ANGLE DEPENDENT MATCH FILTER DESIGN FOR CIRCULATING CODE

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ABSTRACT

Circulating code provides a simple way to achieve transmit diversity with stable gain along wide angular coverage, whereas this property is at costs of poor sidelobe signal suppression ability and degraded range resolution. In this paper, an angle dependent match filter combined with slow-time coding is proposed and investigated to solve the above problem. The angle dependent match filter is designed via convex optimization technique to suppress interfering signals from sidelobes with a controllable penalty in the target signal-to-noise ratio (SNR). Then the slow-time coding technique is used to recover range resolution. Numerical experiments are carried out using simulated data. The performance assessments show that the proposed technique is superior in interference suppression and range resolution improvement.

Index Terms— circulating code, filter design, interference suppression, convex optimization, slow-time coding

1. INTRODUCTION

As a new transmit diversity technique, circulating code has gain much attention in recent years [1-8]. The circulating code employs a small time delay increment across the array elements while the waveform is same for each element, which makes it different from the phased-array. As shown in [3-4], circulating code can generate wide transmit beam with constant gain across the angular coverage while only minimal modification of transmitter hardware (the controlled time shifters) is required.

Although circulating code is a simple way to achieve transmit diversity, the range resolution is compromised since the effective bandwidth for each angle is less than the transmit bandwidth. Specifically, range resolution is degraded by a factor N due to the circulating code, where N is the number of transmitting elements. To improve the range resolution, a short pure spatial (time-invariant) code is applied to array elements while circulating code is used [6]. Another method in [8] uses slow-time coding technique to circulating code, which synthesizes a high resolution range profile with a pulse train. By these hybrid coding methods, the degraded range resolution recovers to the original one. However, for all these circulating code related waveforms, the match filter at receiver is coupled with transmit beamforming. When the filter is matched to a given direction θ_0, relatively poor suppression would happen if echoes from sidelobe exist. Thus, the strong echoes from sidelobe would cause false alerts in the given direction.

In this paper, it is aimed to suppress interfering echoes from sidelobe while recovering the degraded range resolution. Inspired by mismatch filter design methods using convex optimization [9-11], an angle dependent match filter is designed via convex optimization technique to suppress interfering signal from sidelobes with a controllable penalty in the target signal-to-noise ratio (SNR). Then the slow-time coding technique is used to recover range resolution. The effectiveness of the proposed method is verified by the simulation results.

The rest of the paper is organized as follows. In Section 2, the signal model of circulating code is given. Section 3 presents the angle dependent match filter combined with slow time coding technique. In Section 4, simulation results are provided to validate the effectiveness of the proposed method. Finally, conclusions are given in Section 5.

2. CIRCULATING CODE MODEL

According to the circulating signal principle [3], for a transmit array with N antennas, the waveform $s_n(t)$ of the nth antenna can be written as

$$s_n(t) = s(t-(n-1)\Delta t) \tag{1}$$

where $s(t)$ is the same waveform transmitted by each antenna. Δt is the time increment across the array elements which equals to $1/B$. B is the signal bandwidth.

For a given direction θ_0, the synthetic signal of a uniform linear array (ULA) with N antennas shown in Fig.1 can be expressed as

$$s_T(t,\theta_0) = \sum_{n=1}^{N} e^{j2\pi \frac{f_0 d_T}{c}(n-1)\sin\theta_0} e^{j2\pi f_0 t} s_n(t) \tag{2}$$

where d_T is the interspacing of transmit antennas. c is the speed of light. f_0 is the carrier frequency. The first phase term is caused by the wave path difference.

At the receiver, for the purpose of simplicity, a single receiving channel is considered. The echo from a target at angle θ_0 is received by the channel. After down conversion the received signal is given by

$$\begin{aligned} s_R\left(t,\theta_0\right) &= A_0 e^{j2\pi f_0\tau_0}\sum_{n=1}^{N} e^{j2\pi\frac{f_0 d_T}{c}(n-1)\sin\theta_0} e^{j2\pi f_d\left(t-\tau_0\right)} \\ &\times s'\left(t-\tau_0-(n-1)\Delta t\right)+n(t) \end{aligned} \tag{3}$$

where A_0 is the unknown complex target echo coefficient. τ_0 is the propagation time delay. f_d is the Doppler frequency. $n(t)$ is the noise at time *t*.

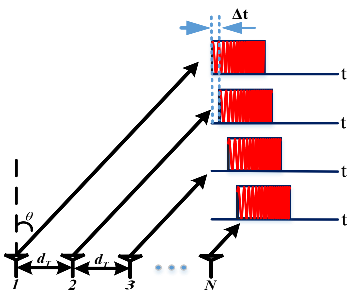

Fig. 1. The transmit array configuration

From equation (3), it can be seen that angle related phase term is coupled with time-shifted waveform in received signal. Usually, a joint transmit beamforming and match filter (TBMF) procedure after down conversion is used which can be defined as

$$r(t) = s_R\left(t,\theta_0\right)\otimes s_T^*\left(-t,\theta'\right) \tag{4}$$

where $r(t)$ is the target echo after TBMF. $(\cdot)^*$ denotes complex conjugation. $\otimes$ represents the convolution process. θ' is the assumed arrive angle. When mismatch between θ' and θ_0 exists, the defocusing problem results in the relatively poor suppression of the interfering signals from sidelobes.

3. ANGLE DEPENDENT MATCH FILTER DESIGN

As aforementioned, the match filter and transmit beamforming for circulating code are coupled at the receiver. the strong echoes from sidelobes would cause interfering peaks. In this section, the angle dependent match filter is designed to suppress strong false peaks. We consider the discrete transmit signal $s_{T\theta_0}(l) = s_{T\theta_0}(t)\big|_{t=(l-1)T_s}$, $l = 1,2,\ldots,L$, where $T_s = 1/B$ is the sample time, $L = T_p/T_s$, T_p is the time duration. The angle dependent match filter in response to the transmitted signal is given by

$$y_{\theta_0}(l) = \sum_{m=1}^{2L-1} h(m) s_{T\theta_0}(l-m+1) \tag{5}$$

where $\mathbf{h}$ is the designed filter vector. $y_{\theta_0}(l)$ is the *l*th range coefficient of the filter output for the angle θ_0. The peak value of $y_{\theta_0}(l)$ is at $l = L-1$. We stack the real and imaginary components of $\mathbf{h}$ into a real vector which can be expressed as

$$\mathbf{h}_p = \left[\Re\{h(1)\},\cdots,\Re\{h(L)\},\Im\{h(1)\},\cdots,\Im\{h(L)\}\right]^T \tag{6}$$

where $\Re\{\cdot\}$, $\Im\{\cdot\}$ denotes taking the real and imaginary components. $(\cdot)^T$ denotes the transpose function.

To compute the response of $y_\theta(L-1)$ for an arbitrary angle θ, we construct constraint matrices $\mathbf{X}_\theta$ from $\mathbf{s}_{T\theta}$ as

$$\begin{aligned} &\mathbf{X}_\theta(1,i) = \Re\{s_{T\theta}(i)\} \\ &\mathbf{X}_\theta(1,i+L) = -\Im\{s_{T\theta}(i)\} \\ &\mathbf{X}_\theta(2,i) = \Im\{s_{T\theta}(i)\} \\ &\mathbf{X}_\theta(2,i+L) = \Re\{s_{T\theta}(i)\} \end{aligned} \quad i = 1,\ldots L \tag{7}$$

Then, the output $y_\theta(L-1)$ of the filter follows

$$y_\theta(L-1) = \mathbf{X}_\theta(1,:)\mathbf{h}_p + j\mathbf{X}_\theta(2,:)\mathbf{h}_p \tag{8}$$

where $\mathbf{X}_\theta(i,:)$ is defined as the *i*th row element of $\mathbf{X}_\theta$.

Furthermore, the signal energy of an arbitrary angle at $l = L-1$ can be expressed as

$$\left\|y_\theta\right\|^2 = \left\|\mathbf{X}_\theta\mathbf{h}_p\right\|^2 = \mathbf{h}_p^T\mathbf{Q}_\theta\mathbf{h}_p \tag{9}$$

where $\mathbf{Q}_\theta = \mathbf{X}_\theta^T\mathbf{X}_\theta$.

The problem of filter design is to suppress signals from sidelobes while achieving a controllable penalty in the desired angle SNR. In this paper, this leads to the following optimization problem:

$$\min_{\mathbf{h}_p} \quad \varepsilon \tag{10a}$$

$$\text{s.t.} \quad \mathbf{h}_p^T\mathbf{Q}_{\theta_0}\mathbf{h}_p / G^2 \geq \delta \tag{10b}$$

$$\mathbf{h}_p^T\mathbf{h}_p = G \tag{10c}$$

$$\mathbf{h}_p^T\mathbf{Q}_{\theta_i}\mathbf{h}_p / G^2 \leq \varepsilon, \theta_i \in \Phi \tag{10d}$$

where $G = \mathbf{s}_{T\theta_0}\mathbf{s}_{T\theta_0}^H$, $(\cdot)^H$ denotes the conjugate transpose function. δ is the lower bound for the output of desired θ_0. θ_i is an interfering angle from the range Φ. ε is the desired maximum acceptable gain level of Φ. In practice, the range Φ can be set based on *a prior* knowledge like the covariance matrix analysis and so on. Here, we assume that the Φ is known.

To take a further analysis to equation (10), we can see that equation (10b) restricts the main peak loss with a lower bound. Equation (10c) normalizes the filter so that the norm of the coefficients is equal to that of the transmit synthetic signal. Finally, equation (10d) restricts the magnitude of interfering signal from unexpected angle range Φ. The formulation of (10) is a quadratically constrained quadratic program (QCQP) because of the quadratic equality of (10c) and the quadratic greater-than inequality constraint of (10b).

Unfortunately, QCQP problems are intrinsically hard to solve. With some relaxation, a potential solution to (10) can be attained by solving the following second-order cone programming (SOCP) problem:

$$\min_{\mathbf{h}_p} \quad \varepsilon \tag{11a}$$

$$\text{s.t.} \quad \mathbf{X}_{\theta_0}(1,:)\mathbf{h}_p / G \geq \sqrt{\delta} \tag{11b}$$

$$\mathbf{X}_{\theta_0}(2,:)\mathbf{h}_p / G = 0 \tag{11c}$$

$$\mathbf{h}_p^T \mathbf{h}_p \leq G \tag{11d}$$

$$\mathbf{h}_p^T \mathbf{Q}_{\theta_i} \mathbf{h}_p / G^2 \leq \varepsilon, \theta_i \in \Phi \tag{11e}$$

We can see from (11b) and (11c) that the main peak loss constrain is loosened to having a purely real and positive value. Besides, the equality constraint of (10c) is relaxed to set an upper bound to the inner product of the $\mathbf{h}_p$ vector.

A complex vector $\mathbf{h}$ is formed from the top and bottom halves of $\mathbf{h}_p$ to give the optimal complex vector $\mathbf{h}_{op}$. If the equality does not hold in (11d), the resulting vector needs to be normalized which is given by

$$\mathbf{h}_{op} = \left(\frac{G}{\mathbf{h}\mathbf{h}^H} \right)^{1/2} \mathbf{h} \tag{12}$$

So far the designed filter is aimed to suppress the interfering signals from sidelobes. To jointly recover the range resolution, the slow-time coding technique proposed in [8] is used. When applying this technique, the transmitted signal of the array changes from pulse to pulse. Therefore, a set of filters which corresponds to each pulse should be designed by solving optimization problem in (11). Note that the proposed filters would not cause any phase difference when compared to TBMF since the main peak is constrained to be a purely real and positive value. Therefore, the proposed filter can be directly combined with slow-time coding technique by changing the TBMF into the proposed filter. Then the high resolution range profile can be synthesized through the pulse train.

4. DESIGN EXAMPLES AND SIMULATION RESULTS

In this section, a uniform array with 11 antennas has been considered. The carrier frequency is 1GHz and the bandwidth B is 5MHz. $T_p = 50us$. $s(t)$ is chosen to be linear frequency modulated (LFM) signal. In the first simulation, the pointed angle of the filter is set to be 0^0. The interfering angle range Φ is $[25^0, 35^0]$. δ is set to be 0.891. Fig.2(a)-(b) shows the response after match filter and range profile synthesis in range-angle domain. From Fig.2(a), we can see that a suppression less than 30dB is achieved in the desired area when using TBMF filter. However, the interfering angle range Φ of peak range bin are suppressed over -60dB when using the proposed filter combined with slow-time coding. Therefore, when strong coherent interference comes from Φ, the proposed filter can get better performance.

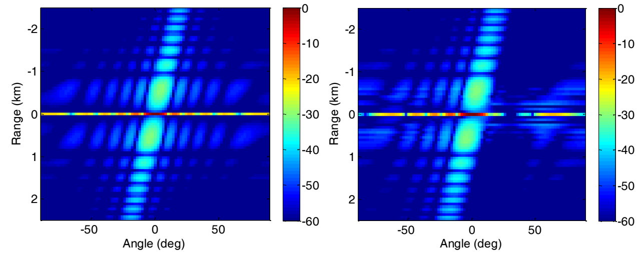

Fig.2 The 0^0 response after match filter and range profile synthesis in range-angle domain (a) TBMF (b) proposed filter

In the next simulation, the pointed angle of the filter is set to be 60^0. The interfering angle range Φ is $[-10^0, 10^0]$. δ is set to be 0.891. Similar to first case, in Fig.3, the TBMF based method cannot achieve a wide deep null in the desired angle range. However, the proposed filter combined with slow-time coding provides nulls over -60dB. We can see that our method is effective to an arbitrary angle.

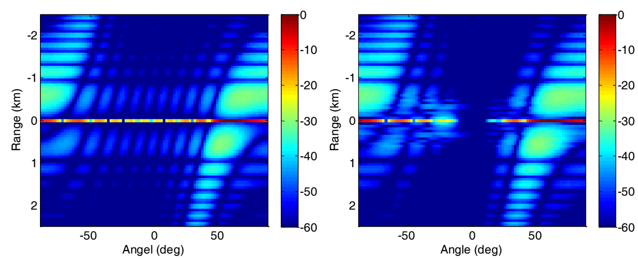

Fig.3 The 60^0 response after match filter and range profile synthesis in range-angle domain (a) TBMF (b) proposed filter

The last example is devoted to comparing the range profiles of multiple targets for different types of circulating codes. In this example, two stationary targets located at 60km and 65km, at angles 0^0 and 30^0 are considered. The observed range profiles were estimated in the absence of noise. The energy of target from 30^0 is 20dB higher than target from 0^0. Fig.4 plots the normalized range profiles of several codes. Since both TBMF and the proposed filter are angle dependent, when filers are matched with 0^0, the target from 30^0 in the simulation would be the interfering target. For circulating code with TBMF, the true peak at 0^0 is much wider than other cases due to the partially bandwidth illuminated to each angle. Besides, false peaks around 65km is 4dB higher than true target at 60km. In case of the hybrid code in [6], the narrow range peak is achieved compared with circulating code. Nevertheless, the increased range resolution is at a cost of poor interfering signal suppression performance since false peaks are over 12dB higher than the matched target in Fig.4(b). As for the circulating code with slow-time coding shown in Fig.4(c), when using TBMF, the narrow peak is achieved with relatively better interfering suppression ability since the interfering peak is much thinner. However, the false peak is still high compared with true target. When slow-time coding technique is combined with the proposed filter shown in Fig.2(b), it can be seen that the false peaks have been suppressed effectively because no obvious false peaks can be seen when compared with range sidelobes. Hence, the proposed filter is effective to suppress coherent interfering signals.

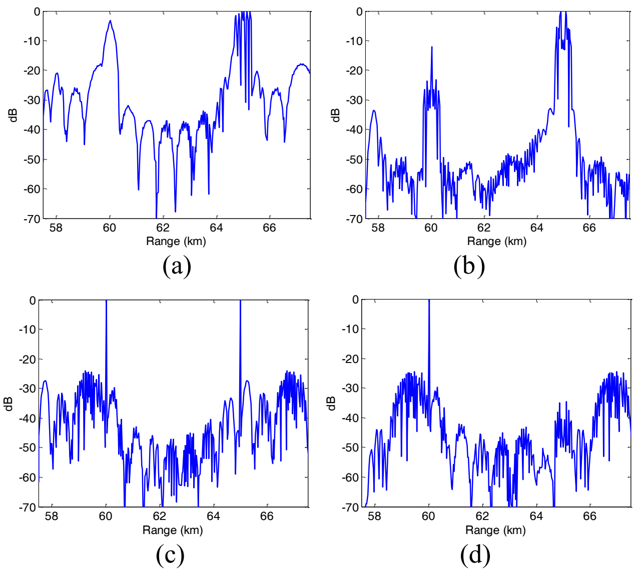

Fig.4 The normalized range profile in angular direction 0^0 (a) circulating code with TBMF, (b) hybrid code in [6] with TBMF, (c) hybrid code in [8] with TBMF (d) proposed method

5. CONCLUSION

In this paper, an angle dependent match filter has been designed for coherent interference suppression. The proposed design is directly formulated as a QCQP which is nondeterministic polynomial-time hard. By loosen quadratic equality and quadratic greater-than inequality constraints, the problem can be efficiently solved by SOCP. Then, the filter is combined with slow-time coding technique to jointly improve the range resolution. Compare with the conventional TBMF based receive processing procedure, the proposed receive processing chain can effectively suppress coherent interference or strong clutters coming from sidelobes without any range resolution loss.

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REFERENCES

[1] G. Babur, P. Aubry, and F. Le Chevalier, “Space-time codes for active antenna systems,” Research Disclosures (www.researchdisclosure.com), 2013, pp. 1–14
[2] F. Le Chevalier, “Space-time transmission and coding for airborne radars,” *CIE J. Radar Sci. Technol.*, vol. 6, no. 12, pp. 411–421, 2007.
[3] G. Babur, P. Aubry, and F. Le Chevalier, “Space-time radar waveforms: circulating codes,” *J. Electr. Comput. Eng., Special Issue on ‘Adv. Radar Technol.’*, vol. 2013, pp. 1–8, 2013.
[4] K. Roussel, G. Babur, and F. Le Chevalier, “Optimization of low sidelobes radar waveforms: circulating codes,” *Proc. Int. Radar Conf.*, Lille, France, pp. 1–6, Oct. 2014.
[5] G. Babur, F. Le Chevalier, and P. Aubry, “Space-time codes for active antenna systems: comparative performance analysis,” *Proc. IET Int. Radar Conf.*, Xi’an, China, pp. 1–6, Apr. 2013.
[6] G. Babur, P. Aubry, and F. Le Chevalier, “Simple transmit diversity technique for phased array radar,” *IET Radar, Sonar & Navigation*, vol. 10, no. 6, pp. 1046-1056, 2016.
[7] L. Lan, G. Liao, J. Xu, Y. Zhang and Y. Xu, “Low sidelobes optimization with subarrays for space-time circulating LFMs” *IEEE Radar Conference*, IEEE, pp. 73-77, Apr. 2017.
[8] S. Li，L. Zhang, N. Liu, J. Zhang, and S. Tang, “Transmit Diversity Technique Based on Joint Slow-time Coding with Circulating Code,” *IET Radar, Sonar & Navigation*, vol. 11, no. 8, pp. 1243-1250, 2017.
[9] P. Kajenski, “Mismatch filter design via convex optimization,” *IEEE Transactions on Aerospace & Electronic Systems*, vol. 52, no. 4, pp. 1587-1591, 2016.
[10] T. Aittomaki, V. Koivunen, “Mismatched filter design for radar waveforms by semidefinite relaxation,” *IEEE International Conference on Acoustics, Speech and Signal Processing*, IEEE, pp. 2739-2743, 2015.
[11] W. Blake, “Multi-dimensional mismatch filter design optimization for radar waveforms,” *Radar Conference*, IEEE, pp. 1-4, 2016.