OPPORTUNISTIC SYNCHRONISATION OF MULTI-STATIC STARING ARRAY RADARS VIA TRACK-BEFORE-DETECT

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ABSTRACT

In this work, we consider the problem of synchronising separately located transmitters and a staring array receiver that also has a local transmitter. The acknowledged benefits of using separate transmitters in active sensing are often undermined by the difficulty in accurate synchronisation of the receiver and the transmitters. In this work, we propose a solution that is based on measurements from non-cooperative objects in the illuminated region. We formulate the problem as parameter estimation in a state space model with individual transmitter channel data cubes as measurements. For maximum likelihood estimation, we use an expectation maximisation type iterative bound optimisation using distributions found by track-before-detect together with explicit formulae derived here for the related score function for chirp waveforms. We demonstrate that the proposed approach is capable of achieving very high accuracy with errors on the order of small fractions of the pulse width thereby enabling coherent processing in bi-static channels.

***Index Terms*—** Multi-static radar, synchronisation, marginal likelihood, track-before-detect, expectation maximisation.

1. INTRODUCTION

Active sensing systems with geographically dispersed transmitters such as widely separated multiple-input multiple-output (MIMO) radars [1] have the potential to deliver significantly improved performance in detecting objects and resolution in locating them owing to the diversity in the aspect angles the objects are illuminated and reflections are observed from [2]. These highly desirable features can only be practically enabled if key configuration parameters can be found easily and accurately. An important challenge in this respect is the synchronisation of separated transmitters and receiver elements [3], i.e., finding the differences between the receiver time reference and that of the transmitters so as to accurately map the receiver time axis onto spatial locations.

In this work we address synchronisation of a staring array receiver with remote transmitters which emit orthogonal probing waveforms thereby inducing independent bi-static reflection channels at the receiver. There is also a co-located transmitter in-synch with the receiver in the setup we consider, together with which a multi-static MIMO configuration is obtained.

The processing at the array signal separates the reflection channels by demodulation, matched filtering and sampling. Each sample corresponds to a time-of-flight value for the probing waveform. If the time reference shift of the transmitters with respect to the receiver are known precisely, these values can be further mapped to precise spatial (bi-static) range values [3]. Ambiguity in these quantities significantly deteriorates the system performance [4].

Our approach is based on a state space model that captures the generation of these tensor valued measurements (see, e.g. [5]) in consecutive coherent processing intervals (CPIs). Estimation of respective parameters including synchronisation terms in this model can be treated as parameter estimation in multi-sensor state space models [6, 7] in which each channel of the multi-static setup acts as a distinct sensor with the co-located transmitter's channel (i.e., the mono-static channel) setting the reference. The parameter likelihood of the synchronisation terms has the kinematic state trajectory of the reflector object and the reflectivity coefficients as the latent variables. We find the ML estimate using expectation maximisation (EM) [8] in which we approximate the expectation using particle representations of the state distributions conditioned on the data cubes. This is known as track-before-detect (see, for example, [9, 10]), and, here implies that the synchronisation process uses the kinematic object trajectory as a reference for estimating time reference shifts in different reflection channels. We derive explicit formulae for the gradient of the expectation to be maximised and use gradient ascent iterations at each EM step (see, e.g., [11] for details of EM in state space models, and, [12] in sensor registration)

Other alternatives include the use of atomic clocks and/or external references such as GPS signals (see, e.g., [13]). Such a process is tedious and requires expensive equipment, yet, prone to errors due to, e.g., inaccuracies in locating transmitter and receiver elements [3]. A data driven solution consisting of processing at the receiver side is more preferable. One such approach with array receivers is to digitally divert a beam towards a transmitter to recover the transmitted signal (see, e.g., [14, 15]). This necessitates transmit power be spent to guarantee a direct link towards the receiver which might not be desirable or feasible. The proposed approach removes such restrictions and operates using reflections from opportunistic objects in the illuminated region and local processing only.

Synchronisation terms can be decomposed into a delay of an integer multiple of the pulse width and a sub pulse-width portion which has a phase ambiguity effect even in the perfect knowledge of the first term. The phase ambiguity undermines the benefits of coherent processing [4], so, its estimation has also been the topic of recent research. For example, in [16], the carrier phase differences between remote transmitters and a local receiver is estimated using consensus based algorithms. Our approach provides a more comprehensive form of synchronisation and estimates both the shift in pulse triggering times and carrier phase differences – with the latter blended into the phase of the complex reflection coefficients which are esti-

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mated during track-before-detect. As a result, the level of accuracy achieved is on the order of a small fraction of the pulse width which is sufficient for coherent processing in the bi-static channels [15].

The article is organised as follows: Sec. 2 gives the mathematical statement of the problem. In Sec. 3, we detail the proposed synchronisation approach. Sec. 4 explains state and reflectivity estimation with the radar data cubes. We demonstrate the proposed algorithm in Sec. 5. Then, we conclude in Sec. 6.

2. PROBLEM STATEMENT

Let us consider the geometry of the problem as illustrated in Fig. 1 and detail the array signal at the receiver. Here, transmitters use mutually orthogonal waveforms of pulse duration T_p and bandwidth B. N pulses are emitted with a pulse repetition interval (PRI) of T. The reflector objects in the scene are assumed to remain coherent (i.e., reflectivity values in each channel remain constant) during this overall NT seconds period known as a CPI. These transmission characteristics are fully known at the receiver except the time reference shift of the remote transmitters with respect to the receiver clock which will be denoted by Δt_m for the mth channel.

The receiver array has L elements spaced by half the carrier wavelength λ_c, i.e., $d = \lambda_c/2$. Each element collects a superposition of the noise background and reflected signals originating from the local (mono-static) transmitter and the remote (bi-static) transmitters. The processing chain for the mth channel begins with demodulation followed by matched filtering with the mth probing waveform which completely suppresses the contributions of the other channels owing to the orthogonality of the waveforms used. The output for the array elements are then sampled with a period that is equal to the the pulse duration T_p. A total of Γ samples are collected for each of the N pulse at each of the L elements. For the range bin r, we stack the columns of the data cube and form a $LN \times 1$ data vector which is a function of the synchronisation term Δt_m and the kinematic state X:

$$\mathbf{Z}_m(r) = \alpha_m \mathbf{s}_m(r, X, \Delta t_m) + \mathbf{n}_m(r), \quad (1)$$

where $\mathbf{s}_m$ is the reflected signal model that will be detailed later in this section, α_m is the complex reflection coefficient in the mth channel, and, $X = [x, y, \dot{x}, \dot{y}]^T$ is the kinematic state of the reflector where $[x, y]^T$ is its location, $[\dot{x}, \dot{y}]^T$ is its velocity, and T denotes vector transpose. The noise background is modelled with a circular symmetry complex Gaussian random vector $\mathbf{n}_m(r) \sim \mathcal{CN}(.; \mathbf{0}, \Sigma_m)$ of zero mean and covariance Σ_m.

The problem we consider is to estimate the vector of unknown time reference shifts as denoted by the synchronisation term, i.e.,

$$\Delta\mathbf{t} \triangleq [\Delta t_1 = 0, \Delta t_2, \ldots, \Delta t_M] \quad (2)$$

using the receiver measurements collected in all M channels for a time window of length t, i.e.,

$$\begin{aligned} \mathbf{Z} &\triangleq [\mathbf{Z}_{1,1:t}, \cdots, \mathbf{Z}_{M,1:t}], \\ \mathbf{Z}_{m,k} &\triangleq [\mathbf{Z}_{m,k}(1), \cdots, \mathbf{Z}_{m,k}(\Gamma)], \end{aligned} \quad (3)$$

where the last line denotes the mth channel measurements at time k as a concatenation of the measurement vectors in (1).

In the rest of this article, we detail the synchronisation term estimation by solving

$$\Delta\hat{\mathbf{t}} = \arg\max_{\Delta\mathbf{t}} l(\mathbf{Z}|\Delta\mathbf{t}), \quad (4)$$

where the marginal likelihood $l(\mathbf{Z}|\Delta\mathbf{t})$ further decomposes as

$$l(\mathbf{Z}|\Delta\mathbf{t}) = \prod_{k=1}^{t} l(\mathbf{Z}_k|\mathbf{Z}_{1:k-1}, \Delta\mathbf{t}) \quad (5)$$

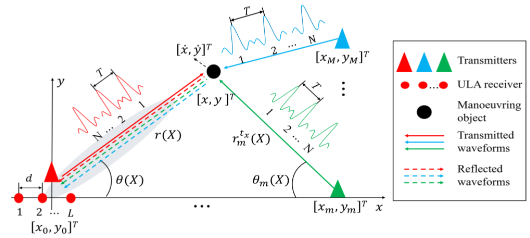


Fig. 1: Geometry of the problem: Both polar and Cartesian coordinate variables are depicted.

by the chain rule of probabilities.

Before we continue with further elaboration on the maximisation of (5) in Sec. 3, we provide details of the signal model in (1).

2.1. Signal model

The reflected signal model in (1) is given by

$$\begin{aligned} \mathbf{s}_m(r, X, \Delta t_m) \triangleq\ & \mathrm{e}^{-j\omega_c \Delta t_m} \mathbf{s}_s(\theta(X)) \otimes \mathbf{s}_t(\tau_m(X), \Omega_m(X)) \\ & \times \Lambda_m(rT_p - \tau_m(X) - \Delta t_m) \end{aligned} \quad (6)$$

$$\mathbf{s}_s(\theta) = \left[1, \mathrm{e}^{-j\pi \sin\theta}, \ldots, \mathrm{e}^{-j(L-1)\pi\sin\theta}\right]^T. \quad (7)$$

$$\mathbf{s}_t(\tau_m, \Omega_m) \triangleq \mathrm{e}^{-j\omega_c \tau_m} \times \left[1, \mathrm{e}^{j\Omega_m}, \ldots, \mathrm{e}^{j(N-1)\Omega_m}\right]^T, \quad (8)$$

where $\theta(X)$, $\tau_m(X)$, and $\Omega_m(X)$ are the angle of arrival, the time-of-flight (TOF), and the angular Doppler shift, respectively, associated with X (Fig. 1). Here, $\mathbf{s}_s$ is the spatial steering vector, $\mathbf{s}_t$ is the temporal steering vector, and, $\otimes$ denotes the Kronecker product operator. $\Lambda_m(\cdot)$ is the auto-correlation of the mth waveform.

3. MAXIMUM LIKELIHOOD ESTIMATOR FOR SYNCHRONISATION

It is not straightforward to find a tractable solution for (4), for example, find the score function for (5) and use gradient descent for solving the ML estimation problem. Instead, lower bounds can be maximised iteratively such that the gap with the logarithm of the original likelihood tends to zero with the maximisation leading to the ML estimate looked for [8]. This approach is known as expectation maximisation and involves iteratively solving the optimisation problem given by

$$\Delta t^{(j+1)} = \arg\max_{\Delta t} Q(\Delta t, \Delta t^{(j)}) \quad (9)$$

$$\begin{aligned} Q(\Delta t, \Delta t^{(j)}) \propto & \int\!\!\int \log l(\mathbf{Z}_{1:t}|X_{1:t}, \boldsymbol{\alpha}_{1:t}, \Delta\mathbf{t}) \\ & \times p(X_{1:t}, \boldsymbol{\alpha}_{1:t}|\mathbf{Z}_{1:t}, \Delta t^{(j)})\, \mathrm{d}X_{1:t}\, \mathrm{d}\boldsymbol{\alpha}_{1:t}, \\ \propto & \sum_{k=1}^{t} \int\!\!\int \log l(\mathbf{Z}_k|X_k, \boldsymbol{\alpha}_k, \Delta\mathbf{t}) \\ & \times p(\boldsymbol{\alpha}_k|\mathbf{Z}_k) p(X_k|\mathbf{Z}_{1:t}, \Delta t^{(j)})\, \mathrm{d}X_k\, \mathrm{d}\boldsymbol{\alpha}_k. \end{aligned} \quad (10)$$

The measurement likelihood term in (10) factorises into channel likelihoods due to the independence of noise terms. It also satisfies a locality property in that the number of range bins associated with X_k is limited by the support of the auto-correlation Λ_m in (6) which is of $2T_p$ seconds width , i.e.,

$$\begin{aligned} & l(\mathbf{Z}_k|X_k, \boldsymbol{\alpha}_k, \Delta\mathbf{t}) \\ & \propto \prod_{m=1}^{M} \prod_{r \in \mathcal{E}_m(X_k, \Delta t_m)} l(\mathbf{Z}_{m,k}(r)|X_k, \alpha_{m,k}, \Delta t_m), \end{aligned} \quad (11)$$

where $\mathcal{E}_m$ is the set of range bins associated the reflector kinematic state X_k and given by

$$\tilde{\mathcal{E}}_m(X_k, \Delta t_m) = \begin{cases} \{r_m, r_m+1\} & r_m T_p < \tau_m(X_k) + \Delta t_m \\ \{r_m\} & r_m T_p = \tau_m(X_k) + \Delta t_m \\ \{r_m, r_m-1\} & r_m T_p > \tau_m(X_k) + \Delta t_m \end{cases}, \tag{12}$$

and

$$r_m = \left\lceil \frac{\tau_m(X_k) + \Delta t_m}{T_p} \right\rceil$$

is the range bin corresponding to the TOF $\tau_m(X_m)$ and time reference shift Δt_m.

Thus, the mth channel likelihood can easily be found by using the noise distribution in the signal model in (1), i.e.,

$$\begin{aligned} &l\ (\mathbf{Z}_{m,k}(r)|X, \alpha_{m,k}, \Delta t_m) \\ &= \mathcal{CN}\Big(\mathbf{Z}_{m,k}(r); \alpha_{m,k}\mathbf{s}_m(r_k, X_k, \Delta t_m), \Sigma_m\Big). \end{aligned} \tag{13}$$

The equation above relates the signal model and the unknowns including the synchronisation terms through (11) and (12) to the expectation in (10). Evaluation of this latter term, however, is not straightforward due to the marginalisations involved. We tackle with marginalisation over the state variable using Monte Carlo integration [17] with samples generated from $p(X_k|\mathbf{Z}_{1:k}, \Delta t^{(j)})$ (instead of the smoothing distribution $p(X_k|\mathbf{Z}_{1:t}, \Delta t^{(j)})$ in (10)) by using Bayesian recursive filtering with the radar data cube likelihoods which is detailed in Sec. 4. With regards to the reflection coefficients, we use the Empirical Bayesian [18] perspective: We find the ML estimate $\hat{\boldsymbol{\alpha}}_k$ using expectation maximisation (EM) within Bayesian state estimation (Sec. 4) and select [15]

$$p(\boldsymbol{\alpha}_k|\mathbf{Z}_k) \leftarrow \delta_{\hat{\boldsymbol{\alpha}}_k}(\boldsymbol{\alpha}_k), \tag{14}$$

where δ is Dirac's delta distribution, and, $\leftarrow$ denotes assignment (i.e., the distribution on the left hand side is asserted to be the Dirac's delta on the right hand side).

Given this empirical prior together with a set of particles $\{X_k^{(p)}, \zeta_k^{(p)}\}_{p=1}^P$ representing the state estimation, and, considering (13), the expectation in (10) is (stochastically) approximated by

$$\begin{aligned} &\tilde{Q}(\Delta t, \Delta t^{(j)}) \propto \frac{1}{P}\sum_{k=1}^{t}\sum_{p=1}^{P}\sum_{m=1}^{M} \\ &\sum_{r\in\mathcal{E}(X_k^{(p)}, \Delta t_m)} \log \mathcal{CN}(\mathbf{Z}_{m,k}(r); \hat{\alpha}_{m,k} s_m(r, X_k^{(p)}, \Delta t_m), \Sigma_m), \end{aligned} \tag{15}$$

where $\hat{\alpha}_{m,k}$ is the ML estimate of $\alpha_{m,k}$ in (16) (see, the top of next page). Here, $\xi^{(i-1)}$ is a state posterior conditioned on the previously found value of the reflection coefficient in [15].

Next, we consider maximisation of $\tilde{Q}(\Delta t, \Delta t^{(j)})$ in (9) and find the gradient of (15) with respect to the synchronisation vector for chirp waveforms [3]. Then, we use iterative gradient descent [19] with this gradient. The iterations at the j^{th} step of EM start with an initial synchronisation vector $\Delta\mathbf{t}^{(j,0)} = \{\Delta t_m^{(j,0)}\}_{m=2}^M$, and updates $\Delta\mathbf{t}^{(j,i)}$ by

$$\Delta\mathbf{t}^{(j,i)} = \Delta\mathbf{t}^{(j,i-1)} + \mu\nabla\tilde{Q}(\Delta\mathbf{t}, \Delta\mathbf{t}^{(j)})|_{\Delta\mathbf{t}=\Delta\mathbf{t}^{(j,i-1)}}, \tag{17}$$

where μ is a step size parameter, and $\nabla\tilde{Q}$ is given by using the partial differentials

$$\nabla\tilde{Q} = [\frac{\partial\tilde{Q}}{\partial\Delta t_m}]_{m=2}^M \tag{18}$$

which are derived for chirp auto-correlations and given explicitly in (19) at the top of next page. In this expression,

$$\begin{aligned} &\tilde{\mathbf{s}}_m(\Delta t_m) \triangleq \mathbf{s}_m(r, X, \Delta t_m), \\ &\mathbf{s}_m^{'}(X) \triangleq \mathbf{s}_s(\theta(X)) \otimes \mathbf{s}_t(\tau_m(X), \Omega_m(X))\Lambda_m(t^{'}), \\ &t^{'} \triangleq (rT_p - \tau_m(X) - \Delta t_m), \quad \text{and} \\ &\Lambda_m(t^{'}) = (1 - \frac{|t^{'}|}{T_p})\mathrm{sinc}(\pi B t^{'}(1 - \frac{|t^{'}|}{T_p})), \end{aligned}$$

where $\Lambda_m(t^{'})$ is the auto-correlation of a chip waveform, and sinc denotes the sinc function. These iterations are repeated until the condition $\| \Delta\hat{\mathbf{t}}^{(j,i)} - \Delta\hat{\mathbf{t}}^{(j,i-1)} \| < \epsilon$, is satisfied, where $\|.\|$ denotes the Euclidean norm.

4. BAYESIAN FILTERING OF THE RADAR DATA CUBES

In this section, we consider sampling from the filtering distribution $p(X_k|\mathbf{Z}_{1:k}, \Delta t^{(j)})$ for evaluating (15) as explained in Sec. 3. The object trajectory $X_{1:t}$ is modelled as a Markov state space model [20]. Then, X_k is sequentially estimated by using Bayesian recursive filtering [20] which consists of a prediction and an update stage. At time step k, the prediction stage is given by

$$\begin{aligned} &p(X_k|\mathbf{Z}_{1:k-1}, \Delta t^{(j)}) = \\ &\int p(X_k|X_{k-1})p(X_{k-1}|\mathbf{Z}_{1:k-1}, \Delta t^{(j)})\mathrm{d}X_{k-1}. \end{aligned} \tag{20}$$

Here, $p(X_k|X_{k-1})$ is the Markov transition density selected as $p(X_k|X_{k-1}) = \mathcal{N}(X_k; FX_{k-1}, Q)$, where F models constant velocity motion, and Q is the covariance matrix specifying the level of the process noise modelling unknown manoeuvres.

In the update stage this prediction is multiplied with the measurement likelihood and all other variables are marginalised out, i.e.,

$$\begin{aligned} p(X_k|\mathbf{Z}_{1:k}, \Delta\mathbf{t}^{(j)}) \propto &\int_{\boldsymbol{\alpha}_k} l(\mathbf{Z}_k|X_k, \boldsymbol{\alpha}_k, \Delta\mathbf{t}^{(j)}) \\ &\times p(\boldsymbol{\alpha}_k)p(X_k|\mathbf{Z}_{1:k-1}, \Delta\mathbf{t}^{(j)})\mathrm{d}\boldsymbol{\alpha}_k, \end{aligned} \tag{21}$$

where $p(\boldsymbol{\alpha}_k)$ is the a priori density for the reflection coefficient. The marginalisation of the reflection coefficients, however, is not straightforward and a reasonable prior is not always available. Instead, we use an empirical Bayes approach [18] as discussed in Sec. 3, and rewrite (21) as follows:

$$\begin{aligned} &p(X_k|\mathbf{Z}_{1:k}, \Delta\mathbf{t}^{(j)}) = \\ &\int_{\boldsymbol{\alpha}_k} p(X_k|\mathbf{Z}_{1:k}, \boldsymbol{\alpha}_k, \Delta\mathbf{t}^{(j)})p(\boldsymbol{\alpha}_k|\mathbf{Z}_k)\mathrm{d}\boldsymbol{\alpha}_k. \end{aligned} \tag{22}$$

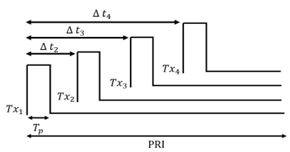
Here, the second term inside the integration is similar to a prior for the reflection coefficients. Because this prior is conditioned on the measurements, more probability mass should be concentrating around the maximum likelihood (ML) estimate of this value which is the rationale behind (14).

For realising the recursive filtering, a sequential Monte Carlo (SMC) approach known as the particle filter is used [21]. In particular, a bootstrap filtering approach is used for estimating the object trajectory. Suppose we have a set of weighted samples, or, particles, representing the state posterior in the previous step i.e., $\left\{X_{k-1}^{(p)}, \zeta_{k-1}^{(p)}\right\}_{p=1}^P$. In the prediction stage at the time step k, we obtain P particles as the set of $\left\{X_{k|k-1}^{(p)}, \zeta_{k|k-1}^{(p)}\right\}_{p=1}^P$ by using $X_{k|k-1}^{(p)} \sim p(\cdot|X_{k-1}^{(p)})$ sampled from the Markov transition realising the prediction stage in (20).

$$\hat{\alpha}_{m,k} = \frac{\sum_{p=1}^{P} \sum_{r \in \mathcal{E}(X_k^{(p)})} \xi_p^{(i-1)} \mathbf{s}_{m,k}(r, X_{k|k-1}^{(p)}, \Delta t_m)^H \Sigma_m^{-1} \mathbf{Z}_{m,k}(r)}{\sum_{p=1}^{P} \sum_{r \in \mathcal{E}(X_k^{(p)})} \xi_p^{(i-1)} \mathbf{s}_{m,k}(r, X_{k|k-1}^{(p)}, \Delta t_m)^H \Sigma_m^{-1} \mathbf{s}_{m,k}(r, X_{k|k-1}^{(p)}, \Delta t_m)}. \quad (16)$$

$$\frac{\partial \tilde{Q}(\Delta \mathbf{t}, \Delta \mathbf{t}_m^{(j)})}{\partial \Delta t_m} = \frac{1}{P} \sum_{p=1}^{P} \sum_{k=1}^{t} \sum_{r \in \mathcal{E}(X_k^{(p)})} \Big[\frac{2 \frac{\partial \tilde{\mathbf{s}}_m(\Delta t_m)^H}{\partial \Delta t_m} \Sigma_m^{-1} \mathbf{Z}_m(r) \times (\tilde{\mathbf{s}}_m(\Delta t_m)^H \Sigma_m^{-1} \mathbf{Z}_m(r))}{\tilde{\mathbf{s}}_m(\Delta)^H \Sigma_m^{-1} \tilde{\mathbf{s}}_m(\Delta)^H}$$
$$- \frac{|\tilde{\mathbf{s}}_m(\Delta t_m)^H \Sigma_m^{-1} \mathbf{Z}_m(r)|^2}{(\tilde{\mathbf{s}}_m(\Delta)^H \Sigma_m^{-1} \tilde{\mathbf{s}}_m(\Delta)^H)^2} \times (2 \frac{\partial \tilde{\mathbf{s}}_m(\Delta t_m)^H}{\partial \Delta t_m} \Sigma_m^{-1} \tilde{\mathbf{s}}_m(\Delta t_m)) \Big] \quad (19)$$

$$\frac{\partial \tilde{\mathbf{s}}_m}{\partial \Delta t_m} = -j\omega_c \mathrm{e}^{-j\omega_c \Delta t_m} \mathbf{s}_m^{'}(X_{k|k-1}^{(p)}) \Lambda_m(t^{'}) + \mathrm{e}^{-j\omega_c \Delta t_m} \mathbf{s}_m^{'}(X_{k|k-1}^{(p)}) \Big[\frac{\sin(\pi B(t^{'})(1 - \frac{|t^{'}|}{T_p}))}{\pi B(t^{'})^2} + \frac{\cos(\pi B(t^{'})(1 - \frac{|t^{'}|}{T_p}))(\frac{|t^{'}| + (t^{'})t^{'}}{T_p} - 1)}{t^{'}} \Big]$$



Parameter	Value
Carrier frequency f_c	10e+9 Hz
Bandwidth B	1e+6 Hz
Pulse duration T_p	1e-6 s
Pulse repetition interval PRI	100e-6 s
Number of pulses during a CPI	20
Number of elements on ULA	20
Number of transmitters (Tx)	4

Fig. 2: Transmitted signal parameters for $M = 4$ transmitters: Δt_2, Δt_3 and Δt_4 are the synchronisation terms for remote channels.

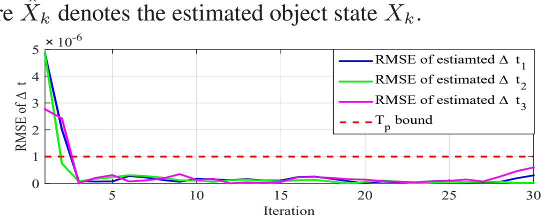
In the update stage, the same sample set is used to represent the state posterior in (21). The weights of these samples need to be adjusted using the measurement likelihood, i.e.,

$$\zeta_k^{(p)} = \frac{\tilde{\zeta}_k^{(p)}}{\sum_{p'=1}^{P} \tilde{\zeta}_k^{(p')}}, \quad \tilde{\zeta}_k^{(p)} = \zeta_{k|k-1}^{(p)} l(\mathbf{Z}_k | X_k^{(p)}, \hat{\boldsymbol{\alpha}}_k, \Delta \hat{\mathbf{t}}^{(j)}), \quad (23)$$

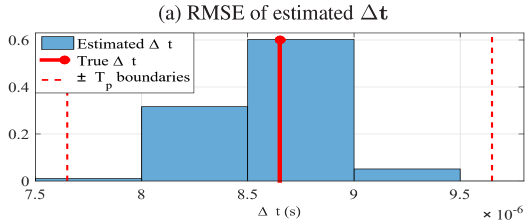
After finding the normalised weights in (23), we test degeneracy of the weighted particles. The degeneracy test is performed by finding the number of effective particles and comparing it with a threshold. When this test value is less than the threshold, we perform resampling (see, e.g., [21]). Using the above particle filter, the object state X_k at the kth CPI is estimated by using the empirical weighted average, i.e.,

$$\hat{X}_k = \sum_{p=1}^{P} \zeta_k^{(p)} X_{k|k-1}^{(p)}, \quad (24)$$

where $\hat{X}_k$ denotes the estimated object state X_k.



(a) RMSE of estimated $\Delta \mathbf{t}$



(b) Histogram of $\Delta \hat{\mathbf{t}}$

Fig. 3: Example: (a) RMSE of estimates of $\Delta \mathbf{t}$, (b) Histogram of $\Delta \hat{\mathbf{t}}$ for 100 experiments in comparison with the true $\Delta \mathbf{t}$ (red line) and $\pm T_p$ boundaries (dashed red lines).

5. EXAMPLE

In this section, we demonstrate the proposed algorithm through an example. We consider a scenario with $M = 4$ transmitters, one of which is co-located with a ULA receiver at the origin of the 2D Cartesian plane, and, the other transmitters are located at $[0, 500]$, $[0, 1000]$, and $[500, 0]$, respectively. The parameters of the transmitted pulses are shown in Fig. 2. Here, $\Delta \mathbf{t} = \{\Delta t_m\}_{m=2}^{M=4}$ denote the synchronisation terms. In the surveillance region, an object with an initial state of $X_0 = [1000\text{m}, 1000\text{m}, 10\text{m/s}, 50\text{m/s}]$ follows an unknown trajectory. The receiver collects measurements in accordance with the signal model in (1) from $M = 4$ channels.

We use the proposed algorithm to estimate the synchronisation term $\Delta \mathbf{t}$ with $P = 400$ particles. These particles are initially selected as a 20×20 element uniform grid within a resolution bin that contains X_0. The reflection coefficient for each channel is generated by a complex Gaussian density leading to an expected SNR of 10dB. Here, $\Delta \mathbf{t}$ is a uniform random variable in $0 < \Delta t < PRI$.

Fig. 3(a) illustrates the root mean square error (RMSE) of a typical run of our algorithm detailed in Sec. 3 and 4. We compare these error values with the T_p bounds (dashed red line). It is seen that the errors of all the estimates of $\Delta \mathbf{t}$ (see, the RMSEs of estimated $\Delta \hat{t}_2$ (solid blue line), $\Delta \hat{t}_3$ (solid green line), and $\Delta \hat{t}_4$ (solid magenta line), respectively, for $M - 1 = 3$ remote channels) stay within a small fraction of the total pulse width T_p after just a few iterations.

Next, we generate 100 measurement and estimate the synchronise term using the proposed approach with a time window length of $t = 50$ CPIs (i.e., 1000 pulses). We repeat this for the 100 iterations. Fig. 3(b) illustrates the histogram of the synchronisation term estimation in comparison with the ground truth value of $\Delta \mathbf{t}$ (solid red line) and the $\pm T_p$ pulse width boundaries (dashed red lines). Here, the percentage of the estimates of $\Delta \hat{\mathbf{t}}$ that are very close to the true $\Delta \mathbf{t}$ is 60 percent of the entire experiments, and, the ratio of the estimation error with respect to T_p is 0.269.

6. CONCLUSION

In this work, we have proposed a novel approach for synchronisation of remote transmitters and a staring array receiver using only local data processing at the receiver end. The reference of synchronisation is trajectories of reflectors in the illuminated scene as acquired through a co-located transmitter. The problem is solved jointly for a multi-static radar configuration. Our approach avoids the use of external reference signals such as GPS signals. The algorithm is built upon ML parameter estimation in state-space models. In order to realise the ML strategy, we have derived explicit formulae for the gradients used in expectation maximisation iterations and developed sequential Monte Carlo samplers for track-before-detect with the radar data cubes. We demonstrate that the proposed approach is capable of achieving very high accuracy with errors on the order of small fractions of the pulse width.

7. REFERENCES

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