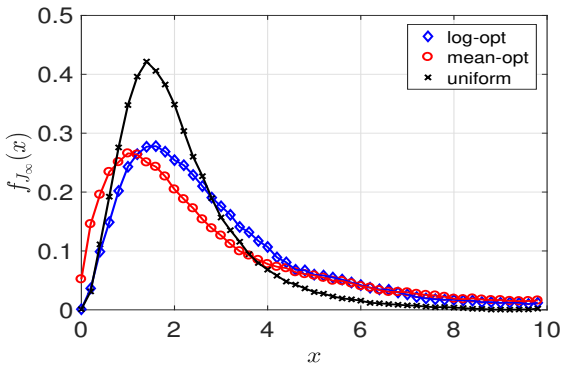

Fig. 3. Second experiment. Probability density functions $f_{J_\infty}(x)$ of the limiting random variable $J_\infty(\mathbf{p})$, for the three energy apportionment policies $\mathbf{p} = \mathbf{p}^*, \bar{\mathbf{p}}, \mathbf{u}$.

log-optimal apportionment vector), $\mathbf{p} = \bar{\mathbf{p}}$ (mean-optimal), and $\mathbf{p} = \mathbf{u}$ (uniform). The shown curves are obtained by averaging 5000 Monte Carlo realizations of $\frac{1}{N} \log J_N(\mathbf{p})$ for different values of N.

The parameters of the Gamma distribution for the second experiment are shown in Table 2. Now we have $\mathbb{E}[\log R(\mathbf{p}^*)] \approx -0.385 < 0$ and $\mu_{\max} = 0.8 < 1$. According to the theorem we know that information does *not* grow exponentially. Instead, it is the quantity $J_N(\bar{\mathbf{p}})$ that approaches an a.e. limit in the long run. Then, the best energy allocation is $\bar{\mathbf{p}}$, for which the expected value of the long-run limit is maximum. These theoretical predictions are confirmed by the computer experiments (obtained again by 5000 Monte Carlo runs) reported in Fig. 2.

s	ξ_s	θ_s	$p^*(s)$	$\bar{p}(s)$
1	1	.5	0	0
2	1	.6	.016	0
3	1	.7	.145	0
4	1	.75	.213	0
5	1	.8	.279	.444
6	1.25	.64	.347	.556

Table 2. Parameters of the second computer experiment.

It is worth noting that $\bar{\mathbf{p}}$ is not degenerate. Indeed, there are two sensors ($s = 5, 6$) for which $\mathbb{E}[X_n(s)] = \xi_s \theta_s$ attains its maximum $\mu_{\max} = 0.8$. In this case, all the energy must be supplied to these two sensors, and we choose to allocate the energy in a way inversely proportional to the variances $\mathbb{V}[X_n(s)] = \xi_s \theta_s^2$, $s = 5, 6$, of the random variables that characterize those sensors. This choice ensures that, aside from maximizing $\mathbb{E}[J_\infty(\mathbf{p})]$, the apportionment vector $\bar{\mathbf{p}}$ also minimizes the variance $\mathbb{V}[J_\infty(\mathbf{p})]$, see [16] for details.

Recall that $\lim_{N\to\infty} J_N(\mathbf{p})$ is now a finite random variable, not a deterministic value. Accordingly, Fig. 3 shows the estimated probability density functions for $J_N(\mathbf{p})$, $N = 200$, corresponding to the three different choices of the energy apportionment.

4. CONCLUSIONS

Designing an efficient energy management protocol for practical inferential sensor networks is a very interdisciplinary and intricately multifaceted problem. Here we adopt an admittedly highly-idealized model that neglects many important factors, two of which are worth mentioning: the specific (wireless, presumably) energy charging mechanism between the mobile agent and the sensors, and the specific information gathering procedure employed by the nodes to make inference. Nevertheless, the consequent level of abstraction allows us to emphasize some key signal processing aspects of the problem, which is our goal.

The adopted model is basically founded on the following general assumptions: (i) the information gathered by the sensors can be precisely measured and quantified, and is additive; (ii) the amount of such information is somehow proportional to the amount of charge received by the sensor; and (iii) at time slot $(n+1)$ the whole network is awarded with a total energy (to be entirely apportioned among the sensors) proportional to the total information gathered in the n-th time slot.

This last assumption is the "capitalist" one: it is assumed that there are many sensor networks and that each is rewarded by its success. And, perhaps arguably when discussed over coffee: the game is not a "zero-sum" pie-slicing exercise. The pie can (in proper conditions) grow exponentially.

The total information gathered by the network can be modeled as a random process, known as perpetuity, whose asymptotic analysis (long-run behavior) is conducted. In one regime the total information grows without bound at an exponential rate, and so does the total energy required (again, we pay no attention to impose upper bounds on these quantities, which certainly will occur soon or later in a practical system). The growth rate is deterministic and the system optimization goal is obvious: make such rate as large as possible. In another regime the perpetuity converges to a random limit. We want to maximize this limit but here there may be debates: what is the maximum of a random quantity? We limit ourselves to the simplest approach of maximizing the statistical average (and in some cases minimizing the variance).

In a sense, the latter regime leads to a kind of obvious policy: energy should be apportioned to the sensors that work better, in expectation. Is such an energy management policy so obvious? The former regime reveals that this is not the case. There are situations in which the optimal energy apportionment is more tricky, departs significantly from the "best-in-expectation" rule, and is much less intuitive. This is not moot: Numerical experiments show that the use of the appropriate energy apportionment is critical to ensure the best mode of operation of the network, and the losses otherwise incurred can be remarkable.

Because of the high idealization level, we do not provide examples of specific existing systems that *exactly* fit our model. For the same reason, however, the model may be adapted to rather different scenarios. For instance, we believe that some smart grid applications can be cast in the developed framework, provided that the concept of information is replaced by some measure of efficiency of the users' energy usage.

5. REFERENCES

[1] J. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "A survey on sensor networks," *IEEE Commun. Mag.*, vol. 40, no. 8, pp. 102–114, Aug. 2002.

[2] B. M. Sadler, "Fundamentals of energy-constrained sensor network systems," *IEEE Aerosp. Electron. Syst. Mag.*, vol. 20, no. 8, Aug. 2005.

[3] P. Addesso, S. Marano, and V. Matta, "Sequential sampling in sensor networks for detection with censoring nodes," *IEEE Trans. Signal Process.*, vol. 55, no. 11, pp. 5497–5505, Nov. 2007.

[4] P. Braca, S. Marano, and V. Matta, "Single-transmission distributed detection via order statistics," *IEEE Trans. Signal Process.*, vol. 60, no. 4, pp. 2042–2048, Apr. 2012.

[5] S. Basagni, M. Conti, S. Giordano, and I. Stojmenovic, *Wireless Sensor Networks with Energy Harvesting*, 2nd ed., Mobile Ad Hoc Networking: Cutting Edge Directions, Ed. John Wiley & Sons, 2013.

[6] F. Shaikh and S. Zeadally, "Energy harvesting in wireless sensor networks: A comprehensive review," *Renewable and Sustainable Energy Reviews*, vol. 55, pp. 1041–1054, Mar. 2016.

[7] S. Sudevalayam and P. Kulkarni, "Energy harvesting sensor nodes: Survey and implications," *IEEE Communications Surveys and Tutorials*, vol. 13, no. 3, pp. 443–461, 2011.

[8] A. Kansal, J. Hsu, S. Zahedi, and M. Srivastava, "Power management in energy harvesting sensor networks," *ACM Transactions on Embedded Computing Systems (TECS)*, vol. 6, no. 4, pp. 1–38, 2007.

[9] L. Tong, Q. Zhao, and S. Adireddy, "Sensor networks with mobile agents," in *Proceedings of MILCOM 2003*, vol. 1, Boston MA, Oct. 2003, pp. 688–693.

[10] S. Marano, V. Matta, P. Willett, and L. Tong, "DOA estimation via a network of dumb sensors under the SENMA paradigm," *IEEE Signal Process. Lett.*, vol. 12, no. 10, pp. 709–712, Oct. 2005.

[11] S. Marano, V. Matta, and L. Tong, "Secrecy in cooperative SENMA with unauthorized intrusions," in *Proceedings of IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC 2006)*, Cannes, France, July 2-5 2006.

[12] Y. Shu, P. Cheng, Y. Gu, J. Chen, and T. He, "Minimizing communication delay in RFID-based wireless rechargeable sensor networks," in *Proc. of Aleventh Annual IEEE International Conference on Sensing, Communication, and Networking (SECON)*, Singapore, June 30- July 3 2014, pp. 441–449.

[13] Y. Shu, H. Yousefi, P. Cheng, J. Chen, Y. J. Gu, T. He, and K. G. Shin, "Near-optimal velocity control for mobile charging in wireless rechargeable sensor networks," *IEEE Trans. on Mobile Computing*, vol. 15, no. 7, pp. 1699–1713, Jul. 2016.

[14] F. Hu and Q. Hao, Eds., *Intelligent Sensor Networks*. Boca Raton, Florida, USA: CRC Press, Taylor & Francis Group, 2013.

[15] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. New Jersey, USA: Wiley-Interscience, 2006.

[16] S. Marano and P. Willett, "Resource allocation in energy-harvesting sensor networks," *IEEE Trans. Signal and Info. Process. over Networks*, to appear. DOI: 10.1109/TSIPN.2018.2800645.

[17] C. M. Goldie and R. Grubel, "Perpetuities with thin tails," *Advances in Applied Probability*, vol. 28, no. 2, pp. 463–480, Jun. 1996.

[18] P. Hitczenko and J. Wesolowski, "Renorming divergent perpetuities," *Bernoulli*, vol. 17, no. 3, pp. 880–894, 2011.

[19] T. M. Cover, "An algorithm for maximizing expected log investment return," *IEEE Trans. Inf. Theory*, vol. 30, no. 2, pp. 369–373, Mar. 1984.