AN EVENT-TRIGGERED AVERAGE CONSENSUS ALGORITHM WITH PERFORMANCE GUARANTEES FOR DISTRIBUTED SENSOR NETWORKS

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ABSTRACT

This paper proposes a distributed guaranteed-performance event-triggered average consensus (GP-ETAC) algorithm for multi-agent/sensor networks. The proposed GP-ETAC approach is distributed and event-triggered in the sense that the agents selectively limit their transmissions to local neighbourhoods when certain triggering conditions are satisfied. Using the Lyapunov stability theorem, a novel cost function is optimized to compute consensus design parameters (namely, the overall control gain and local event-triggering thresholds). The proposed cost function provides a structured trade-off between the number of local transmissions and the rate of consensus convergence. The performance of the GP-ETAC approach is evaluated through Monte-Carlo simulations.

Index Terms–Average consensus, Distributed networks, Event-triggered communication, Guaranteed performance.

1. INTRODUCTION

Consensus algorithms have been studied extensively in distributed signal processing applications [1, 2]. A vast majority of consensus techniques require continuous participation of all nodes constituting the multi-agent network with constant communications between the nodes to achieve consensus on a set of pre-defined parameters [3]. To preserve energy and prolong the life of the nodes, several strategies [4–7] have lately been developed to reduce the number of information transfers between nodes. These strategies can be classified into periodic communication (time-triggered) approaches [8, 9] for first-order integrators and event-triggered approaches [10]. The later offer additional savings in the number of consensus transmissions and is the focus of this paper.

Existing event-triggered, average consensus approaches [11–13] focus on the selection of trigger functions that ensure the stability of the consensus framework and prevent the Zeno behaviour. In practice, however, it is highly desirable to include a performance guarantee on the convergence rate of the consensus algorithm. To this end, we propose a guaranteed-performance event-triggered average consensus (GP-ETAC) algorithm that confirms a minimum rate for consensus convergence. This performance specification is incorporated in the design of the consensus framework by utilizing the Lyapunov stability theorem [14–17]. The optimal design parameters satisfying the predefined performance objectives are then obtained by solving a linear matrix inequality (LMI) optimization problem [18–20].

Although, the Lyapunov-based consensus approaches have attracted much attention lately in control, application of such approaches to optimal event-triggered average consensus is still very much in its infancy. The paper addresses this gap with a focus on developing a Lyapunov-based GP-ETAC algorithm for distributed signal processing applications in multi-agent/sensor networks. We extend the event-triggered consensus framework developed in our previous work [6] by incorporating a communication constraint and the rate of consensus convergence in a novel cost function used to compute the consensus design parameters, namely the local event-triggering thresholds and common control gain. Such an approach provides a flexible trade-off between the consensus convergence rate and number of local nodal communications.

The paper is organized as follows. Section 2 defines the notation used in the paper and introduces the problem statement. In Section 3, we develop the GP-ETAC algorithm used to compute the consensus design parameters. Simulation results are included in Section 4. Section 5 concludes the paper.

2. PRELIMINARIES

Notation: We use the following notation throughout the paper. Matrix I is the identity matrix of the appropriate order; Superscript † denotes Pseudo-inverse of the matrix. For matrix $A = \{a_{ij}\} \in \mathbb{R}^{m \times n}$, matrix $|A|$ is the element-wise absolute values of A; row vector $a_{(i,\bullet)}$ denotes row i of matrix A, i.e., $a_{(i,\bullet)} = [a_{i1}, \ldots, a_{in}]$. $A > 0$ specifies that A is a symmetric positive definite matrix; $\mathrm{Tr}(A)$ is the trace of A; Asterisk $*$ in the lower block triangle of symmetric matrices denotes the transpose of the corresponding block from the upper triangle. For two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ of order n, $\boldsymbol{u} \leq \boldsymbol{v}$ refers to n element-wise inequalities, i.e., $u_i \leq v_i$, for $(1 \leq i \leq n)$.
Graph Theory: Matrix $\mathcal{A} = \{a_{ij}\}_{N \times N}$ is the weighted adjacency matrix for graph $\mathcal{G}$; L is the Laplacian matrix; and $\mathcal{N}_i$ is the neighbouring set for node i. See [21] for more details.

Consider a sensor network system with N nodes that require average consensus on parameter $x(t)$ associated with the system. Prior to the average consensus process, each node has a different estimate of $x(t)$, denoted by $x_i(0)$. Similar to [2], we utilize the first-order integrator multi-agent model given below to reach average consensus on $x_i(t)$

$$\dot{x}_i(t) = u_i(t), \quad (1 \leq i \leq N). \tag{1}$$

The goal of average consensus is to distributively achieve the mean value on $x_i(0)$ at all nodes [2], i.e.,

$$\lim_{t \to \infty} \left| x_i(t) - 1/N \sum_{j=1}^{N} x_j(0) \right| = 0, \quad (1 \leq i \leq N), \tag{2}$$

where t is the consensus time index. We note that the time

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scale for the system state model is different from t. The control signal $u_i(t) \in \mathbb{R}$ is generated from a proposed distributed control law. The agents share their states $x_i(t)$ within their local neighbourhoods through an undirected connected network to reach average consensus. To reduce the amount of transmissions and control updates $u_i(t)$, efficient local event-triggering functions are of great interest. Let $t^i_0, t^i_1, \dots$ denote the time instants at which node i transmits its values under fulfillment of the event-triggering condition $\mathcal{T}_i$. For the interval between two consecutive triggering instants, we denote the most recently transmitted state of node i by $\hat{x}_i(t) \triangleq x_i(t^i_k),\ t \in [t^i_k, t^i_{k+1})$. To reach consensus, the distributed protocol depending only on last transmitted values is proposed

$$u_i(t) = -k \sum_{j \in \mathcal{N}_i} a_{ij} (\, \hat{x}_i(t) - \hat{x}_j(t) \,), \quad (1 \le i \le N), \tag{3}$$

where $k \in \mathbb{R}$ is the control gain parameter to be designed. We note that the control law (3) is a more general protocol for achieving average consensus as compared to the one frequently used in existing literature where $k=1$ [19]. Since the choice of k affects the convergence rate of the algorithm, its design is unavoidable in situations where maintaining a minimum rate of consensus convergence is important. Let $e_i(t) = \hat{x}_i(t) - x_i(t)$ denote the difference between the most recently transmitted state and its instantaneous value for agent i. For further analysis, we define global vectors as $\boldsymbol{x}(t) = [\, x^T_1(t), \dots, x^T_N(t) \,]^T$, $\hat{\boldsymbol{x}}(t) = [\, \hat{x}^T_1(t), \dots, \hat{x}^T_N(t) \,]^T$, $\boldsymbol{u}(t) = [\, u^T_1(t), \dots, u^T_N(t) \,]^T$, and $\boldsymbol{e}(t) = [e^T_1(t), \dots, e^T_N(t)]^T$. It holds that $\boldsymbol{e}(t) = \hat{\boldsymbol{x}}(t) - \boldsymbol{x}(t)$. Combining (3) with (1) leads to the following augmented system

$$\dot{\boldsymbol{x}}(t) = -kL\,(\boldsymbol{x}(t) + \boldsymbol{e}(t))\,. \tag{4}$$

Motivated by the cost function

$$J = \int_0^\infty \boldsymbol{x}^T(t) R \boldsymbol{x}(t) + \boldsymbol{u}^T(t) Q \boldsymbol{u}(t) dt \tag{5}$$

used for parameter design in control theory [22], Definition 1 proposes a modified function to incorporate the convergence rate and local transmission load in the consensus framework.

Definition 1. Let $J = \int_0^\infty (\boldsymbol{x}^T(t) R \boldsymbol{x}(t) + \boldsymbol{e}^T(t) Q \boldsymbol{e}(t)\,) dt$, where R and Q are given positive definite weighting matrices. If there exists a positive scalar J^* such that the value of associated cost J with the event-triggered average consensus in (4) for any initial values $\boldsymbol{x}(0)$ satisfies $J \le J^*$, then J^* is said to be the guaranteed cost for such a process.

In cost function J, matrices R and Q, respectively, assign desired weights to the state trajectories $\boldsymbol{x}(t)$ (to control the convergence rate to the average value) and to the difference vector $\boldsymbol{e}(t)$ (to control the number of transmission events). An objective function (introduced later) is also associated with the optimization framework to derive consensus parameters with optimal norms. Definition 1 is exclusive to our approach.

3. THE PROPOSED GP-ETAC ALGORITHM

To benefit from the Lyapunov stability theorem that incorporates performance guarantees in the design procedure, we convert the consensus problem for system (4) to the stability problem of an equivalent system. Let $\hat{L} \in \mathbb{R}^{(N-1)\times N}$ denote the reduced Laplacian matrix obtained by removing any arbitrary row of L. The following transformation [6] is proposed

$$\boldsymbol{x}_{\mathrm{r}}(t) = \hat{L}\boldsymbol{x}(t). \tag{6}$$

According to [6], the consensus problem for system (4) is equivalent to the stability problem of the system (6). According to [23], average consensus is reached when $\boldsymbol{x}_{\mathrm{r}}(t) = 0$. Without loss of generality, we, therefore, remove row N from L to derive $\hat{L}$ resulting in the following transformed system

$$\dot{\boldsymbol{x}}_{\mathrm{r}}(t) = -k\,\mathbb{L}\,(\,\boldsymbol{x}_{\mathrm{r}}(t) + \boldsymbol{e}_{\mathrm{r}}(t)\,)\,, \tag{7}$$

where $\boldsymbol{e}_{\mathrm{r}}(t) = \hat{L}\boldsymbol{e}(t)$, with $\mathbb{L} = \hat{L}L\hat{L}^\dagger$. It also follows from (6) that $\boldsymbol{e}_{\mathrm{r}}(t) = \hat{\boldsymbol{x}}_{\mathrm{r}}(t) - \boldsymbol{x}_{\mathrm{r}}(t)$, where $\hat{\boldsymbol{x}}_{\mathrm{r}}(t) = \hat{L}\hat{\boldsymbol{x}}(t)$.

3.1. Event-Triggering Scheme

Let $\mathbb{X}_i(t) = l_{(i,\bullet)}\hat{\boldsymbol{x}}(t)$ define the instantaneous disagreement between the last transmitted value of node i and the last received values from its neighbours, where $l_{(i,\bullet)}$ is the i-th row of L. Moreover, $\mathbb{X}(t) = [\,\mathbb{X}^T_1(t), \dots, \mathbb{X}^T_N(t)\,]^T$ denotes the stacked disagreement vector. Given t^i_k, the decision on next transmission for agent i is made locally based on the condition

$$t^i_{k+1} = \inf\{\, t > t^i_k : \mathcal{T}_i \ge 0\}, \tag{8}$$

where $\mathcal{T}_i = |\boldsymbol{e}_i(t)| - \phi_i |\mathbb{X}_i(t)|, (1 \le i \le N)$. The scalar $\phi_i > 0$ is the local event-triggering threshold to be designed for agent i. To incorporate the design of ϕ_i's along with control gain k in the cost function J, the event-triggering conditions (9) are expressed as a function of $\boldsymbol{x}_{\mathrm{r}}(t)$ and $\boldsymbol{e}_{\mathrm{r}}(t)$. Derived from (8), the following component-wise inequality is, therefore, considered

$$\boldsymbol{e}_{[\mathrm{abs}]} \le \Phi\, \mathbb{X}_{[\mathrm{abs}]}, \tag{9}$$

with $\boldsymbol{e}_{[\mathrm{abs}]} = [|\boldsymbol{e}_1(t)|, \dots, |\boldsymbol{e}_N(t)|]^T$, $\mathbb{X}_{[\mathrm{abs}]} = [|\mathbb{X}_1(t)|, \dots, |\mathbb{X}_N(t)|]^T$, and $\Phi = \mathrm{diag}(\phi_1, \dots, \phi_N)$. The following lemmas are introduced to express (9) as a function of $\boldsymbol{x}_{\mathrm{r}}(t)$ and $\boldsymbol{e}_{\mathrm{r}}(t)$.

Lemma 1. If matrix Φ with certain values ϕ_i $(1 \le i \le N)$ satisfies (9), the following entry-wise inequality is also satisfied

$$\hat{L}\boldsymbol{e}_{[\mathrm{abs}]} \le \Phi\,|\hat{L}|\,\mathbb{X}_{[\mathrm{abs}]}. \tag{10}$$

Lemma 2. Let $\boldsymbol{\alpha} = [\,\alpha_1, \dots, \alpha_{N-1}\,] = l_{(N,\bullet)}\hat{L}^\dagger$, and

$$M = \sqrt{\lambda_{\max}(\hat{L}^T\hat{L})} \begin{bmatrix} I_{N-1} \\ -\boldsymbol{\alpha} \end{bmatrix}.$$

If matrix Φ with certain values ϕ_i $(1 \le i \le N)$ satisfies the following entry-wise inequality

$$\boldsymbol{\psi}_e \le \Phi\,\boldsymbol{\psi}_{\hat{x}}, \tag{11}$$

it also satisfies inequality (10). The undefined vectors in (11) are $\boldsymbol{\psi}_{\hat{x}} = \left[\,|\,m_{(1,\bullet)}\hat{\boldsymbol{x}}_{\mathrm{r}}(t)|\,, \dots, |m_{(N-1,\bullet)}\hat{\boldsymbol{x}}_{\mathrm{r}}(t)|\,\right]^T$, where $m_{(i,\bullet)}$ is row i of matrix M, and $\boldsymbol{\psi}_e = \left[\,|l_{(1,\bullet)}\boldsymbol{e}(t)|\,, \dots, |l_{(N-1,\bullet)}\boldsymbol{e}(t)|\,\right]^T$.

Lemmas 1 and 2 can be proved by applying the sub-additive property to the reverse triangle inequality in Euclidean space. Lemma 2, in short, suggests that a Φ satisfying (11) also satisfies (10). Based on Lemma 1, inequality (10) is equivalent to triggering condition (9). Further, inequality (11) can be expressed as the quadratic constraint $\boldsymbol{e}^T_{\mathrm{r}}(t)\boldsymbol{e}_{\mathrm{r}}(t) \le \hat{\boldsymbol{x}}^T_{\mathrm{r}}(t) M^T \Phi^2 M \hat{\boldsymbol{x}}_{\mathrm{r}}(t)$. Replacing $\hat{\boldsymbol{x}}_{\mathrm{r}}(t)$ with its equal term $\boldsymbol{e}_{\mathrm{r}}(t) + \boldsymbol{x}_{\mathrm{r}}(t)$ results in the expression

$$\boldsymbol{e}^T_{\mathrm{r}}(t)\boldsymbol{e}_{\mathrm{r}}(t) \quad \le \quad (\,\boldsymbol{e}_{\mathrm{r}}(t) + \boldsymbol{x}_{\mathrm{r}}(t)\,)^T M^T \Phi^2 M (\,\boldsymbol{e}_{\mathrm{r}}(t) + \boldsymbol{x}_{\mathrm{r}}(t)\,). \tag{12}$$

Given the dependence of inequality (12) on the transmission thresholds ϕ_i's, this inequality provides a constraint on the number of transmissions. Once the ϕ_i's are obtained, the distributed event-triggering mechanism (8) determines the triggering instants for each agent. According to Lemmas 1 and 2, inequality (12) is guaranteed.

3.2. Parameter Design

The following theorem computes the optimal control gain k and event-triggering threshold ϕ_i form the cost function J and objective function f used in the optimization problem.

Theorem 1. Given R and Q for the cost function J, the optimal event-triggering threshold ϕ_i's and control gain k are computed from

$$k = \mathcal{P}^{-1}\mu, \quad \text{and} \quad \phi_i = \sqrt{\tau^{-1}\gamma_i{}^{-1}} \ \ (1 \le i \le N), \tag{13}$$

which are conditioned on the existence of positive scalars $\mathcal{P}$, τ, μ, ω_τ, ω_k, $\omega_\mathcal{P}$, γ_i, and ω_{γ_i} $(1 \le i \le N)$, satisfying the following minimization problem with constraints expressed in terms of linear matrix definiteness inequalities

$$\min_{\mu,\tau,\mathcal{P},\gamma_i,\omega_\tau,\omega_\mu,\omega_\mathcal{P},\omega_{\gamma_i}} f = \omega_\tau + \omega_k + \omega_\mathcal{P} + \mathcal{P} + \mathrm{Tr}(\boldsymbol{\omega}_\Gamma) \tag{14}$$

$$\text{such that } \ \Pi \triangleq \begin{bmatrix} -\mu\mathbb{L}-\mu\mathbb{L}^T+R & -\mu\mathbb{L} & M^T \\ * & -\tau I+Q & M^T \\ * & * & -\Gamma \end{bmatrix} < 0, \begin{bmatrix} \omega_\mathcal{P} & 1 \\ * & \mathcal{P} \end{bmatrix} > 0,$$

$$\begin{bmatrix} -\boldsymbol{\omega}_\Gamma & \Gamma \\ * & -I \end{bmatrix} < 0, \begin{bmatrix} -\omega_\tau & \tau \\ * & -1 \end{bmatrix} < 0, \text{ and } \begin{bmatrix} -\omega_\mu & \mu \\ * & -1 \end{bmatrix} < 0,$$

where $\Gamma = \mathrm{diag}\,(\gamma_1, \ldots, \gamma_N)$, and $\boldsymbol{\omega}_\Gamma = \mathrm{diag}\,(\omega_{\gamma_1}, \ldots, \omega_{\gamma_N})$. The associated cost J for the average consensus process using the obtained ϕ_i's and k from (13), collectively, guarantees $J \le J^*$, where $J^* = \boldsymbol{x}_\mathrm{r}(0)^T\mathcal{P}\boldsymbol{x}_\mathrm{r}(0)$. The consensus process is accomplished by minimizing the objective function given in (14) for which the following inequalities are guaranteed

$$k^2 \le \omega_\mu\omega_\mathcal{P}^2, \quad \phi_i \ge (\omega_\tau\omega_{\gamma_i})^{\frac{-1}{4}}, \ (1 \le i \le N). \tag{15}$$

Proof. Consider $V(t) = \boldsymbol{x}_\mathrm{r}^T(t)\mathcal{P}\boldsymbol{x}_\mathrm{r}(t)$ as the Lyapunov candidate for system (7). According to the Lyapunov stability theorem, system (7) remains stable if $\dot{V}(t) < 0$. However, in order to incorporate the cost function J with the stability condition, we consider the following inequality

$$\dot{V}(t) + \boldsymbol{x}_\mathrm{r}^T(t)R\,\boldsymbol{x}_\mathrm{r}(t) + \boldsymbol{e}_\mathrm{r}^T(t)Q\boldsymbol{e}_\mathrm{r}(t) < 0. \tag{16}$$

If (16) is satisfied, then the time derivative of $V(t)$ is negative, i.e., $\dot{V}(t) < 0$. Therefore, (7) is stable which implies that $\lim \boldsymbol{x}_\mathrm{r}(t) = 0$ as $t \to \infty$. On the other hand, integrating (16) results in $V(\infty) - V(0) + \int_0^\infty (\boldsymbol{x}_\mathrm{r}^T(t)R\boldsymbol{x}_\mathrm{r}(t) + \boldsymbol{e}_\mathrm{r}^T(t)Q\boldsymbol{e}_\mathrm{r}(t)\,)\,dt < 0$, which is equivalent to $J < [V(0) = \boldsymbol{x}_\mathrm{r}^T(0)\mathcal{P}\boldsymbol{x}_\mathrm{r}(0)]$. Denoting $J^* = \boldsymbol{x}_\mathrm{r}^T(0)\mathcal{P}\boldsymbol{x}_\mathrm{r}(0)$, the cost of the event-triggered average consensus process for a given network and certain initial values is guaranteed not to exceed J^*, i.e., $J < J^*$. Now according to the reduced order system (7), we expand (16). Denoting $\Omega = [\,\boldsymbol{x}_\mathrm{r}^T(t), \boldsymbol{e}_\mathrm{r}^T\,]^T$, one obtains $\Omega^T\,\Pi_1\,\Omega < 0$ from (16), where Π_1 is defined as follows

$$\Pi_1 = \begin{bmatrix} -k\mathcal{P}\mathbb{L}-k\mathcal{P}\mathbb{L}^T+R & -k\mathcal{P}\mathbb{L} \\ * & Q \end{bmatrix}. \tag{17}$$

The event-triggered constraint (12) is equivalent to $\Omega^T\Pi_2\,\Omega < 0$,

$$\Pi_2 = \begin{bmatrix} -M^T\Phi^2M & -M^T\Phi^2M \\ * & I-M^T\Phi^2M \end{bmatrix}. \tag{18}$$

According to the S-procedure lemma [24], if there exists a positive scalar τ such that $\Pi = \Pi_1 - \tau\Pi_2 < 0$, then both $\Omega^T\,\Pi_1\,\Omega < 0$ and $\Omega^T\,\Pi_2\,\Omega < 0$ are guaranteed. Therefore, we incorporate the two inequalities by obtaining Π. Applying Schur complement [24] for Π leads to the following inequality

$$\begin{bmatrix} -k\mathcal{P}\mathbb{L}-k\mathcal{P}\mathbb{L}^T+R & -k\mathcal{P}\mathbb{L} & \tau M^T\Phi \\ * & -\tau I+Q & \tau M^T\Phi \\ * & * & -\tau I \end{bmatrix} < 0. \tag{19}$$

We pre- and post-multiply (19) with $H = \mathrm{diag}(I, I, \tau^{-1}\Phi^{-1})$. The resulting inequality is not linear due to the product of decision variables. To derive a linear matrix constraint, we define alternative variables $\Gamma = \tau^{-1}\Phi^{-2}$ and $\mu = k\mathcal{P}$. The objective function for constraint Π would maximize the event-triggering thresholds (to minimize the number of transmissions) and minimize the control gain (to minimize the control effort). The change of variables used to derive Π preserves the original problem but makes the objective function nonlinear. Motivated by [25], an objective function which minimizes the decision variables involved in obtaining k and ϕ_i's is developed. In this regard, we consider inequalities $\mathcal{P}^{-1} < \omega_\mathcal{P}$, $\omega_\mathcal{P} > 0$, $\mu^2 < \omega_\mu$, $\omega_\mu > 0$, $\tau^2 < \omega_\tau$, $\omega_\tau > 0$, $\gamma_i^2 < \omega_{\gamma_i}$, $\omega_{\gamma_i} > 0$, $(1 \le i \le N)$, for the minimized sum of $\omega_\mathcal{P}$, ω_{γ_i}, ω_τ, and ω_μ. The Schur complement converts the above inequalities into LMIs. To minimize the guaranteed cost J^*, scalar $\mathcal{P}$ is considered in the convex objective function f. Once (14) is solved, consensus parameters are computed from (13). □

Algorithm 1. The GP-ETAC Algorithm

Input: Adjacency Weighting Matrix $\mathcal{A} = \{a_{ij}\}$, Initial conditions $x_i(0)$, and Weighting Matrices $\{R, Q\}$.

Output: Event-triggered Average Consensus with Guaranteed Performance

Preliminaries: (P1 – P2)

P1. Remove the N^{th} row of L to determine $\hat{L}$ and $\mathbb{L} = \hat{L}L\hat{L}^\dagger$.

P2. Given $\hat{L}$, determine $\boldsymbol{\alpha}$ and matrix M from Lemma 2.

Optimization and Parameter Design Steps: (D1–D2)

D1. Using a convex optimization solver, solve the minimization problem (14) for given parameters $\{R, Q\}$.

D2. Using (13), compute transmission threshold ϕ_i $(1 \le i \le N)$ and control gain k.

Consensus Steps: (C1 – C4)

C1. Each sensor sends its initial value $x_i(0)$ to its neighbours.

C2. In each consensus iteration, the state of node i is excited by control law (3) with k computed from D2.

C3. In each consensus iteration, the event-triggering condition (8) is locally monitored with the designed ϕ_i to determine when to transmit $x_i(t)$ to the neighbours.

C4. Steps C2 and C3 continue until average consensus (i.e., $u_i(t) \to 0$ in (3)) is achieved among agents.

In an event-triggering scheme there must exist a positive lower bound for any two consecutive triggering moments. Otherwise, the triggering function exhibits Zeno behaviour [26]. It can be proved that the inter-event interval for agent i is strictly positive and lower bounded by $\phi_i k^{-1}$, i.e., $t^i_{k_i+1} - t^i_{k_i} \ge \phi_i k^{-1}$, $(1 \le i \le N)$, which rules out the Zeno behaviour. The guaranteed performance event-triggered average consensus (GP-ETAC) approach is outlined in Algorithm 1. There may be scenarios where Algorithm 1 does not optimize to a solution. In such cases, R and Q are modified to have reduced norm values and Algorithm 1 is repeated. Although Algorithm 1 assumes a globally known topology, its extension to uncertain topologies is presented in [27].

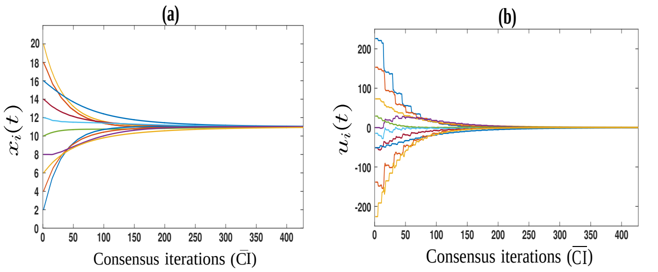

Fig. 1: a) Average consensus on $x_i(t)$. (b) Control input $u_i(t)$.

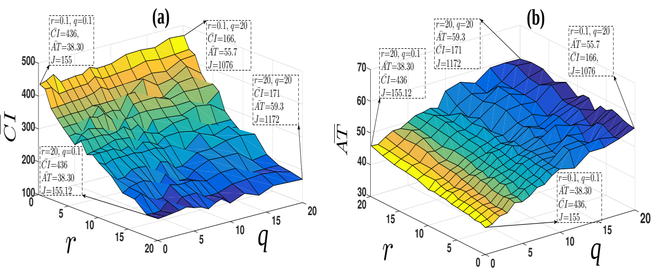

Fig. 2: The effect of r and q on: (a) $\overline{\mathrm{CI}}$, and: (b) $\overline{\mathrm{AT}}$ ($N = 10$).

4. NUMERICAL SIMULATIONS

The performance of the GP-ETAC algorithm is assessed by running Monte-Carlo simulations on random sensor networks with various choices of N, R, and Q. The second eigenvalue of L in each randomly generated network follows a normal distribution with a mean of 2 and variance of 0.2. Only connected networks are selected. All non-zero adjacency weights a_{ij} are set to 1. To show detailed results for Algorithm 1, one randomly selected Monte-Carlo realization is chosen as an example. The non-zero elements in the adjacency matrix A of this network are $\{a_{14}, a_{16}, a_{17}, a_{19}, a_{1,10}, a_{24}, a_{26}, a_{29}, a_{2,10}, a_{37}, a_{39}, a_{45}, a_{48}, a_{5,10}, a_{67}, a_{68}, a_{6,10}, a_{78}, a_{7,10}, a_{9,10}\}$. To initialize the convex optimization (14), we set $R = rI$ and $Q = qI$ with $r = 1$ and $q = 1$. Using the YALMIP parser and SDPT3 solver [28], we solve (14). The resulting consensus parameters are $k = 3.939$, ϕ_1=0.055, ϕ_2=0.065, ϕ_3=0.034, ϕ_4=0.035, ϕ_5=0.050, ϕ_6=0.046, ϕ_7=0.041, ϕ_8=0.032, ϕ_9=0.056, and ϕ_{10}=0.020. The computed value of the guaranteed cost J^* is 824.76. For a sampling time $T_s = 0.001$ sec, the evolutions of the states $x_i(t)$ and control inputs (3) for the ten nodes are shown in Fig. 1. With a termination value of 0.005, i.e., $\|\boldsymbol{x}_{\mathrm{r}}(t)\| \leq 0.005\|\boldsymbol{x}_{\mathrm{r}}(0)\|$, it takes 462 consensus iterations ($\overline{\mathrm{CI}}$) to reach average consensus in this experiment. The ten nodes, respectively, make 29, 26, 47, 44, 33, 34, 37, 47, 27, and 71 transmissions, leading to an average transmission ($\overline{\mathrm{AT}}$) value of 39.50 times per agent. The cost of consensus process is $J = 151.67$, which certifies that $J < J^*$.

Scenario 1: investigates the effect of different choices of $\{r, q\}$ (R=rI and Q=qI) on the average consensus performance in the aforementioned network. Based on the results summarized in Table 1, with a fixed q, increasing r accelerates the convergence rate (smaller $\overline{\mathrm{CI}}$) at the expense of higher average transmission $\overline{\mathrm{AT}}$ and increased cost J. We note that increasing R in J implies assigning a higher penalty to the deviation of the states from their mean value. Therefore, the optimization framework attempts to accelerate the convergence rate. On the other hand, decreasing q for a fixed r leads to a smaller $\overline{\mathrm{AT}}$ among agents. It is consistent with the definition of J since decreasing Q assigns a lower penalty on error $\boldsymbol{e}_{\mathrm{r}}(t)$ allowing for larger gaps between triggering moments.

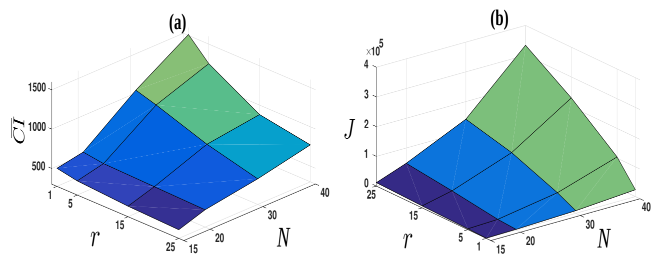

Fig. 3: The effect of r and N on: (a) $\overline{\mathrm{CI}}$, and: (b) J.

Table 1: Impact of weighting matrices r and q on GP-ETAC.

r	q	k	mean(ϕ_i)	$\overline{\mathrm{CI}}$	$\overline{\mathrm{AT}}$	J	J^*
4	1	5.0731	0.0404	333	43.90	450.15	794.71
8	1	6.0091	0.0370	282	47.50	756.35	858.43
1	20	3.8875	0.0373	434	45.70	272.53	795.11
1	0.1	3.9256	0.0441	428	38.80	144.01	849.31

Scenario 2: studies the performance of GP-ETAC over random networks with $N = 10$ for various selections of r and q. Let $\mathbb{S} = \{0.1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 18, 20, 25, 30\}$. For each pair $(r, q) \in \mathbb{S}\times\mathbb{S}$, we solve (14) for twenty random connected networks of size 10. The corresponding values for the total number of consensus iterations $\overline{\mathrm{CI}}$ and average local transmissions $\overline{\mathrm{AT}}$ are shown in Fig 2. In addition to the observations made under Scenario 1, we note that: (i) Parameter r is more relevant in controlling the convergence rate ($\overline{\mathrm{CI}}$) than q, whereas parameter q is more influential in controlling the average transmission $\overline{\mathrm{AT}}$; (ii) The fastest (slowest) convergence rate is roughly equivalent to the largest (smallest) amount of average transmission $\overline{\mathrm{AT}}$ and happens when both r and q are large (small), and; (iii) The effect of a change in q on $\overline{\mathrm{CI}}$ and $\overline{\mathrm{AT}}$ is stronger when r is small.

Scenario 3: studies the scalability of GP-ETAC for random networks of different size. Let $\mathbb{N} = \{15, 20, 30, 40\}$ and $\mathbb{W} = \{1, 5, 10, 15\}$. We fix $q = 1$, and select $(r, N) \in \mathbb{W}\times\mathbb{N}$. For each triplet (q, r, N) chosen from the above sets, we solve (14) for a set of 100 randomly generated networks. From the resulting values of $\overline{\mathrm{CI}}$ and J included in Fig 3, we conclude that: (i) Higher values of r starting from $r = 1$ have a greater impact on the convergence rate in larger networks, and; (ii) The operation cost J increases as N is increased. These observations corroborates that GP-ETAC provides a structured framework to control the consensus convergence rate and amount of data transmissions with a guaranteed cost of operation.

5. SUMMARY AND FUTURE WORK

This paper proposes a guaranteed performance, event-triggered average consensus (GP-ETAC) approach for distributed multi-agent networks. The event-triggered consensus problem is converted to an equivalent stability problem. The Lyapunov stability theorem is used to develop a novel cost function to compute the consensus design parameters. The optimal gains guaranteeing the minimum cost for the process is obtained through convex optimization. Future work will extend the results to time-varying networks.

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