SPATIAL ARRAY THINNING FOR INTERFERENCE CANCELLATION UNDER CONNECTIVITY CONSTRAINTS

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ABSTRACT

Array spatial thinning is employed to select the most effective antenna elements in a large phased array for optimum performance concerning hardware and computational costs, in conjunction with managing element failure and radio interference mitigation. We formulate spatial array thinning under connectivity constraints to make the thinning applicable in large arrays. By introducing graph optimization, the problem is recast as a k-clique version of a generalized minimum clique problem. Furthermore, by studying optimum clustering for the proposed formulation, we show by an example that the unconstrained thinning performance is achievable, even with connectivity constraints.

Index Terms— Array thinning, antenna selection, beamforming, semidefinite programming, convex optimization, graph optimization, generalized minimum clique problem.

1. INTRODUCTION

Many applications such as communications, radar systems, radio telescopes, and automotive sensing stand to benefit from large phased arrays [1–4]. With the increasing the size of phased arrays, the role of antenna selection and array thinning is becoming more important [5,6]. Antenna selection aims to thin a large array, taking advantage of the redundancy inherent to it in order to reduce hardware cost, alleviate heating problems [7], decrease dimensionality of post processing, and mitigate radio interference [8,9]. Furthermore, a thinned array can be viewed as a full array with some non-operating antenna elements due to failure of the electronic front-ends.

Several methods e.g statistical spatial tapering [10], regular grid-based thinning [11], iterative Fourier technique [12], soft-thresholding-based optimization, and convex optimization [8,13] have been proposed to reduce the number of active elements while maintaining performance in a phased array. Achieving full array reconfigurability, where any subset of antennas may be selected, requires that every front-end be connected via switches to every antenna in the array. Although this permits the selection algorithm to choose the best subarray for any scenario, the complexity of connectivity, routing, and RF multiplexing makes this approach impractical. In this paper, we propose employing constraints on the accessible antenna elements for each front-end during the array thinning process to simplify the connections, and routing and RF circuitry, and solve the array thinning problem using graph optimization. To this end, we revisit antenna selection in the case of single interference cancellation defining it as a minimum k-clique problem in graph optimization [14]. We then propose a k-clique version of a generalized minimum clique (GMCP) problem to address connectivity constraints [15].

The connectivity constraints are formulated as a given set of clusters of antenna elements. After relaxing and lifting the problem by a semidefinite method and finding the lower bound for this formulation, we proceed to obtain the optimum clusters. To solve this problem, we cast it as a M-densest subgraph. Two different density matrices including inverse Euclidean with a Gaussian core and the signal of interestinterference correlation is used to divide the full array into M clusters. We show through simulations that the proposed optimum clustering technique performs very close to unconstrained antenna selection despite the added connectivity constraints. Furthermore, in addition to the dimensionality reduction provided by antenna selection, thinning under connectivity constraints is more computationally efficient in the optimization phase as it operates over a smaller subspace of possible solutions.

2. PROBLEM FORMULATION

The signal-to-interference-plus noise ration (SINR) at the output of the optimum beamformer for a phased array containing N elements is given by [8]

$$SINR_{out} = P_s \mathbf{v}_s^H \mathbf{R}_n^{-1} \mathbf{v}_s$$

$$\approx NSNR \left(1 - |\alpha_{js}|^2\right), \qquad (1)$$

where SNR is the signal-to-noise ratio. P_s and \mathbf{v}_s are the power and steering vector of the signal of interest, and $|\alpha_{js}|^2$ is the spatial separability between the signal of interest and interference measured by the spatial correlation coefficient

(SCC)

$$\alpha_{js} = \frac{\mathbf{v}_{j}^{H}\mathbf{v}_{s}}{\|\mathbf{v}_{j}\| \|\mathbf{v}_{s}\|}$$
$$= \frac{\mathbf{v}_{j}^{H}\mathbf{v}_{s}}{\sqrt{\mathbf{v}_{j}^{H}\mathbf{v}_{j}}\sqrt{\mathbf{v}_{s}^{H}\mathbf{v}_{s}}} = \frac{\mathbf{v}_{j}^{H}\mathbf{v}_{s}}{N^{2}}, \qquad (2)$$

where \mathbf{v}_{j} is the steering vector of the interference.

It can be seen from (1) that a smaller correlation between the signal and interference (that is smaller $|\alpha_{js}|^2$) translates to a higher the SINR at the output. Thus, through antenna selection we cast the problem as the combinatorial optimization

$$\min_{\mathbf{c}} f(\mathbf{c})$$

s.t. $c_i(c_i - 1) = 0 \quad i = 1...N$
and $\mathbf{c}^T \mathbf{c} = k$, (3)

where $f(\mathbf{c})$ is the weighted norm of the selection vector \mathbf{c}

$$f(\mathbf{c}) = \left|\alpha_{js}\right|^2 = \frac{\|\mathbf{c}\|_{\mathbf{W}_r}^2}{k^2} = \frac{\mathbf{c}^T \mathbf{W}_r \mathbf{c}}{k^2}.$$
 (4)

 \mathbf{W}_r is a $N \times N$ matrix comprising the real part of the correlation coefficients of different antenan elements

$$\mathbf{W}_r = \operatorname{real}(\mathbf{v}_{js}\mathbf{v}_{js}^H),\tag{5}$$

where

$$\mathbf{v}_{js} = \mathbf{v}_s \odot \mathbf{v}_j^*. \tag{6}$$

Thus, (3) seeks the k least correlated elements of the correlation vector \mathbf{v}_{js} . It is worth noting that this formulation can be extended to multiple interference scenarios [16, 17].

The optimization problem (3) is a non-convex quadrature programming with quadratic constraints. Due to the binary constraints, it is intractable and no exact solution is available. Nevertheless, the solution can be approximated via heuristic methods e.g. correlation measurements, difference of two convex sets (DCS), and randomized semidefinite programming [8, 13]. In this work, we approach antenna selection from a different perspective. Noting that W_r is a similarity matrix, the array can be modelled as a weighted complete graph. The optimization (3) is then recast as finding a k-clique of minimum weight sum [18] as shown in Fig. 1(a).

In addition to reducing the problem dimensionality, antenna selection can be employed to reconfigure the array to manage the failure of a RF front-end. For a large array, the necessary connections among the RF front-ends and antenna elements can grow rapidly. Managing RF front end failure for k front-ends and N elements requires a fully connected array, or C = kN connections. On the other hand, to ensure there is at least one RF path for every element, the number of necessary connections is

$$\mathcal{C} = kN - \sum_{l=1}^{k-1} l. \tag{7}$$



Fig. 1. (a): k-clique problem (b): k-clique GMCP.

To satisfy the connectivity criteria, connectivity constraints are added to optimization in (3) to limit the number of antenna elements that each front-end is permitted to serve. In this case, the clique problem mentioned earlier becomes a kclique version of the generalized minimum clique problem. The complete graph is divided into some clusters and a kclique GMCP is devised to find a clique containing exactly k vertices from each cluster such that the cost of the induced subgraph is minimized (Fig. 1(b)). Let $G = (\mathbf{V}, \mathbf{E}, \mathbf{W}_r)$ where V is the set of vertices and E the edges be a similarity graph representing the full array observing the correlation steering vector \mathbf{v}_{si} for a specific scenario. We assume that there are M clusters defined by $\{\mathbf{v}_1, ..., \mathbf{v}_M\}$ as the characteristic vectors of disjoint subsets of V. Letting c be the characteristic vector of the final k-clique, the GMCP for antenna selection under the connectivity constraints is

$$\min_{\mathbf{c}} \frac{1}{k^2} \mathbf{c}^T \mathbf{W}_r \mathbf{c}$$

s.t. $c_i (c_i - 1) = 0 \quad i = 1...N,$
 $\mathbf{c}^T \mathbf{V}_i \mathbf{c} = k_i \quad i = 1...M,$ (8)

where $\mathbf{V}_i = \text{diag}(\mathbf{v}_i)$ and k_i is the number of vertices in cluster *i*.

The optimization problem (8) is one of quadratic programing with convex quadratic and binary constraints. It can be relaxed by semidefinite programming and the solution can be approximated via constrained randomized sampling as proposed in [13]. By introducing a new matrix variable $\mathbf{C} \in \mathbb{R}^{N \times N}$, the semidefinite relaxed (SDR) version of (8) is:

$$\min_{\mathbf{C}, \mathbf{c}} \frac{1}{k^2} \operatorname{Tr}(\mathbf{C}\mathbf{W}_r)$$

s.t. diag(\mathbf{C}) = \mathbf{c}
Tr($\mathbf{C}\mathbf{V}_i$) = k_i $i = 1...M$,
 $\begin{bmatrix} \mathbf{C} & \mathbf{c} \\ \mathbf{c}^T & 1 \end{bmatrix} \succeq 0.$ (9)

The optimum solutions ($\mathbf{C}^*, \mathbf{c}^*$) provided by (9) consitute a lower bound for the primal problem in (8) [19]. Note the formulation presented here implies the precise of the directions

of arrival (DOA) of the signal and interference. Although beyond the scope of this work, the problem may be reformulated to account for the uncertainty in DOA estimation.

3. OPTIMUM ARRAY ELEMENT CLUSTERING

Given the partitioning into M clusters, the previous section sought to determine the opimum antenna selection. In what follows, we aim to determine the optimum clustering configuration itself for a particular number of cluster M. This, then, would allow the antenna selection for optimal interference cancellation to be determined as per section 2.

Considering the full array as an undirected weighted graph $G = (\mathbf{V}, \mathbf{E}, \mathbf{W})$, various weights can be adopted for clustering depending on the purpose. For instance, if minimum routing length (e.g. for minimum delay) is required, then clusters with geometrically-closed members are needed. In this case, the Euclidean distance can be employed as the cost function. If, on the other hand, the goal is to manage front-end failure, then clusters with geometrically-distant members are more effective as electronic failures are more likely to occur in adjacent front-ends.

An induced subgraph of G whose set of vertices consists of k disjoint cliques is called a k-disjoint-clique (KDC) subgraph. Finding a KDC subgraph is a graph clustering problem [20] (we use M instead of k for notation consistency). To divide G into M clusters, we need to find the M-densest region in terms of **W**. For M disjoint cliques in a graph, the normalized partition matrix is formed as,

$$\mathbf{X} = \left(\frac{\mathbf{v}_1}{\|\mathbf{v}_1\|}, \frac{\mathbf{v}_2}{\|\mathbf{v}_2\|}, ..., \frac{\mathbf{v}_M}{\|\mathbf{v}_M\|}\right).$$

The total density of the clusters induced by $\mathcal{P}_M = {\mathbf{v}_1, ..., \mathbf{v}_M}$ is then calculated as

$$\mathcal{D}(\mathcal{P}_M) \triangleq \sum_{i=1}^{M} \frac{\mathbf{v}_i^T \mathbf{W} \mathbf{v}_i}{\mathbf{v}_i^T \mathbf{v}_i} = \operatorname{Tr}(\mathbf{X}^T \mathbf{W} \mathbf{X}).$$
(10)

Hence, the problem becomes finding \mathcal{P}_M that maximizes the total density. This problem is expressed as

$$\max_{\mathcal{P}_{M}} \mathcal{D}(\mathcal{P}_{M})$$

s.t. $[\mathbf{X}_{i}]_{j} \in \{0, \frac{1}{\|\mathbf{v}_{j}\|}\}^{N}, \quad j = 1, ..., M$
rank $(\mathbf{X}) = M,$
 $\mathbf{X}\mathbf{1}_{M} = \mathbf{1}_{N},$
 $\mathbf{X}^{T}\mathbf{1}_{N} = \mathbf{1}_{M}.$ (11)

Taking the advantage of orthogonality of the characteristic vectors, we introducing the new matrix variable

$$\mathbf{Y} = \sum_{i=1}^{M} \frac{\mathbf{v}_i \mathbf{v}_i^T}{\mathbf{v}_i^T \mathbf{v}_i}.$$
 (12)



Fig. 2. (a): Clustering by maximizing the inverse Euclidean distance. (b): Clustering by minimizing the inverse Euclidean distance. (c): Clustering by maximizing $\mathbf{W}_{r,T}$.

After lifting and rank constraint relaxation, the SDR version of (9) is formulated as [21]

$$\max_{\mathbf{Y}} \operatorname{Tr}(\mathbf{YW})$$
s.t. $\mathbf{Ye} \leq \mathbf{e},$
 $\operatorname{Tr}(\mathbf{Y}) = M,$
 $\mathbf{Y} \geq 0,$
 $\mathbf{Y} \succeq 0.$ (13)

The solution \mathbf{Y}^* given by (13) can then be used to recover the KDC subgraph. To this end, a rank reduction scheme followed by an optimum rounding is required.

3.1. Choosing the weight matrix

Assuming that the criteria for cluster enabling are satisfied, we can find the optimum clusters for the particular weight matrix. As mentioned earlier, the cost function is chosen depending on the aim of the clustering. For instance, the actual physical distance of the elements can be used to ensure shorter and less complicated routing. To illustrate this, consider a 4×4 rectangular array with uniform half wavelength inter-element spacing. Also define the similarity measure as

$$\mathbf{W}_{i,j} = e^{-\|p_i - p_j\|^2}.$$
 (14)

Here p_i and p_j denote the actual positions of elements *i* and *j*. Thus, elements that are physically close are considered more similar. Figs. 2(a) and 2(b) show the results of clustering the array into M = 4 clusters using (13) and (14) respectively. Notice that, as expected, the first strategy leads to contiguous clusters, whereas the second one gives the maximal spread.

Another useful weight matrix is composed of the correlation coefficients corresponding to different correlation steering vectors \mathbf{v}_{js} . In this way, antenna elements that are most correlated with each other are spread across different clusters. This guarantees that the loss of a front-end imposes the smallest loss when the the selection is implemented using (9). To demonstrate, assume we have a set of *l* scenarios of interst where each scenario specifies a combination of signal of interest and interference DOAs. The corresponding correlation matrices are denoted $\mathbf{W}_{r,i}$, i = 1, ..., l. Now we define the optimum clustering as that which maximizes the total density over the entire set of sample points. That is

$$\max_{\mathbf{Y}} \operatorname{Tr}(\mathbf{Y}\mathbf{W}_{r,T}),$$
$$\mathbf{W}_{r,T} = \sum_{i=1}^{L} \mathbf{W}_{r,i}.$$
(15)

The effectiveness of this clustering scheme will be studied in the next section.

4. SIMULATION

In this section, we present simulation examples to illustrate the performance of the antenna selection under connectivity constraints and optimum clustering. We employ a uniformly spaced rectangular array comprising 4×4 antennas. The elevation and azimuth of the signal of interest are fixed at $\phi_s = 0.15\pi, \theta_s = 0.25\pi$. The azimuth of the interference, ϕ_j , varies from 0 to $\frac{\pi}{2}$, whereas the elevation is fixed at $\theta_j = 0.4\pi$. In the first scenario we consider three cases involving the selection of k = 4, 8, and 12 antennas from total 16 elements, arranged in M = 4 clusters each consisting of $k_M = 1, 2$ and 3 elements respectively. The clustering is implemented according to the strategies shown in Figs. 2(a), 2(b), 2(c), which are denoted as configurations 1, 2 and 3 respectively. The correlation matrix $\mathbf{W}_{r,i}$ is sampled over $\phi = 0, ..., \pi$ with steps of $\frac{\pi}{180}$. For each case, two different mean-square-errors (MSE) are calculated. The first is the MSE between the optimum value obtained by exhaustive search and the SDR lower bound proposed in (8). The second is the MSE between the minimum achievable values of the SCC squared for the clustered and unconstrained arrays. The SDR lower bound is obtained using CVX [22]. The results are displayed in Fig. 3. The bottom curves give the distance of each solution from its respective lower bound (determined using an exhaustive search). It is clear that the SDR lower bounds are tight. The top curves, which exhibit the MSE of each configuration with respect to the unconstrained solution, show that Conf. 3 (that is the clustering based on maximizing $\mathbf{W}_{r,T}$ is closest the unconstrained selection. This is due to the fact that, in the general case, where no other criteria such as minmal routing are provided, (15) allows the maximal amount of information to be retained by the clustering.

In the second example, the DOAs are fixed and the performance comparison is done in terms of output SINR for different cluster cardinalities. Although all configurations are capable of providing acceptable SINRs compared to the full array, the clustering based on (15) outperforms the other two for all considered cardinality values. Therefore, by adopting connectivity constraints and optimum clustering we are able to achieve the unconstrained selection with a much smaller number of connections. For instance, to select k = 12 elements, C = 192 required connections in an unconstrained



Fig. 3. Top: MSE between clustered selection and unconstrained selection. Bottom: MSE between the optimum value obtained by exhaustive search and SDR.



Fig. 4. The maximum achievable output *SINR* corresponding to different cluster cardinalities for unconstrained spatial thinning and spatial thinning under different connectivity constraints.

selection is decreased to C = 48 connections by dividing the array into M = 4 clusters. Moreover, the solution space for the unconstrained selection which is $\binom{16}{12} = 1820$, is reduced to $\binom{4}{2}^4 = 256$.

5. CONCLUSION

In this paper, we studied spatial array thinning for interference cancellation under connectivity constraints. We formualted the problem was a generalized clique problem in graph optimization and use semidefinite relaxation to solve it. We proposed appropriate connectivity contraints for different criteria and evalutated their performance using simulations. We demonstrated that the relaxed solutions attain their respective lower bounds. Furhtermore, we showed that the unconstrained thinning performance was achieved by optimum clustering scheme resulting in smaller number of required connections and lower computational complexity.

6. REFERENCES

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