SPARSITY-BASED SPACE-TIME ADAPTIVE PROCESSING FOR AIRBORNE RADAR WITH COPRIME ARRAY AND COPRIME PULSE REPETITION INTERVAL

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ABSTRACT

In this paper, we present a sparsity-based space-time adaptive processing (STAP) algorithm with coprime array and coprime pulse repetition interval (PRI). The considered space-time coprime configuration can significantly save the cost. However, the direct STAP does not exploit the advantage of the large aperture brought by coprime configuration and the recently developed spatial-temporal smoothed-based STAP requires a large number of training snapshots. To solve these issues, we propose a sparsity-based STAP algorithm by using the spacial-temporal sparsity of clutter in virtual domain. Simulation results show that the proposed algorithm can obtain a much higher output signal-to-interference-plus-noise ratio and improve the convergence speed.

***Index Terms*—** Sparsity-based space-time adaptive processing (STAP), clutter suppression, coprime array, coprime pulse repetition interval

1. INTRODUCTION

The restless search for low cost radar with relatively high performance is always considered in research and practical application areas. Compared with the uniform linear array (ULA), coprime arrays have attracted tremendous attention by its ability which can provide a larger array aperture and achieve $O(MN)$ degrees of freedom (DoFs) by using only $M+N-1$ physical sensors (M and N are coprime integers). Indeed, coprime arrays provide an effective method for reducing the radar cost requirements. Exploiting these advantages, the directional-of-arrival (DOA) estimation achieves significant enhancement in the number of sources detected and accuracy of DOA estimation [1, 2]. Moreover, researchers applied the idea of the coprime sampling to the area of the space-time adaptive processing (STAP) [3–7].

There are two types of coprime sampling structure in STAP processor, one is the space-fast-time coprime sampler, and the other is the space-slow-time coprime sampler. For the former one, the joint angle-Doppler estimation can obtain a much higher resolution [3]. For the latter one, the airborne radar transmits a coherent pulse sequence with coprime pulse repetition intervals (PRIs) and receives returns by using a ULA. It was shown that a much higher SINR performance is achieved than the uniform transmitting configuration with the same number of pulses, but it only considers the case of the ULA [4]. A STAP algorithm for the nested array (a special case of coprime array) was proposed for clutter suppression, where deep nulls along clutter ridge and a narrow mainlobe in the desired direction were achieved [5]. However, the directions and Dopplers of all jamming and clutter sources are required to be prior known. In addition, the minimum redundancy STAP was proposed by arranging the joint space-slow-time samplers to achieve larger DoFs with the minimum redundancy sampler, and the space-time resolution and slow moving target detection performance were greatly increased [6]. However, the constructing procedure is complex for the case of large arrays. Compared with minimum redundancy sampler design, the coprime sampler design is very simpler. [7] developed a spacial-temporal smoothed-based STAP with space-slow-time samplers and showed a much higher signal-to-interference-plus-noise ratio (SINR) compared with direct STAP. However, since the errors are incorporated by the virtual covariance matrix estimation, this kind of approaches require a much larger number of snapshots for training. In this paper, we focus on the space-slow-time sampler and try to develop STAP algorithms to reduce the number of space-time snapshots.

Motivated by the sparsity-based STAP with significant performance improvement in a very small number of snapshots [8–10], we propose a novel sparsity-based STAP algorithm by considering coprime arrays and coprime PRI. Specifically, the proposed algorithm can be divided into three steps: (i) introduce the virtual space-time snapshot construction; (ii) formulate the sparse signal model for virtual space-time snapshot and apply the least absolute shrinkage and selection operator (LASSO) [11] method to obtain the clutter spectrum estimate; (iii) design a STAP filter with the clutter spectrum estimate. Furthermore, we analyze error of the virtual space-time snapshot constructed by using finite snapshots. Simulation results are presented to demonstrate the effectiveness of the proposed algorithm.

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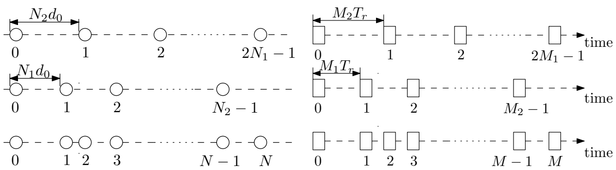

(a) coprime array (b) coprime PRI

Fig. 1. Coprime array and coprime PRI configuration.

2. SPACE-SLOW-TIME COPRIME SAMPLING MODEL

Consider a pulse-Doppler side-looking airborne radar with an N-sensors coprime array, which is a interleaving of two U-LAs: one ULA has $2N_1$ sensors with intersensor spacing N_1d and the other ULA has N_2 sensors with intersensor spacing N_2d, as shown in Fig.1(a). Therefore, the sensor locations are $\{n_{i1}d, i1 = 1, \cdots, N-1\}$ with $N = 2N_1 + N_2 - 1$ (N_1 and N_2 being coprime integers satisfying $N_1 < N_2$). The radar transmits $M = 2M_1 + M_2 - 1$ coherent pulses in a coherent processing interval (CPI) with the coprime PRI set $\{m_{i2}T_r, i2 = 1, \cdots, M-1\}$, as shown in Fig.1(b). Here M_1 and M_2 are also coprime integers satisfying $M_1 < M_2$, and T_r is the minimal PRI. The transmitter carrier frequency is f_c. Ignoring the impact of range ambiguities, the received signal in the target-free range bins can be formulated as

$$\mathbf{x} = \mathbf{x}_c + \mathbf{u}, \tag{1}$$

where $\mathbf{u}$ denotes the thermal noise, and $\mathbf{x}_c$ denotes an $MN \times 1$ clutter space-time snapshot with the form

$$\mathbf{x}_c = \sum_{i=1}^{N_c} \alpha_{i,c}\mathbf{v}(\omega_{i,c}, \phi_{i,c}) = \mathbf{V}\boldsymbol{\alpha}_c. \tag{2}$$

Here, $\alpha_{i,c}$ is the unknown complex amplitude of the ith clutter patch, N_c is the number of statistically independent clutter patches in each iso-range, $\mathbf{V} = [\mathbf{v}(\phi_{1,c}, \omega_{1,c}), \cdots, \mathbf{v}(\phi_{N_c,c}, \omega_{N_c,c})]$ denotes the clutter space-time steering matrix, and $\mathbf{v}(\omega_{i,c}, \phi_{i,c}) = \mathbf{b}(\omega_{i,c}) \otimes \mathbf{a}(\phi_{i,c})$ is the $MN \times 1$ space-time steering vector at the normalized Doppler frequency $\omega_{i,c}$ and spatial frequency $\phi_{i,c}$. Here, $\mathbf{b}(\omega_{i,c})$ and $\mathbf{a}(\phi_{i,c})$ are the temporal and spatial steering vectors, given by

$$\mathbf{b}(f_{d,i}) = [1, \cdots, \exp(j2\pi m_{M-1}\omega_{i,c})]^T, \tag{3}$$

$$\mathbf{a}(f_{s,i}) = [1, \cdots, \exp(j2\pi n_{N-1}\phi_{i,c})]^T, \tag{4}$$

where the superscript T denotes the transpose.

Then, by assuming mutual independence of noise components, the corresponding covariance matrix of received signal $\mathbf{x}$ is calculated by

$$\begin{aligned}\mathbf{R} &= E[\mathbf{x}\mathbf{x}^H] \\ &= \sum_{i=1}^{N_c} \sigma_{i,c}^2 \mathbf{v}(\phi_{i,c}, \omega_{i,c})\mathbf{v}^H(\phi_{i,c}, \omega_{i,c}) + \sigma_u^2\mathbf{I},\end{aligned} \tag{5}$$

where $\sigma_{i,c}^2 = E[|\alpha_{i,c}|^2]$, σ_u^2 is the variance of the noise vector $\mathbf{u}$, $\mathbf{I}$ is the identity matrix, $E[\cdot]$ denotes the expected value, and the superscript H stands for transpose-conjugate of matrices. Since $\mathbf{R}$ is unknown in practice, the secondary data is employed to estimate the interference (clutter-plus-noise) covariance matrix. Assume these secondary data is noted as $\mathbf{x}_k$, where $k = 1, 2, \cdots, K$. The covariance matrix is calculated by

$$\hat{\mathbf{R}} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{x}_k{\mathbf{x}_k}^H, \tag{6}$$

where K is the number of the secondary data, and $\hat{\mathbf{R}}$ is the estimate of $\mathbf{R}$.

3. PROPOSED SPARSITY-BASED STAP ALGORITHM

In this section, we first detail the process of virtual space-time snapshot construction, then design the propose sparsity-based STAP filter, and finally analyze the errors of virtual construction by using finite snapshots.

3.1. Virtual Space-Time Snapshot Construction

By using the property of Kronecker product $(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = (\mathbf{AC}) \otimes (\mathbf{BD})$, we express the term $\mathbf{v}(\phi_{i,c}, \omega_{i,c})\mathbf{v}(\phi_{i,c}, \omega_{i,c})^H$ in (5) as

$$\left(\mathbf{b}(\omega_{i,c})\mathbf{b}(\omega_{i,c})^H\right) \otimes \left(\mathbf{a}(\phi_{i,c})\mathbf{a}(\phi_{i,c})^H\right). \tag{7}$$

Note that the entries in $\mathbf{b}(\omega_{i,c})\mathbf{b}(\omega_{i,c})^H$ are taken the form of $e^{j2\pi\omega_{i,c}\breve{m}}$, where the set $\breve{m}$ contains unique integers of $(3M_1M_2 + M_1 - M_2)$ [3], [12]. Hence, there exists an arrangement that converts $\mathbf{b}(\omega_{i,c})\mathbf{b}(\omega_{i,c})^H$ to a new steering vector $\breve{\mathbf{b}}(\omega_{i,c}) = [e^{j2\pi\omega_{i,c}\breve{m}_1}, \cdots, e^{j2\pi\omega_{i,c}\breve{m}_{(3M_1+M_1-M_2)}}]$. So, the $\breve{\mathbf{b}}(\omega_{i,c})$ can be regarded as steering vector corresponding to $(3M_1M_2 + M_1 - M_2)$-pulses in one CPI. The same argument holds true in the spatial domain, where a new steering vector $\breve{\mathbf{a}}(\phi_{i,c})$ is viewed as the steering vector corresponding to $3N_1N_2 + N_1 - N_1$ sensors. According to the equivalence of space-time snapshot and a slice matrix for a give range bin, the virtual space-time snapshot $\mathbf{Z}$ is constructed from $\mathbf{R}$ by

$$\mathbf{Z} = \sum_{i}^{N_c} \sigma_{i,c}^2 \breve{\mathbf{a}}(\phi_{i,c})\breve{\mathbf{b}}(\omega_{i,c})^T + \sigma_u^2\mathbf{e}_1\mathbf{e}_2^T, \tag{8}$$

Vectorizing the virtual space-time snapshot $\mathbf{Z}$, we get

$$\mathbf{z} = \mathrm{vec}(\mathbf{Z}) = \breve{\mathbf{V}}\mathbf{p} + \sigma_u^2\mathbf{e}_u', \tag{9}$$

where $\breve{\mathbf{V}} = [\breve{\mathbf{b}}(\omega_{1,c}) \otimes \breve{\mathbf{a}}(\phi_{1,c}), \cdots, \breve{\mathbf{b}}(\omega_{N_c,c}) \otimes \breve{\mathbf{a}}(\phi_{N_c,c})]$ denotes the clutter space-time steering matrix, $\mathbf{p} = [\sigma_{1,c}^2, \cdots,$

$\sigma^2_{N_c,c}]^T$, and $\mathbf{e}'_u$ is a column vector of all zeros except for a one at the central position. Comparing (9) with (1), we can note that $\mathbf{z}$ behaves like an equivalent received signal from a virtual array with much longer array aperture whose corresponding steering matrix is defined by $\breve{\mathbf{V}}$. The equivalent source signal vector is represented by $\mathbf{p}$ and the noise becomes a deterministic vector given by $\sigma^2_u\mathbf{e}'_u$.

Since the virtual space-time snapshot becomes a single snapshot of $\mathbf{p}$, the rank of $\mathbf{z}$ is one. As such, the spatial-temporal smoothed-based method is developed to obtain a positive estimate of the interference covariance matrix but sacrificing some DoFs of the virtual snapshots [6], [7]. In the following, we propose a sparsity-based STAP without any reducing of the DoFs.

3.2. Sparsity-based STAP Filter Design

If we discrete the whole angle-Doppler plane into $N_d = \rho_d M$ and $N_s = \rho_s N$ ($\rho_d, \rho_s > 1$) grids, where N_d and N_s are the number of Doppler bins and the number of angle bins, respectively [8], by ignoring the mismatch between the assumed clutter space-time steering vectors and the true clutter space-time steering vectors, (9) can be rewritten as

$$\bar{\mathbf{z}} = \bar{\mathbf{V}}\bar{\mathbf{p}} + \sigma^2_u\mathbf{e}'_u, \tag{10}$$

where the $N_dN_s \times 1$ vector $\bar{\mathbf{p}}$ stands for the angle-Doppler spectrum, and $\bar{\mathbf{V}}$ is a sparsity representation dictionary, given by

$$\begin{aligned}\bar{\mathbf{V}} = \{&\breve{\mathbf{b}}(\omega_{1,c}) \otimes \breve{\mathbf{a}}(\phi_{1,c}), \cdots, \breve{\mathbf{b}}(\omega_{1,c}) \otimes \breve{\mathbf{a}}(\phi_{N_d,c}),\\ &\cdots, \breve{\mathbf{b}}(\omega_{N_s,c}) \otimes \breve{\mathbf{a}}(\phi_{N_d,c})\}.\end{aligned} \tag{11}$$

It is noted that the elements of clutter angle-Doppler spectrum only occupy some of the whole angle-Doppler spectrum, which results in sparsity. This has also been illustrated by previous researches about the sparsity-based STAP [9], [10].

In practice, we can only estimate the $\mathbf{R}$ by a finite number of snapshots, which will result in estimate error. That is to say, $\bar{\mathbf{z}}$ is bias-contaminated, i.e.,

$$\hat{\bar{\mathbf{z}}} = \bar{\mathbf{V}}\bar{\mathbf{p}} + \sigma^2_u\mathbf{e}'_u + \bar{\boldsymbol{\epsilon}}. \tag{12}$$

Here, the vector $\bar{\mathbf{p}}$ can be derived by solving the following optimization problem

$$\hat{\bar{\mathbf{p}}} = \arg\min_{\bar{\mathbf{p}}} \|\bar{\mathbf{p}}\|_0 \quad s.t.\ \|\hat{\bar{\mathbf{z}}} - \bar{\mathbf{V}}\bar{\mathbf{p}} - \sigma^2_u\mathbf{e}'_u\|_2 \le \zeta \tag{13}$$

where $\|\cdot\|_i (i = 0, 2)$ denotes the l_i-norm, ζ is a noise error allowance which equals to the square root of variance of the $\bar{\boldsymbol{\epsilon}}$. It is well known that a number of effective methods has been proposed to solve this type of problem in compressive sensing area. In this paper, we adopt LASSO [11] method to solve the sparse vector $\hat{\bar{\mathbf{p}}}$. Once the $\hat{\bar{\mathbf{p}}}$ is solved, the covariance matrix of the virtual space-time snapshot can also be estimated by

$$\hat{\bar{\mathbf{R}}} = \bar{\mathbf{V}}\text{diag}(\hat{\bar{\mathbf{p}}})\bar{\mathbf{V}}^H + \hat{\sigma}^2_u\mathbf{I}. \tag{14}$$

Then, the STAP filter is designed based on the derived virtual covariance matrix $\hat{\bar{\mathbf{R}}}$ by maximizing the output SINR. So the STAP filter vector is

$$\mathbf{w} = \gamma\hat{\bar{\mathbf{R}}}^{-1}\mathbf{v}(\phi_t, \omega_t), \tag{15}$$

where $\gamma = 1/(\mathbf{v}(\phi_t, \omega_t)^H\bar{\mathbf{R}}^{-1}\mathbf{v}(\phi_t, \omega_t))$, and ω_t and ϕ_t are the normalized Doppler frequency and the spatial frequency of the target, respectively.

3.3. Analysis of Errors of Virtual Construction by Using Finite Snapshots

In order to set the value of the ζ, we derive distribution of $\bar{\boldsymbol{\epsilon}}$ by evaluating the effect of the finite sampling in the following. First, we denote the estimate of vectorization of covariance matrix of direct received signal due to finite sampling as

$$\begin{aligned}\hat{\mathbf{r}} &= \text{vec}(\hat{\mathbf{R}})\\ &= \sum_{i=1}^{N_c} \sigma^2_{i,c}\mathbf{v}^*(\phi_{i,c}, \omega_{i,c}) \otimes \mathbf{v}(\phi_{i,c}, \omega_{i,c}) + \sigma^2_u\mathbf{i}_u + \boldsymbol{\epsilon},\end{aligned} \tag{16}$$

where $\mathbf{i}_u = [\mathbf{e}_1^T, \mathbf{e}_2^T, \cdots, \mathbf{e}_{MN}^T]^T$ with $\mathbf{e}_i (i = 1, 2, \cdots, MN)$ denoting a $MN \times 1$ column vector of all zeros except a 1 in the ith entry, and $\boldsymbol{\epsilon} = (\hat{\mathbf{r}} - \mathbf{r})$ denotes the vectorization of the bias of clutter covariance matrix induced by the effect of finite number of space-time snapshots. The distribution of $\boldsymbol{\epsilon}$ is [13]

$$\boldsymbol{\epsilon} \sim \mathit{CN}(0, \frac{1}{N}\mathbf{R}^T \otimes \mathbf{R}). \tag{17}$$

With consideration the process of derivation of $\mathbf{z}$ in (9), we can easily note that the distribution of the $\bar{\boldsymbol{\epsilon}}$ is different from $\boldsymbol{\epsilon}$ in $\hat{\mathbf{r}}$ of (16). It relies not only on the entries of $\mathbf{R}$, but also the arrangement transformation from $\mathbf{r}$ to $\mathbf{z}$. Therefore, we can derive the distribution of $\bar{\boldsymbol{\epsilon}}$ by establishing the relation between $\mathbf{r}$ and $\mathbf{z}$.

First, we formulate the following expression

$$\boldsymbol{\epsilon} = \mathbf{H}\bar{\boldsymbol{\epsilon}}. \tag{18}$$

where $\mathbf{H} \in \{0, 1\}^{(MN)^2 \times [(3M_1M_2+M_1-M_2)(3N_1N_2+N_1-N_2)]}$ denotes the arrangement transformation which involves virtual and equivalent process.

In reference with [12], the matrix $\mathbf{H}$ can be characterized as

$$\mathbf{H} = \mathbf{J}^{-1}(\mathbf{F}_t \otimes \mathbf{F}_s), \tag{19}$$

where $\mathbf{J} \in \{0, 1\}^{(MN)^2 \times (MN)^2}$ is a matrix with the ith row being all zeros but a single 1 at position $i + kNM - lN^2 + (l - k)N$. Here, $k = \lfloor\frac{mod(mod(i-1, N^2M), N^2)}{N}\rfloor$, $l = \lfloor\frac{mod(i-1, N^2M)}{N^2}\rfloor$, $\lfloor\cdot\rfloor$ and $mod(\cdot)$ denote the round down to the nearest integer and module operator, respectively.

$\mathbf{F}_t = (\mathbf{\Phi}_t \otimes \mathbf{\Phi}_t)\mathbf{G}_t\mathbf{\Psi}_t^{-1}$, $\mathbf{F}_s = (\mathbf{\Phi}_s \otimes \mathbf{\Phi}_s)\mathbf{G}_s\mathbf{\Psi}_s^{-1}$ with $(\cdot)^{-1}$ denoting the Moore-Penrose inverse of matrix. Here, by assuming $M' = M_2(2M_1 - 1)$, $\mathbf{\Phi}_t \in \{0,1\}^{M\times(M'+1)}$ is a matrix with the ith row being all zeros but a single 1 at the (m_i+1) position, and $\mathbf{G}_t \in \{0,1\}^{(M'+1)^2\times(2M'+1)}$ takes the form

$$\mathbf{G}_t = \begin{bmatrix} \mathbf{0}_{(M'+1)\times M'} & & \mathbf{I}_{M'+1} \\ \mathbf{0}_{(M'+1)\times(M'-1)} & \mathbf{I}_{M'+1} & \mathbf{0}_{(M'+1)\times 1} \\ \vdots & \vdots & \vdots \\ \mathbf{I}_{M'+1} & & \mathbf{0}_{(M'+1)\times(M')} \end{bmatrix}. \quad (20)$$

In addition, $\mathbf{\Psi}_t \in \{0,1\}^{(M'+M_1M_2+M_1)\times(2M'+1)}$ is a matrix with the ith row containing all zeros but a single 1 at the $(\breve{m}_i + M' + 1)$. Meanwhile, $\mathbf{\Phi}_s \in \{0,1\}^{N\times(N'+1)}$, $\mathbf{G}_s \in \{0,1\}^{(N'+1)^2\times(2N'+1)}$ with $N' = N_2(2N_1-1)$, and $\mathbf{\Psi}_s \in \{0,1\}^{(N'+N_1N_2+N_1)\times(2N'+1)}$ are derived by the similar process in time domain. Now, we combine (17) with (18) to obtain

$$\bar{\boldsymbol{\epsilon}} \sim CN(0, \frac{1}{N}\mathbf{H}^{-1}(\mathbf{R}^T \otimes \mathbf{R})(\mathbf{H}^{-1})^H). \quad (21)$$

The variance of the $\bar{\boldsymbol{\epsilon}}$ is the trace of covariance matrix $\frac{1}{N}\mathbf{H}^{-1}(\mathbf{R}^T \otimes \mathbf{R})(\mathbf{H}^{-1})^H$.

4. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed algorithm (virtual SR) in terms of the output SINR performance. In simulations, the parameters are: $v_p = 125$m/s, $f_c = 2.4$GHz, $N_1 = M_1 = 2$, $N_2 = M_2 = 3$, and PRF $= 4000$Hz. The clutter is assumed to be distributed as an independent zero-mean complex-valued Gaussian distribution. In the sparsity-based STAP, $\rho_d = \rho_s = 4$, and ζ is set as the square root of variance of $\bar{\boldsymbol{\epsilon}}$. Simulation results are obtained by averaging 100 independent Monte Carlo runs.

In the first simulation, we plot the SINR versus the number of snapshots used for training, as shown in Fig. 2(a). In the examples, the target is assumed to be located at a range of 32km with normalized Doppler frequency of 0.3 and signal-to-noise-ratio (SNR) of 0dB. The proposed algorithm shows fastest convergence and best performance among the simulated STAP algorithms. This is because the proposed algorithm can fully exploit the large aperture of the space-slow-time coprime samplers and also provide a high resolution of the clutter spectrum without loss of the virtual DoFs.

In the second simulation, as shown in Fig. 2(b), the stable output SINR performance of above mentioned algorithms are compared. The proposed algorithm achieves the theoretical performance with only 60 snapshots(denoted as snps). In the third simulation, as shown in Fig. 2(c), the snapshots of 45 is considered for all simulated algorithms, the proposed STAP algorithm still shows better SINR performance compared with others due to increased DoFs in virtual domain and high resolution characteristic of the sparse recovery methods.

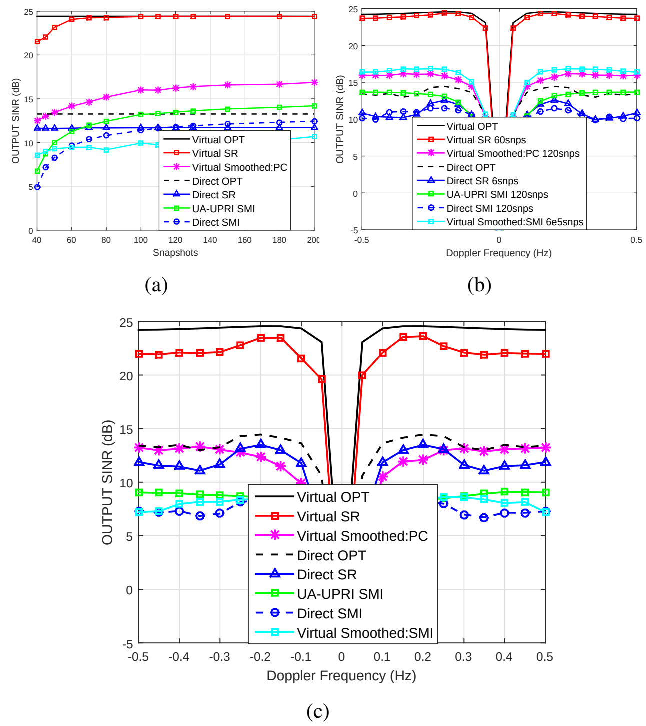

Fig. 2. Output SINR comparisons for direct physical STAP, sparsity recovery STAP in virtual domain, and spatial-temporal smoothed based STAP.

5. CONCLUSION

We have proposed a sparsity-based STAP method by using the spatial-temporal sparsity of clutter in virtual domain for airborne radar with the coprime array and coprime PRI. The proposed algorithm first rearranges the received space-slow-time coprime samplers into a large virtual snapshot, formulates it as a sparse signal model, estimates the clutter spectrum using the sparse recovery method, and designs the STAP filter by using the recovered clutter spectrum. It is found that the advantages of the proposed algorithm lie in fully utilizing of the DoFs offered by the virtual construction and high resolution characteristic of the sparse recovery methods. Additionally, the analysis of errors of virtual construction by using finite snapshots is conducted for the setting of the parameters of the sparse recovery algorithms. Simulation results have shown that the proposed algorithm outperforms the virtual smoothed-based algorithm in terms of the output SINR performance and convergence speed for the case of finite number of snapshots.

6. REFERENCES

[1] P. Pal and P. P. Vaidyanathan, "Nested arrays: A novel approach to array processing with enhanced degrees of freedom," *IEEE Transactions on Signal Processing*, vol. 58, pp. 4167 – 4181, April 2010.

[2] P. P. Vaidyanathan and P. Pal, "Sparse sensing with coprime samplers and arrays," *IEEE Transactions on Signal Processing*, vol. 59, pp. 573 – 586, April 2011.

[3] Ch. Liu and P. P. Vaidyanathan, "Coprime arrays and samplers for space-time adaptive processing," in *2015 IEEE International Conference Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, 2015, pp. 2364 – 2368.

[4] Y. Ma, Z. Yang, J. Huang, J. Huang, and L. Kang, "Space-time adaptive processing airborne radar with coprime pulse repetition interval," in *2016 CIE International Conference on Radar (RADAR)*. CIE, 2016, pp. 1 – 4.

[5] P. Vouras, "Fully adaptive space-time processing on nested arrays," in *2015 IEEE Radar Conference (RadarConf)*. IEEE, 2015, pp. 0858 – 0863.

[6] R. Li, Y. Wang, Z. He, J. Li, and G. Sun, "Minimum redundancy space-time adaptive processing utilizing reconstructed covariance matrix," in *2017 IEEE Radar Conference (RadarConf)*. IEEE, 2017, pp. 0722 – 0726.

[7] X. Wang, Z Yang, and J. Huang, "Space-time adaptive processing for clutter suppression in coprime array and coprime pulse repetition interval airborne radar," in *2017 IEEE International Symposium on Intelligent Signal Processing and Communication Systems (ISPACS)*. IEEE, 2017, pp. 380 –385.

[8] I. W. Selesnick, S. U. Pillai, K. Y. Li, and B. Himed, "Angle-doppler processing using sparse regularization," in *IEEE International Conference Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, 2010, vol. II, pp. 2750–2753.

[9] Z. Yang, H. Wang X. Li, and R. Fa, "Knowledge-aided stap with sparse-recovery by exploiting spatio-temporal sparsity," *IET Signal Processing*, vol. 10, pp. 150 – 161, March 2016.

[10] Z. Yang, R. C. Lamare, and W. Liu, "Sparsity-based stap using alternating direction method with gain/phase errors," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 53, pp. 2756 – 2768, June 2017.

[11] R. Tibshirani, "Regression shrinkage and selection via the lasso," *Journal of the Royal Statistical Society Series B-methodological*, vol. 58, pp. 267 – 288, 1996.

[12] J. Yang, Y. Yang, G. Liao, and B. Lei, "A super-resolution direction of arrival estimation algorithm for coprime array via sparse bayesian learning inference," *Circuit Systems and Signal Processing*, vol. 9, pp. 1–28, August 2017.

[13] B. Ottersten, P. Stoica, and R. Roy, "Covariance matching estimation techniques for array signal processing applications," *Digital Signal Processing*, vol. 8, pp. 185–210, July 1998.