TOEPLITZ MATRIX-BASED TRANSMIT COVARIANCE MATRIX OF COLOCATED MIMO RADAR WAVEFORMS FOR SINR MAXIMIZATION

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ABSTRACT

Focusing on the signal-to interference-plus-noise ratio (SINR) maximization in colocated multiple-input multiple-output (MIMO) radars, using the covariance matrix design of transmitted waveforms, we propose a kind of transmit covariance matrix (TCM) $\mathbf{R}_{pm}$ with the form of symmetrical Toeplitz matrix, whose full rank characteristic firstly can sufficiently exploit the waveform diversity advantage of MIMO radar to further suppress the maximum number of interfering sources. Meanwhile, the positive semi-definition characteristic of $\sin((\pi/2)\mathbf{R}_{pm})$ guarantees that these TCMs can be synthesized with binary phase shift keying (BPSK) waveforms in closed form. Furthermore, employing certain proposed TCM, higher SINR level can be yielded, and lower sidelobe levels (SLLs) can be obtained for the unwanted sidelobe interference suppression. Simulation results validate the better performance of our proposed TCMs in comparison with the phased array, omnidirectional MIMO radar and the recently proposed TCMs.

Index Terms—colocated MIMO radar, transmit covariance matrix, Toeplitz matrix, SINR maximization

1. INTRODUCTION

For the multiple-input multiple-output (MIMO) radar, waveform design always is one of the most important issues [1-14], which generally can be classified into the direct or indirect methods. In the direct design approaches [1-6], the transmitted signal symbols are directly calculated and some special waveform characteristics must be considered to be satisfied, such as orthogonality, low peak-to-average power ratio, constant modulus, similarity constraints and so on [6]. However, due to the flexibility and lower system design complexity, the indirect MIMO waveform design has recently received much attention [7-13], where the transmit covariance matrix (TCM) $\mathbf{R}$ of the transmitted signals is first designed, and then the transmitted waveform symbols are generated. As shown in [7][8], for a given positive semi-definite $\mathbf{R}$, binary phase shift keying (BPSK) waveforms can be synthesized to realize $\mathbf{R}$ in closed form. Focusing on the TCM-based waveform design, some methods have been proposed [9-13]. In [9], a TCM $\mathbf{R}_{2x}$ using a cosine Toeplitz matrix is presented and yield gains in SINR level. Though the sidelobe levels (SLLs) of receive beampattern using $\mathbf{R}_{2x}$ are lower, the rank of $\mathbf{R}_{2x}$ is only 2, that is, the most important degree of freedom (DOF) advantage cannot be exploited for the interference suppression. Moreover, $\sin((\pi/2)\mathbf{R}_{2x})$ is not positive semi-definite [9][10], which cannot guarantee to synthesize $\mathbf{R}_{2x}$ with BPSK in closed form. Considering the full-rank constraint of $\mathbf{R}$ and positive semi-definition of $\sin((\pi/2)\mathbf{R})$, two Toeplitz matrices $\mathbf{R}_{p1}$ and $\mathbf{R}_{p2}$ are proposed as TCMs to achieve higher SINR levels in [10], however, the full rank property of these two TCMs is not proofed rigorously and the achieved SINR is not the optimum with the prior knowledge of locations.

In this paper, we have extended the two matrices in [10] to a kind of more general symmetrical Toeplitz matrix $\mathbf{R}_{pm}$ as the TCMs, where $0 < m < 3M_t/(M_t+1)$ is the control parameter and $M_t \geq 2$ denotes the number of transmit antennas. It is demonstrated that $\mathbf{R}_{pm}$ is full-ranked and $\sin((\pi/2)\mathbf{R}_{pm})$ is also positive semi-definite. Moreover, the $\mathbf{R}_{pm}$ with smaller m yields higher SINR level, and even gets closed to the one of phased array. While the $\mathbf{R}_{pm}$ with certain larger m (*e.g.* m =1.5 or 2) could obtain the receive

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beampattern with lower SLLs, which is beneficial to suppress the unwanted sidelobe interferences.

2. PROBLEM FORMULATION

Consider a colocated MIMO radar system equipped with a transmit uniform linear array (ULA) of M_t elements and a receive ULA of M_r elements. Each transmit element emits a distinct waveform $\mathbf{s}_m(n)$ and $\mathbf{s}(n)=\left[\mathbf{s}_1(n),\mathbf{s}_2(n),\cdots,\mathbf{s}_{M_t}(n)\right]^T$ is assumed as a $M_t\times 1$ vector of transmitted waveforms at the *n*th snapshot, then the signal received by the *i*th source at a spatial location ϕ_i is $\mathbf{a}^T(\phi_i)\mathbf{s}(n)$, where $\mathbf{a}(\phi_i)$ is the transmit steering vector. Denoting the receive steering vector with $\mathbf{b}(\phi_i)$, when there is a target located at ϕ_0 and Q signal-dependent interfering sources at ϕ_i ($i=1,2,\cdots,Q$), further the received vector can be written as [9-13]

$$\mathbf{y}(n)=\alpha_0\mathbf{b}(\phi_0)\mathbf{a}^T(\phi_0)\mathbf{s}(n)+\sum_{i=1}^{Q}\beta_i\mathbf{b}(\phi_i)\mathbf{a}^T(\phi_i)\mathbf{s}(n)+\mathbf{v}(n) \quad (1)$$

where α_0 and β_i are the complex reflection coefficients of the target and the *i*th interfering source, respectively, which obey the Swerling II model [13]. $\mathbf{v}(n)\sim N\left(0,\sigma_v^2\right)$ denotes the zero-mean Gaussian noise term with covariance σ_v^2. The received echo is first passed through M_t filters, *i.e.* each one matched to one of the transmitted waveforms. Then the $M_tM_r\times 1$ filters' outputs stacking in one column vector is obtained as [14]

$$\mathbf{z}=\alpha_0\mathbf{b}(\phi_0)\otimes\mathbf{Ra}(\phi_0)+\sum_{i=1}^{Q}\beta_i\mathbf{b}(\phi_i)\otimes\mathbf{Ra}(\phi_i)+\mathbf{v}_c \quad (2)$$

where $\mathbf{v}_c\sim N\left(0,\sigma_v^2\left(\mathbf{I}_{M_r}\otimes\mathbf{R}\right)\right)$ stands for the colored Gaussian noise vector, $\otimes$ symbolizes the Kronecker product, $\mathbf{R}=\left(1/N\right)\sum_{n=1}^{N}\mathbf{s}(n)\mathbf{s}^H(n)\succeq 0$ refers to the TCM, which is directly related with the sampled waveforms. $\mathbf{R}\succeq 0$ denotes that $\mathbf{R}$ is positive semi-definite and N is the sample number.

In view of the monotonic relation between the detection probability and SNR/SINR [15], the design criterion of $\mathbf{R}$ and receive filter $\boldsymbol{\omega}$ is always the SINR maximization. The SINR of signal in (2) is given by [16][17]

$$SINR=\left(\rho\left|\boldsymbol{\omega}^H\mathbf{b}(\phi_0)\otimes\mathbf{Ra}(\phi_0)\right|^2\right)\Big/\left(\boldsymbol{\omega}^H\mathbf{R}_{in}\boldsymbol{\omega}\right) \quad (3)$$

where $\mathbf{R}_{in}=\sum_{i=1}^{Q}\eta_i\left|\mathbf{b}(\phi_i)\otimes\mathbf{Ra}(\phi_i)\right|^2+\left(\mathbf{I}_{M_r}\otimes\mathbf{R}\right)$ denotes the interference-plus-noise covariance matrix, $\mathbf{I}_{M_t}$ refers to a $M_t\times M_t$ identity matrix, $\rho=E\left\{\left|\alpha_0\right|^2\right\}/\sigma_v^2$, $\eta_i=E\left\{\left|\beta_i\right|^2\right\}/\sigma_v^2$ and $E\{\ \}$ is the statistical expectation. By employing the principle of minimum variance distortionless response (MVDR), the optimum receive filter and the maximum (optimum) SINR can be derived as (4) and (5), respectively. It is seen that $\mathbf{R}$ can be optimized for the MIMO radar waveform design and SINR improvement.

$$\boldsymbol{\omega}=\frac{\mathbf{R}_{in}^{-1}\mathbf{b}(\phi_0)\otimes\mathbf{Ra}(\phi_0)}{\mathbf{b}^H(\phi_0)\otimes\mathbf{a}^H(\phi_0)\mathbf{R}^H\mathbf{R}_{in}^{-1}\mathbf{b}(\phi_0)\otimes\mathbf{Ra}(\phi_0)} \quad (4)$$

$$SINR_{opt}=\rho\mathbf{b}^H(\phi_0)\otimes\mathbf{a}^H(\phi_0)\mathbf{R}^H\mathbf{R}_{in}^{-1}\mathbf{b}(\phi_0)\otimes\mathbf{Ra}(\phi_0) \quad (5)$$

3. PROPOSED TCM WITH THE FORM OF TOEPLITZ MATRIX

It is known that employing a proper TCM, the transmitted power can be cohered in the region of interest (ROI) and the power of back scattered signals from target at the receiver can increase [10]. To improve the SINR and consider the SINR level of phased array as an upper limit, a kind of more general symmetrical Toeplitz matrix $\mathbf{R}_{pm}$ is proposed as the TCM for the waveform design of colocated MIMO radars

$$\mathbf{R}_{pm}=\begin{bmatrix} 1 & \frac{M_t-m}{M_t} & \cdots & \frac{M_t-(M_t-1)m}{M_t} \\ \frac{M_t-m}{M_t} & 1 & \cdots & \vdots \\ \vdots & \ddots & \ddots & \frac{M_t-m}{M_t} \\ \frac{M_t-(M_t-1)m}{M_t} & \cdots & \frac{M_t-m}{M_t} & 1 \end{bmatrix} \quad (6)$$

where $0<m<3M_t/\left(M_t+1\right)$ is the control parameter to generate difference TCM. Especially, when $m=1$, $\mathbf{R}_{pm}=\mathbf{R}_{p1}$, when $m=2$, $\mathbf{R}_{pm}=\mathbf{R}_{p2}$ in [10], and $\sin\left(\left(\pi/2\right)\mathbf{R}_{p1}\right)=\mathbf{R}_{2x}$ in [9]. It is obvious that $\mathbf{R}_{pm}$ can be characterized only by its first row or first column. Let $\left\{h(d)\right\}_{d=0}^{M_t-1}=\left\{\left(M_t-dm\right)/M_t\right\}_{d=0}^{M_t-1}$ denote the elements in the first row and assume $\left\{H(k)\right\}_{k=0}^{M_t-1}$ are the frequency domain samples for $\left\{h(d)\right\}_{d=0}^{M_t-1}$ followed with the M_t-point discrete Fourier transformation, then we can proofed that $\mathbf{R}_{pm}$ is full-ranked based on the lemma in [18]. The detailed proof is not shown here for the limited length of paper.

3.1. Maximum SINR for only noise

For the only-noise case without interferences in (5), we have $\mathbf{R}_{in}^{-1}=\left(\mathbf{I}_{M_r}\otimes\mathbf{R}_{pm}\right)^{-1}=\mathbf{I}_{M_r}\otimes\mathbf{R}_{pm}^{-1}$ using $\left(\mathbf{A}\otimes\mathbf{B}\right)^{-1}=\mathbf{A}^{-1}\otimes\mathbf{B}^{-1}$, and the maximum SINR employing $\mathbf{R}_{pm}$ can be formulated as (7) based on $\left(\mathbf{A}\otimes\mathbf{B}\right)\left(\mathbf{C}\otimes\mathbf{D}\right)=\left(\mathbf{AC}\right)\otimes\left(\mathbf{BD}\right)$

$$SINR_{pm}=\rho\mathbf{b}^H(\phi_0)\otimes\mathbf{a}^H(\phi_0)\mathbf{R}_{pm}^H\left(\mathbf{I}_{M_r}\otimes\mathbf{R}_{pm}^{-1}\right)\mathbf{b}(\phi_0)\otimes\mathbf{R}_{pm}\mathbf{a}(\phi_0) \tag{7}$$
$$=\rho\left(\mathbf{b}^H(\phi_0)\otimes\mathbf{a}^H(\phi_0)\mathbf{R}_{pm}^H\right)\left(\left(\mathbf{I}_{M_r}\mathbf{b}(\phi_0)\right)\otimes\left(\mathbf{R}_{pm}^{-1}\mathbf{R}_{pm}\mathbf{a}(\phi_0)\right)\right)$$
$$=\rho\left(\mathbf{b}^H(\phi_0)\mathbf{b}(\phi_0)\right)\otimes\left(\mathbf{a}^H(\phi_0)\mathbf{R}_{pm}^H\mathbf{a}(\phi_0)\right)$$
$$=\rho M_r\left(\mathbf{a}^H(\phi_0)\mathbf{R}_{pm}^H\mathbf{a}(\phi_0)\right)$$

For the simplify of derivation, it is always assumed that the target is located at $\phi_0=0^o$, hence $\mathbf{a}(\phi_0)=[1,1,\cdots,1]^T$ and

$$\mathbf{R}_{pm}\mathbf{a}_t(\phi_0)=\begin{bmatrix}\sum_{i=0}^{M_t-1}\frac{M_t-im}{M_t}+\sum_{i=0}^{0}\frac{M_t-im}{M_t}-1\\ \vdots\\ \sum_{i=0}^{M_t-k}\frac{M_t-im}{M_t}+\sum_{i=0}^{k-1}\frac{M_t-im}{M_t}-1\\ \vdots\\ \sum_{i=0}^{M_t-1}\frac{M_t-im}{M_t}\end{bmatrix} \tag{8}$$

where k is the row index. By employing $\sum_{q=1}^{n}q=n(n+1)/2$ and $\sum_{q=1}^{n}q^2=n^3/3+n^2/2+n/6$, (7) can be derived as

$$\begin{aligned}SINR_{\mathbf{R}_{pm}}&=\rho M_r\sum_{k=1}^{M_t}\left(\sum_{i=0}^{M_t-k}\left(\frac{M_t-im}{M_t}\right)+\sum_{i=0}^{k-1}\left(\frac{M_t-im}{M_t}\right)-1\right)\\&=\rho M_r\sum_{k=1}^{M_t}\left(\left(M_t-k+1-\sum_{i=0}^{M_t-k}\left(\frac{im}{M_t}\right)\right)+k-\sum_{i=0}^{k-1}\left(\frac{im}{M_t}\right)-1\right)\\&=\rho M_r\sum_{k=1}^{M_t}\left(M_t-\frac{m}{M_t}\left(\frac{M_t^2}{2}-kM_t+\frac{M_t}{2}+k^2-k\right)\right)\\&=\rho M_r\left(\left(1-\frac{m}{3}\right)M_t^2+\frac{m}{3}\right)\end{aligned} \tag{9}$$

Note that for $0<m<3M_t/(M_t+1)$, we can arrive at

$$\left(1-\frac{m}{3}\right)M_t^2+\frac{m}{3}>M_t \tag{10}$$

That is, the maximum SINR level using $\mathbf{R}_{pm}$ outperforms the one of omnidirectional MIMO radar ($SINR_m=\rho M_rM_t$). Meanwhile, when $m\to 0$, the SINR level in (9) gets closed to the optimum one of phased array ($SINR_p=\rho M_rM_t^2$). Therefore, we can conclude $SINR_m<SINR_{pm}<SINR_p$. Especially, when $m=1$, $SINR_{p1}=\rho M_r\left((2/3)M_t^2+1/3\right)$, when $m=2$, $SINR_{p2}=\rho M_r\left((1/3)M_t^2+2/3\right)$ in [10].

3.2. Synthesis with BPSK waveforms

In the finite alphabet correlated waveform design [7][8], the desired TCM $\mathbf{R}$ can be built from BPSK samples x's, where $x=sign(r)$ and r's are Gaussian random variables with covariance matrix $\mathbf{R}_s$. Meanwhile, the relationship $\mathbf{R}_s(m,n)=\sin\left(0.5\pi\mathbf{R}(m,n)\right)$ must be satisfied, *i.e.* $\mathbf{R}_s=\sin\left(0.5\pi\mathbf{R}\right)$ [7][8], where $\mathbf{R}(m,n)$ stands for the mth row and nth column element of $\mathbf{R}$. Since $\mathbf{R}_s$ is positive semi-definite for the realization of $\mathbf{R}$ using BPSK waveforms in closed form, therefore $\sin\left(0.5\pi\mathbf{R}\right)$ has to be positive semi-definite [10][15]. Fortunately, we can derive (11) from (12) for the eigenvalue solving.

$$\lambda^{M_t-2}\left(\lambda^2-M_t\lambda+\left(\frac{1}{4}M_t^2-\frac{1-\cos(m\pi)}{8\sin^2\left(\frac{m\pi}{2M_t}\right)}\right)\right)=0 \tag{11}$$

$$\det\left(\sin\left((\pi/2)\mathbf{R}_{pm}\right)-\lambda\mathbf{I}_{M_t}\right)=0 \tag{12}$$

then we can conclude that $\sin\left(0.5\pi\mathbf{R}_{pm}\right)$ has M_t-2 zero eigenvalues and two non-zero eigenvalues as

$$\begin{cases}\lambda_1=\frac{M_t}{2}+\frac{1}{2}\sqrt{\left(1-\cos(m\pi)\right)\Big/\left(1-\cos\left(\frac{m\pi}{M_t}\right)\right)}>0\\ \lambda_2=\frac{M_t}{2}-\frac{1}{2}\sqrt{\left(1-\cos(m\pi)\right)\Big/\left(1-\cos\left(\frac{m\pi}{M_t}\right)\right)}\end{cases} \tag{13}$$

For $0<m<3M_t/(M_t+1)$, it is easily derived that $\lambda_2>0$. Furthermore, $\sin\left(0.5\pi\mathbf{R}_{pm}\right)$. Herein, the detailed proof is also not given for the limited length of paper.

Actually, let $\theta_1=\sin^{-1}\left(m/(2M_t)\right)$ and $\theta_2=\sin^{-1}\left(-m/(2M_t)\right)$, then $\sin\left((\pi/2)\mathbf{R}_{pm}\right)$ is positive semi-definite and can be reformulated as the auto-correlation matrix sum of two orthogonal steering vectors $\mathbf{a}_t(\theta_1)$ and $\mathbf{a}_t(\theta_2)$, *i.e.*

$$\sin\left((\pi/2)\mathbf{R}_{pm}\right)=(1/2)\left(\mathbf{a}_t(\theta_1)\mathbf{a}_t^H(\theta_1)+\mathbf{a}_t(\theta_2)\mathbf{a}_t^H(\theta_2)\right) \tag{14}$$

The angle difference between θ_1 and θ_2 is $2\sin^{-1}\left(m/(2M_t)\right)$, which is the minimum difference for $\mathbf{a}_t(\theta_1)$ and $\mathbf{a}_t(\theta_2)$ to be orthogonal.

4. SIMULATION RESULTS

In our simulation, to evaluate the performance of the representative TCMs $\mathbf{R}_{pm}$, where m is selected from 0.5 to 2.5 with the interval 0.5, two examples are presented in comparison with the phased array, conventional omnidirectional MIMO radar, and the scheme using $\mathbf{R}_{2x}$ in [9]. The target is located at $\phi_0=0^o$, where the power is expected to be cohered. The total transmitted power equals to M_t and the MVDR beamformer is employed as the receiving filter.

In the first example, it is assumed that $M_t = 10$, $M_r = 10$ and there are two signal-dependent interfering sources located at $\phi_1 = -15^o$ and $\phi_2 = 25^o$ with the interference-to-noise ratio (INR) 30 *dB*. Fig. 1 depicts the obtained SINR levels of the compared schemes with different SNR and the receive beampatterns are shown in Fig. 2. It can be seen that: (a) The SINR level using each $\mathbf{R}_{pm}$ is higher than the one of conventional MIMO radar, and higher SINR level can be obtained by employing $\mathbf{R}_{pm}$ with smaller *m*. Meanwhile, $\mathbf{R}_{p0.5}$ outperforms the other MIMO radar schemes and gets closed to the phased array, which benefits from that the transmitted power using $\mathbf{R}_{pm}$ with smaller *m* are more cohered in the target sector of interest (SOI), when there are deep nulls (less than -30 *dB*) in the direction of interferences. (b) Compared to $\mathbf{R}_{2x}$ in [9], whose rank is only 2, $\mathbf{R}_{pm}$ is firstly full-ranked and can sufficiently exploit the benefits of full waveform diversity to reject more interferences. Besides, as a special case, $\mathbf{R}_{p2}$ has comparable low SLLs with $\mathbf{R}_{2x}$ in Fig. 2. The SLLs using $\mathbf{R}_{p1.5}$ are also low. The low SLLs are beneficial to suppress the unwanted interferences out of SOI.

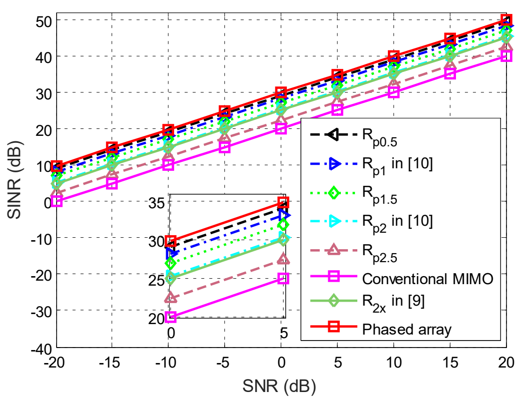

Fig. 1. SINR versus SNR, where $M_t = M_r = 10$.

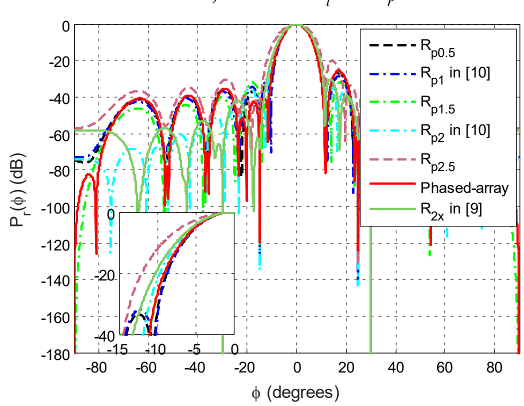

Fig. 2. Receive beampatterns, where $M_t = M_r = 10$.

To show the maximum capability of interference suppression of all schemes, in second simulation, we assume that $M_t = 6$, $M_r = 6$ and there are 12 interfering sources located at 25º, -25º, -85º, 80º, 15º, -10º, 30º, -70º, 55º, 45º, -40º, 50º. Fig. 3 shows the average SINR level for different number of interfering sources. Since $\mathbf{R}_{pm}$ are full-ranked, the colocated MIMO radar using $\mathbf{R}_{pm}$ can suppress $2M_t\text{-}2 = 10$ interfering sources. While the phased array only can suppress $M_t\text{-}1 = 5$ interfering sources and the scheme using $\mathbf{R}_{2x}$ can suppress $M_t + 2 - 1 = 7$ interfering sources. From Fig. 3, it is seen that when the interfering source number $N_i > 5$, the average SINR level of phased-array radar drops obviously, when $N_i > 7$, the one of scheme using $\mathbf{R}_{2x}$ in [9] also degrades. In contrast, the schemes using $\mathbf{R}_{pm}$ can suppress the interferences effectively with the average output SINR>32*dB*. Also, the average SINR level using $\mathbf{R}_{p0.5}$ outperforms the others.

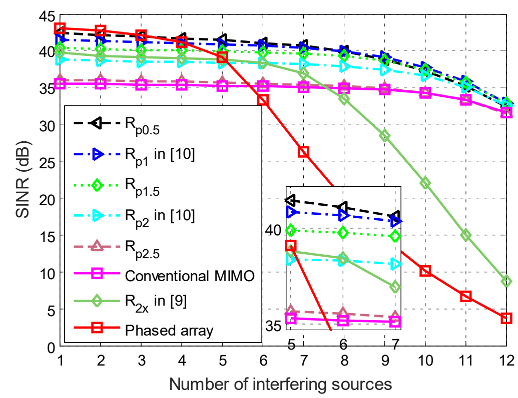

Fig. 3. Average SINR level comparison for different number of interfering sources, where $M_t = M_r = 6$.

5. CONCLUSIONS

In this paper, a kind of TCM $\mathbf{R}_{pm}$ with the form of symmetrical Toeplitz matrix has been proposed for the waveform design of colocated MIMO radars. Compared to the TCM $\mathbf{R}_{2x}$ in [9], the rank of which is only 2, $\mathbf{R}_{pm}$ is full-ranked and $\sin\left((\pi/2)\mathbf{R}_{pm}\right)$ is positive semi-definite, which guarantees that $\mathbf{R}_{pm}$ can sufficiently exploit the advantage of full waveform diversity and also can be synthesized with BPSK waveforms in closed form. The proposed TCMs all yield gains in SINR level compared to the omnidirectional MIMO radar. Simulation results show that when the directions of target and interfering sources are known, the SINR level using $\mathbf{R}_{pm}$ with small *m* is higher than the counterparts. Meanwhile, lower SLLs can be obtained with certain lager *m* (*e.g.* *m* =1.5 or 2) for the unwanted sidelobe interference suppression.

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