

JOINT SPACE-(SLOW) TIME TRANSMISSION WITH UNIMODULAR WAVEFORMS AND RECEIVE ADAPTIVE FILTER DESIGN FOR RADAR

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ABSTRACT

A novel computationally efficient method for jointly designing the space-(slow) time (SST) transmission with unimodular waveforms and receive adaptive filter is developed for different radar configurations. The range sidelobe effect and Doppler characteristics are considered. In particular, we develop a novel approach for jointly synthesizing unimodular SST waveforms and minimum variance distortionless response receive adaptive filter for two cases of known Doppler information and presence of uncertainties on clutter bins. Corresponding non-convex optimization problems are formulated and efficient algorithms are derived. The main ideas of the algorithm developments are to decouple composite objective function of the formulated problems, generate minorizing surrogates, and then solve the joint design problem iteratively, but in closed-form for each iteration by means of minorization-maximization technique. The proposed algorithms demonstrate good performance and have fast convergence speed and low complexity.

Index Terms— Adaptive filter, joint waveform design, minorization-maximization, space-(slow) time, radar.

1. INTRODUCTION

Waveform design has been the research field of significant interest over several decades [1]–[4]. Many past works focus on designing fast-time waveform(s) to achieve various desirable properties [5]–[9]. These works improve the waveform quality when the receiver is fixed as the matched filter. However, in harsh environments involving heterogeneous clutter with Doppler uncertainties and/or active jamming, the receiver should be flexibly adaptive, and therefore, joint transmission and receive filter design (JTRFD) becomes necessary.

Recent works on JTRFD [10]–[16] can be divided into two categories. The first category concentrates on designing fast-time waveform transmission and receive filter with particular constraints on waveform characteristics, which essentially trade off the signal-to-noise ratio (SNR) for signal-to-clutter-plus-noise ratio [10]–[12]. The methods therein normally do not consider Doppler information processing. Differing from the first category, the methods in the second category focus on synthesizing slow-time waveforms (for inter-pulse coding) at transmission while jointly enforcing the receive adaptive filter

[13]–[16]. As a result, they have the potential of coping with Doppler related issues, such as uncertainty, and therefore, can offer enhanced resolution, superior detection, etc. The latter newly emerged trend motivates us to further investigate the joint design with considerations on the range sidelobe effect of fast-time waveforms [17] and the waveform diversity [18] in the multi-input multi-output (MIMO) radar context [19], even with difficult constraints. It also motivates us to use space-time adaptive processing (STAP) technique [20], [21], so that the signal-to-interference-plus-noise ratio (SINR) performance can be improved through multi-dimension adaptive filter.

In this paper, we address the joint space-(slow) time (SST) transmission and receive adaptive filter design problem. We present a generic signal model suitable for different radar configurations while considering the intra-pulse compression (or range sidelobe) effect and Doppler characteristics. The SST waveforms are designed to have unit modulus by maximizing the output SINR, with which a minimum variance distortionless response (MVDR) STAP filter is associated. We devise an efficient approach based on simple iterative procedures, and find closed-form solutions to the sub-problems via minorization-maximization technique [22]. Our strategy is to minorize the composite objective function by properly designing surrogates defined in terms of quadratic form. Both cases of known Doppler information and uncertainties on clutter bins are studied. The solution to the latter case serves as a generic form for the former. Corresponding computationally efficient algorithms with good performance are proposed.

Notations: We use bold upper case, bold lower case, and italic letter to denote matrices, column vectors, and scalars, respectively. Notations $(\cdot)^T$, $(\cdot)^H$, \otimes , \odot , $\mathfrak{D}(\cdot)$, $\text{vec}(\cdot)$, $\mathbb{E}\{\cdot\}$, $|\cdot|$ and $\|\cdot\|$ are respectively the transpose, conjugate transpose, Kronecker product, Hadamard product, diagonalization, column-wise vectorization, expectation, modulus, and Euclidean norm operators. In addition, \mathbb{C} denotes the complex field, and $\mathbf{1}_M$ stands for a length- M vector of all ones.

2. PROBLEM FORMULATION

Consider an airborne colocated MIMO radar equipped with M transmit and N receive elements. At each transmit element, a burst of L pulses encoded by an independent slow-time waveform, denoted by $\phi_m \triangleq [\phi_{m,1}, \dots, \phi_{m,L}]^T \in \mathbb{C}^{L \times 1}$ for the m th element, is launched within one radar coherent processing interval (CPI). An independent fast-time waveform

*Supported in part by the Academy of Finland under Grant 299243.

of length P , denoted by $\mathbf{s}_m \in \mathbb{C}^{P \times 1}$ for the m th antenna, is repeatedly used for all intra-pulse modulations. We denote the SST and space-(fast) time (SFT) waveform matrices for transmission as $\Phi \triangleq [\phi_1, \dots, \phi_M]^T \in \mathbb{C}^{M \times L}$ and $\mathbf{S} \triangleq [\mathbf{s}_1, \dots, \mathbf{s}_M]^T \in \mathbb{C}^{M \times P}$, respectively, and define $\mathbf{R}_S^{(p)} \triangleq \mathbf{S} \mathbf{J}_p \mathbf{S}^H \in \mathbb{C}^{M \times M}$ as the waveform covariance matrix (for pulse compression) at the time lag p ($0 \leq p \leq P-1$), with $\mathbf{J}_p \in \mathbb{C}^{P \times P}$ being the p th lower shift matrix whose entries are ones on the p th off-diagonal ($p=0$ for the main diagonal) and zeros elsewhere.

At the receive end, after intra-pulse compression, i.e., matched filtering to \mathbf{S} at time lag p ($p=0$ for the target), and stacking the data into a vector, the received target vector $\mathbf{y}_t \in \mathbb{C}^{MNL \times 1}$ with a normalized Doppler frequency f_t for the target located at θ_t can be expressed as

$\mathbf{y}_t = \alpha_t \mathbf{a}_R(\theta_t) \otimes \mathcal{D}(\mathbf{d}(f_t)) \otimes ((\mathbf{R}_S^{(0)})^T \mathcal{D}(\mathbf{a}_T(\theta_t))) \phi$ (1)
 where α_t , $\mathbf{a}_T(\theta_t)$, $\mathbf{a}_R(\theta_t)$, and $\mathbf{d}(f_t)$ are the complex reflection coefficient, the transmit, receive, and Doppler steering vectors for the target, respectively, and $\phi \triangleq \text{vec}(\Phi) \in \mathbb{C}^{ML \times 1}$ is the vectorized version of Φ .

The observed clutter is a superposition of echoes from different uncorrelated scatters. Assuming that N_r ($N_r \leq L$) range rings interfere with the range-azimuth bin of interest where the target locates, and each ring consists of N_c discrete azimuth bins, the received clutter vector $\mathbf{y}_c \in \mathbb{C}^{MNL \times 1}$ can be expressed as

$$\mathbf{y}_c = \sum_{i'=0}^{N_r-1} \sum_{i=1}^{N_c} \xi_{i i'} \mathbf{a}_R(\theta_{i i'}) \otimes (\mathbf{J}_{i'} \mathcal{D}(\mathbf{d}(f_{i i'}))) \otimes ((\mathbf{R}_S^{(p)})^T \mathcal{D}(\mathbf{a}_T(\theta_{i i'}))) \phi$$
 (2)

where $\theta_{i i'}$, $f_{i i'}$, and $\xi_{i i'}$ are respectively the azimuth angle, normalized Doppler frequency, and complex reflection coefficient with zero mean, for the (i', i) th range-azimuth bin.

The overall receive data vector \mathbf{y} can be expressed as

$$\mathbf{y} = \mathbf{y}_t + \mathbf{y}_c + \mathbf{y}_{j+n}$$
 (3)

where $\mathbf{y}_{j+n} \in \mathbb{C}^{MNL \times 1}$ is the jamming plus noise vector which is assumed to be independent of the target and clutter components, and its covariance matrix is $\mathbf{R}_{j+n} \triangleq \mathbb{E}\{\mathbf{y}_{j+n} \mathbf{y}_{j+n}^H\}$. To simplify the notations, \mathbf{y}_t can be further expressed as $\mathbf{y}_t = \alpha_t \mathbf{T}_t \phi$, and \mathbf{y}_c can be expressed as

$$\mathbf{y}_c = \sum_{i'=0}^{N_r-1} \sum_{i=1}^{N_c} \xi_{i i'} \mathbf{T}_{i i'}^{(p)} \phi = \sum_{i'=0}^{N_r-1} \sum_{i=1}^{N_c} \xi_{i i'} \tilde{\mathbf{T}}_{i i'}^{(p)} (\tilde{\mathbf{d}}(f_{i i'}) \odot \phi)$$
 (4)

where $\tilde{\mathbf{d}}(f_{i i'}) \triangleq \mathbf{d}(f_{i i'}) \otimes \mathbf{1}_M$, $\mathbf{T}_t \triangleq \mathbf{a}_R(\theta_t) \otimes \mathcal{D}(\mathbf{d}(f_t)) \otimes ((\mathbf{R}_S^{(0)})^T \mathcal{D}(\mathbf{a}_T(\theta_t)))$, $\mathbf{T}_{i i'}^{(p)} \triangleq \mathbf{a}_R(\theta_{i i'}) \otimes (\mathbf{J}_{i'} \mathcal{D}(\mathbf{d}(f_{i i'}))) \otimes ((\mathbf{R}_S^{(p)})^T \mathcal{D}(\mathbf{a}_T(\theta_{i i'})))$, and $\tilde{\mathbf{T}}_{i i'}^{(p)} \triangleq \mathbf{a}_R(\theta_{i i'}) \otimes \mathbf{J}_{i'} \otimes ((\mathbf{R}_S^{(p)})^T \mathcal{D}(\mathbf{a}_T(\theta_{i i'})))$.

Using (4), the clutter covariance matrix $\mathbf{R}_c \triangleq \mathbb{E}\{\mathbf{y}_c \mathbf{y}_c^H\}$ for the case of known Doppler on clutter bins (i.e., $f_{i i'}$ is fixed), denoted in this case as \mathbf{R}_c^I , can be expressed as

$$\mathbf{R}_c^I = \sum_{i'=0}^{N_r-1} \sum_{i=1}^{N_c} \sigma_{i i'}^2 \mathbf{T}_{i i'}^{(p)} \phi \phi^H (\mathbf{T}_{i i'}^{(p)})^H$$
 (5)

with $\sigma_{i i'}^2 \triangleq \mathbb{E}\{|\xi_{i i'}|^2\}$. When $f_{i i'}$ is unknown, but rather distributed with a known probability density function (PDF) in the uncertainty interval $[\bar{f}_{i i'} - \epsilon_{i i'}/2, \bar{f}_{i i'} + \epsilon_{i i'}/2]$ with mean $\bar{f}_{i i'}$ and bounding parameter $\epsilon_{i i'}$, the clutter covariance matrix \mathbf{R}_c , denoted in this case as \mathbf{R}_c^{II} , can be expressed as

$$\mathbf{R}_c^{II} = \sum_{i'=0}^{N_r-1} \sum_{i=1}^{N_c} \sigma_{i i'}^2 \tilde{\mathbf{T}}_{i i'}^{(p)} (\phi \phi^H) \odot (\Upsilon_{i i'} \otimes \mathbf{1}_M \mathbf{1}_M^T) (\tilde{\mathbf{T}}_{i i'}^{(p)})^H$$
 (6)

where we used (4), and $\Upsilon_{i i'} \in \mathbb{C}^{L \times L}$ is a Hermitian matrix determined by the PDF of $f_{i i'}$ (see [13] for the example of uniform distribution).

Finally, the STAP filter with the weight vector $\mathbf{w} \in \mathbb{C}^{MNL \times 1}$ is applied to the received data vector \mathbf{y} . Hence, the SINR at the output of the filter can be expressed as

$$\zeta = \frac{|\alpha_t|^2 \cdot |\mathbf{w}^H \mathbf{T}_t \phi|^2}{\mathbf{w}^H (\mathbf{R}_c + \mathbf{R}_{j+n}) \mathbf{w}}$$
 (7)

The problem considered here is the joint design of SST waveform(s) and receive adaptive filter under the constraint that the waveforms have constant modulus. The design objective is to maximize SINR in (7). Under the condition that the SFT waveform matrix \mathbf{S} is known, the above joint design can be written as the following optimization problem

$$\begin{aligned} \max_{\phi, \mathbf{w}} \quad & \zeta \\ \text{s.t.} \quad & |\phi(n)| = 1, \quad n = 1, \dots, ML \end{aligned}$$
 (8)

where the constraints ensure the constant-modulus property.

3. JOINT SST WAVEFORM AND RECEIVE ADAPTIVE FILTER DESIGN

Using (7), for given ϕ , the solution of the optimization problem (8) with respect to \mathbf{w} can be easily found, and it obeys the following MVDR expression

$$\mathbf{w}_{\text{opt}}(\phi) = \frac{(\mathbf{R}_c + \mathbf{R}_{j+n})^{-1} \mathbf{T}_t \phi}{\phi^H \mathbf{T}_t^H (\mathbf{R}_c + \mathbf{R}_{j+n})^{-1} \mathbf{T}_t \phi}$$
 (9)

Inserting (9) into (7), the SINR metric ζ can be rewritten as

$$\zeta = |\alpha_t|^2 \cdot \phi^H \mathbf{T}_t^H (\mathbf{R}_c + \mathbf{R}_{j+n})^{-1} \mathbf{T}_t \phi$$
 (10)

Therefore, the optimization problem (8) with respect to ϕ only and for give \mathbf{w} can be written as

$$\begin{aligned} \max_{\phi} \quad & \phi^H \mathbf{T}_t^H (\mathbf{R}_c + \mathbf{R}_{j+n})^{-1} \mathbf{T}_t \phi \\ \text{s.t.} \quad & |\phi(n)| = 1, \quad n = 1, \dots, ML \end{aligned}$$
 (11)

The objective in (11) is a composite function of ϕ and $\mathbf{R}_c \in \{\mathbf{R}_c^I, \mathbf{R}_c^{II}\}$, where \mathbf{R}_c is also a function of ϕ . Before proceeding with solving (11), we present the following result.

Lemma 1. *The objective in (11) is minorized by*

$$\begin{aligned} g_1(\phi, \phi^{(k)}) &= (\phi^{(k)})^H \Psi(\phi^{(k)}) \phi^{(k)} + 2\Re\left\{(\phi^{(k)})^H \right. \\ &\times (\Psi(\phi^{(k)}))^H (\phi - \phi^{(k)}) \left. \right\} - (\phi^{(k)})^H \mathbf{T}_t^H (\Omega(\phi^{(k)}))^H \\ &\times (\mathbf{R}_c(\phi) - \mathbf{R}_c(\phi^{(k)})) \Omega(\phi^{(k)}) \mathbf{T}_t \phi^{(k)} \end{aligned}$$
 (12)

where $\phi^{(k)}$ is the SST waveform vector obtained at the k th iteration, and $\Omega(\phi^{(k)}) \in \mathbb{C}^{MNL \times MNL}$ and $\Psi(\phi^{(k)}) \in \mathbb{C}^{NL \times NL}$ are both functions of ϕ , defined as $\Omega(\phi^{(k)}) \triangleq (\mathbf{R}_c(\phi^{(k)}) + \mathbf{R}_{j+n})^{-1}$ and $\Psi(\phi^{(k)}) \triangleq \mathbf{T}_t^H \Omega(\phi^{(k)}) \mathbf{T}_t$.

Proof. Using Taylor's theorem and considering the first order expansion of the objective in (11), it can be straightforwardly proved (but after some derivations that we omit due to space limitation) that (12) minorizes the objective in (11). \square

Let us consider first the case when \mathbf{R}_c is given by (5), i.e., $\mathbf{R}_c = \mathbf{R}_c^I$. Using Lemma 1 and inserting (5) into (12), after some derivations, we can rewrite the minorizing function as

$$g_1^I(\phi, \phi^{(k)}) = (\phi^{(k)})^H \Psi^I(\phi^{(k)}) \phi^{(k)} + 2\Re \left\{ (\phi^{(k)})^H \times (\Psi^I(\phi^{(k)}))^H (\phi - \phi^{(k)}) \right\} - \phi^H \mathbf{E}_c^I(\phi^{(k)}) \phi + (\phi^{(k)})^H \times \mathbf{E}_c^I(\phi^{(k)}) \phi^{(k)} \quad (13)$$

where $\Omega^I(\phi^{(k)}) \triangleq (\mathbf{R}_c^I(\phi^{(k)}) + \mathbf{R}_{j+n})^{-1}$, $\Psi^I(\phi^{(k)}) \triangleq \mathbf{T}_t^H \Omega^I(\phi^{(k)}) \mathbf{T}_t$, and

$$\mathbf{E}_c^I(\phi^{(k)}) \triangleq \sum_{i'=0}^{N_r-1} \sum_{i=1}^{N_c} \sigma_{ii'}^2 (\mathbf{T}_{ii'}^{(p)})^H \Omega^I(\phi^{(k)}) \times \mathbf{T}_t \phi^{(k)} (\phi^{(k)})^H \mathbf{T}_t^H (\Omega^I(\phi^{(k)}))^H \mathbf{T}_{ii'}^{(p)}. \quad (14)$$

Note that the third term in (13) takes a quadratic form with respect to ϕ , to which we can apply a proper minorization function once more.

Before proceeding with the further minorization of (13), we present the following lemma.

Lemma 2. *The quadratic function $f(\phi) = -\phi^H \mathbf{E}_c^I(\phi^{(k)}) \phi$ is minorized by the following function*

$$\tilde{g}(\phi, \phi^{(k)}) = -\frac{1}{2} \phi^H \mathbf{G}^{(k)} \phi - (\phi^{(k)})^H \left(\frac{1}{2} \mathbf{G}^{(k)} - \mathbf{E}_c^I(\phi^{(k)}) \right) \times \phi^{(k)} - 2\Re \left\{ \phi^H \left(\mathbf{E}_c^I(\phi^{(k)}) - \frac{1}{2} \mathbf{G}^{(k)} \right) \phi^{(k)} \right\} \quad (15)$$

if $\mathbf{G}^{(k)} \succeq \mathbf{E}_c^I(\phi^{(k)})$ is satisfied.

Proof. The result is equivalent to majorization of $-f(\phi)$, and it is proved in this form in [23]. \square

Applying Lemma 2 to (13), after some derivations, the minorization function $g_1^I(\phi, \phi^{(k)})$ can be rewritten as

$$g_2^I(\phi, \phi^{(k)}) = -\frac{1}{2} \phi^H \mathbf{G}^{(k)} \phi - 2\Re \left\{ \phi^H \left(\mathbf{E}_c^I(\phi^{(k)}) - \frac{1}{2} \mathbf{G}^{(k)} - \Psi^I(\phi^{(k)}) \right) \phi^{(k)} \right\} + (\phi^{(k)})^H \left(2\mathbf{E}_c^I(\phi^{(k)}) - \Psi^I(\phi^{(k)}) - \frac{1}{2} \mathbf{G}^{(k)} \right) \phi^{(k)}. \quad (16)$$

Choosing $\mathbf{G}^{(k)} = \lambda^{(k)} \mathbf{I}_{ML}$, where $\lambda^{(k)}$ is a properly selected magnitude (e.g., the largest eigenvalue of $\mathbf{E}_c^I(\phi^{(k)})$) such that $\mathbf{G}^{(k)} \succeq \mathbf{E}_c^I(\phi^{(k)})$. It is straightforward to see that the first and third terms in (16) are constant, and therefore,

immaterial for optimization. Ignoring these terms, the minorization problem for (11) can be written as

$$\max_{\phi} -\Re \left\{ \phi^H \left(\mathbf{E}_c^I(\phi^{(k)}) - \frac{1}{2} \mathbf{G}^{(k)} - \Psi^I(\phi^{(k)}) \right) \phi^{(k)} \right\} \quad \text{s.t. } |\phi(n)| = 1, n = 1, \dots, ML. \quad (17)$$

Using the constraint that ϕ has constant modulus and defining $\tau_{\mathbf{I}}^{(k)} \triangleq (\mathbf{E}_c^I(\phi^{(k)}) - \frac{1}{2} \mathbf{G}^{(k)} - \Psi^I(\phi^{(k)})) \phi^{(k)}$, the problem (17) can be equivalently written as

$$\min_{\phi} \|\phi - \tau_{\mathbf{I}}^{(k)}\| \quad \text{s.t. } |\phi(n)| = 1, n = 1, \dots, ML \quad (18)$$

which can be solved in closed-form as

$$\phi(n) = \exp \{ j \cdot \arg(\tau_{\mathbf{I}}^{(k)}(n)) \}, n = 1, \dots, ML. \quad (19)$$

For the case when $\mathbf{R}_c = \mathbf{R}_c^{\text{II}}$, i.e., \mathbf{R}_c is given by (6), the additional difficulty is that we need to deal with the Hadamard product. Let $\Upsilon_{ii'} = \sum_{k=1}^{K^{(ii')}} \lambda_k^{(ii')} \mathbf{q}_k^{(ii')} (\mathbf{q}_k^{(ii')})^H = \sum_{k=1}^{K^{(ii')}} \mathbf{u}_k^{(ii')} (\mathbf{u}_k^{(ii')})^H$ be the eigen decomposition of the matrix $\Upsilon_{ii'}$ in (6), with $K^{(ii')}$ being the rank of $\Upsilon_{ii'}$, $\lambda_k^{(ii')}$ (real-valued) and $\mathbf{q}_k^{(ii')}$ being the k th eigenvalue and eigenvector, respectively, and $\mathbf{u}_k^{(ii')} \triangleq (\lambda_k^{(ii')})^{1/2} \mathbf{q}_k^{(ii')} \in \mathbb{C}^{L \times 1}$. Then \mathbf{R}_c^{II} can be expressed as

$$\mathbf{R}_c^{\text{II}} = \sum_{i'=0}^{N_r-1} \sum_{i=1}^{N_c} \sum_{k=1}^{K^{(ii')}} \sigma_{ii'}^2 \tilde{\mathbf{T}}_{ii'}^{(p)} \mathbf{D}_k^{(ii')} \phi \phi^H (\mathbf{D}_k^{(ii')})^H (\tilde{\mathbf{T}}_{ii'}^{(p)})^H \quad (20)$$

where $\mathbf{D}_k^{(ii')} \triangleq \mathfrak{D}(\mathbf{u}_k^{(ii')} \otimes \mathbf{1}_M) \in \mathbb{C}^{ML \times ML}$ is diagonal.

Applying the same minorization strategies as in the previous case, we obtain the corresponding minorization functions, denoted here by $g_1^{\text{II}}(\phi, \phi^{(k)})$ and $g_2^{\text{II}}(\phi, \phi^{(k)})$, by replacing matrices $\Omega^I(\phi^{(k)})$, $\Psi^I(\phi^{(k)})$, and $\mathbf{E}_c^I(\phi^{(k)})$ in (13) and (16) with $\Omega^{\text{II}}(\phi^{(k)}) \triangleq (\mathbf{R}_c^{\text{II}}(\phi^{(k)}) + \mathbf{R}_{j+n})^{-1}$, $\Psi^{\text{II}}(\phi^{(k)}) \triangleq \mathbf{T}_t^H \Omega^{\text{II}}(\phi^{(k)}) \mathbf{T}_t$, and

$$\mathbf{E}_c^{\text{II}}(\phi^{(k)}) \triangleq \sum_{i'=0}^{N_r-1} \sum_{i=1}^{N_c} \sum_{k=1}^{K^{(ii')}} \sigma_{ii'}^2 (\mathbf{D}_k^{(ii')})^H (\mathbf{T}_{ii'}^{(p)})^H \Omega^{\text{II}}(\phi^{(k)}) \times \mathbf{T}_t \phi^{(k)} (\phi^{(k)})^H \mathbf{T}_t^H (\Omega^{\text{II}}(\phi^{(k)}))^H \mathbf{T}_{ii'}^{(p)} \mathbf{D}_k^{(ii')}. \quad (21)$$

Then the minorization problem for (11) can be written as

$$\max_{\phi} -\Re \left\{ \phi^H \left(\mathbf{E}_c^{\text{II}}(\phi^{(k)}) - \frac{1}{2} \mathbf{G}^{(k)} - \Psi^{\text{II}}(\phi^{(k)}) \right) \phi^{(k)} \right\} \quad \text{s.t. } |\phi(n)| = 1, n = 1, \dots, ML \quad (22)$$

and solved in closed-form as

$$\phi(n) = \exp \{ j \cdot \arg(\tau_{\text{II}}^{(k)}(n)) \}, n = 1, \dots, ML \quad (23)$$

where $\tau_{\text{II}}^{(k)} \triangleq (\mathbf{E}_c^{\text{II}}(\phi^{(k)}) - \frac{1}{2} \mathbf{G}^{(k)} - \Psi^{\text{II}}(\phi^{(k)})) \phi^{(k)}$ and $\mathbf{G}^{(k)}$ is chosen as in the routine used in the previous case. Note that the solution (23) boils down to solution (19), if $\mathbf{u}_k^{(ii')} = \mathbf{d}(f_{ii'})$ and $K^{(ii')} = 1, \forall i, i'$.

Finally, the algorithm for joint SST waveform and receive filter design is summarized in Algorithm 1. It can be accelerated using, for example, the squared iterative method (SQUAREM) of [24], the backtracking line search method (BLSM) [25], etc. We omit the corresponding convergence analyses for our proposed algorithm with accelerations here because of space limitation.

Algorithm 1 Joint Design Algorithm

- 1: Initialization: $\phi^{(0)}$; $\text{mod} \in \{\text{I}, \text{II}\}$
 - 2: **repeat** procedure with respect to $\phi^{(k)}$
 - 3: Calculate $\Omega^{\text{mod}}(\phi^{(k)})$, $\Psi^{\text{mod}}(\phi^{(k)})$, $\mathbf{E}_c^{\text{mod}}(\phi^{(k)})$
 - 4: Construct $\mathbf{G}^{(k)}$ via $\mathbf{E}_c^{\text{mod}}(\phi^{(k)})$
 - 5: $\tau_{\text{mod}}^{(k)} \triangleq (\mathbf{E}_c^{\text{mod}}(\phi^{(k)}) - \frac{1}{2}\mathbf{G}^{(k)} - \Psi^{\text{mod}}(\phi^{(k)}))\phi^{(k)}$
 - 6: $\phi(n) = \exp\{j \cdot \arg(\tau_{\text{mod}}^{(k)}(n))\}$, $n = 1, \dots, ML$
 - 7: $k \leftarrow k + 1$
 - 8: **until** convergence
 - 9: Calculate \mathbf{w}_{opt} , and $\phi_{\text{opt}} = \phi^{(k+1)}$
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4. SIMULATION RESULTS

We evaluate the performance of Algorithm 1 for both modes. The radar platform is set to have $M = 4$ transmit and $N = 3$ receive antenna elements half-wavelength spaced between each other, with moving velocity of 125 m/s. The carrier wavelength is 0.25 m, and $L = 20$ pulses are emitted in one CPI with pulse repetition frequency of 500 Hz. The target is located at $\theta_t = 10^\circ$ with Doppler $f_t = 0.13$, and the SNR is 10 dB. Three acceleration schemes: i) SQUAREM [24]; ii) BLSM [25]; and iii) combination of i) and ii) are used. We choose the absolute SINR difference between the current and previous iterations normalized by SNR as the stopping criterion, and set the tolerance to 10^{-8} . The SFT waveforms are generated via the ISLNew method in [23], and the SST waveforms are initialized by sequences with random phases.

We first consider the scenario of homogeneous environment where $N_r = 10$ range rings interfere with the range-azimuth bin of interest, with each separated into $N_c = 181$ azimuth bins. The Doppler information of clutter bins is known (determined by their relative radial velocities), and the clutter-to-noise ratio (CNR) for each bin is set to 40 dB. Mode I of Algorithm 1 is exploited. It can be seen from Fig. 1 that our proposed algorithm shows good SINR behaviour in terms of the convergence speed. Both the original algorithm and its accelerations i), ii), and iii) demonstrate sharp SINR improvements for few iterations, starting from an initial SINR of 4.74 dB. The corresponding improvements after the first 25 iterations have reached 3.27 dB, 3.77 dB, 4.17 dB, and 4.48 dB (with completion rates 72%, 82%, 91%, and 98% compared to the maximum achievable SINR), respectively. Among the results shown, the smallest number (around 45) of consumed iterations after convergence to tolerance is achieved by acceleration iii), while the others (original and accelerations i) and ii)) consume about 510, 225, 85 iterations, respectively.

We then consider the scenario with discrete heterogeneous environment with Doppler uncertainties on clutter bins. The corresponding parameters are: $N_r = 10$, $N_c = 3$, CNR=50 dB (for each discrete bin). The spatial directions of the three clutter sources at each ring are randomly distributed within the sectors $[-50^\circ, -30^\circ]$, $[-20^\circ, 10^\circ]$, and $[25^\circ, 35^\circ]$, respec-

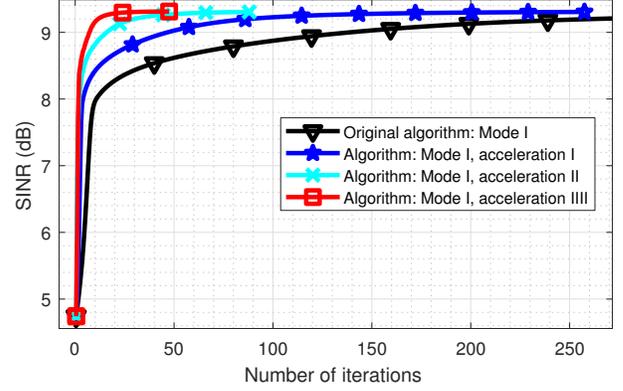


Fig. 1. SINR v.s. iterations: Example 1, cut at 270th iteration.

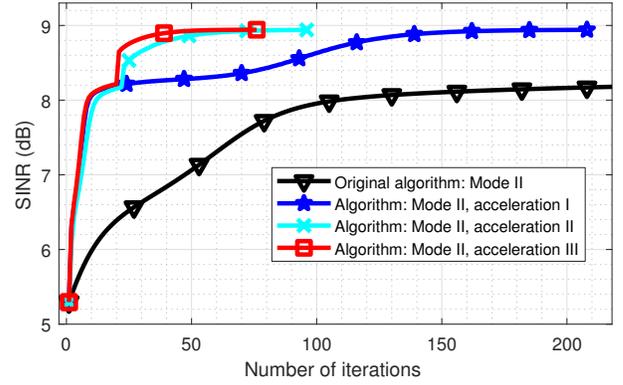


Fig. 2. SINR v.s. iterations: Example 2, cut at 220th iteration.

tively. The Doppler uncertainty parameters are: $\bar{f}_{ii'} = 0$, $\epsilon_{ii'} = 0.35$, $\forall i' \in \{0, \dots, 9\}$, $i \in \{1, 2, 3\}$, and $\Upsilon_{ii'}$ is determined by the PDF of uniform distribution (see [13]). Fig. 2 shows the corresponding SINR performance versus number of iterations consumed. It can be seen that the obtained SINRs for this scenario verify the effectiveness of our proposed algorithm. With the aid of accelerations, the obtained SINR levels are significantly improved after consuming around 10 iterations (above 8 dB), and the number of iterations has been reduced at most to about 70 (by acceleration III).

5. CONCLUSION

We have developed a novel approach for jointly synthesizing unimodular SST waveforms and MVDR receive STAP filter with considerations on the range sidelobe effect and Doppler characteristics. Two cases of known Doppler and presence of uncertainties on clutter bins have been considered. We have formulated the corresponding non-convex optimization problems, and have developed efficient algorithms for addressing them through manipulating the composite objective of the formulated problems and designing minorizing surrogates. The resulting minorized problems can be solved in closed-form via minorization-maximization technique, and the proposed algorithms have low complexity, fast convergence speed, and demonstrate good performance through simulations.

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