NONCIRCULARITY-BASED LOCALIZATION FOR MIXED NEAR-FIELD AND FAR-FIELD SOURCES WITH UNKNOWN MUTUAL COUPLING

Hua Chen[1], *Wei Liu*[2], *Wei-Ping Zhu*[3] *and M.N.S. Swamy*[3]

[1]Faculty of Information Science and Engineering, Ningbo University, Ningbo, 315211, China.
[2]Department of Electronic and Electrical Engineering, University of Sheffield, Sheffield, UK S1 3JD.
[3]Department of Electrical and Computer Engineering, Concordia University, Montreal, Canada.

ABSTRACT

In this paper, a novel noncircularity-based localization method for mixed near-field (NF) and far-field (FF) sources is proposed with a symmetric uniform linear array (ULA) in the presence of unknown mutual coupling (UMC). Based on the principle of rank reduction (RARE), the multiple parameters of the sources including direction of arrival (DOA), range and mutual coupling coefficient (MCC) are decoupled, so that only several one-dimensional (1-D) spectral searches are required for their estimation. Meanwhile, the proposed method can also distinguish the types of sources without any extra processing. Simulation results are provided to demonstrate the effectiveness of the proposed method for the classification and localization of mixed sources under UMC.

Index Terms— DOA estimation, near-field, far-field, noncircular signals, mutual coupling.

1. INTRODUCTION

Recently, simultaneous localization of both near-field (NF) and far-field (FF) signals has drawn a lot of attention in the array signal processing community given its many practical applications such as speaker localization using microphone arrays and guidance (homing) systems. In [1], Liang et al. proposed a two-stage MUSIC method based on two special fourth-order cumulant (FOC) matrices. By using FOC matrices, localization of mixed sources with sparse signal reconstruction was studied in [2] and a mixed-order MUSIC algorithm using a sparse symmetric array was proposed in [3], respectively. However, one common issue with these cumulant-based methods is their high computational complexity to construct FOC matrices. To avoid this issue, a series of second-order statistics (SOS)-based methods were presented in [4–6]. In [4], an oblique projection MUSIC-based algorithm was proposed to separate the NF and FF sources, which unfortunately results in extra estimation errors. To overcome the extra estimation errors in NF localization [4], Zuo et al. developed an alternating iterative method in [5] by recalculating the oblique projector without eigendecomposition. By resorting to the spatial differencing technique, a mixed localization method was presented in [6] by eliminating the FF and noise components from the covariance matrix of the array observation. It is to be noted that to avoid phase ambiguities, all the abovementioned mixed source localization methods require the inter-sensor spacing to be constrained within **a quarter wavelength**, which inevitably results in mutual coupling effect between closely located elements [7, 8]. Only in [9], a mixed source localization method was put forward in the presence of mutual coupling. On the other hand, strictly noncircular or rectilinear signals [10–18], including amplitude modulated (AM) and binary phase shift keying (BPSK) signals, are usually encountered in the context of radio communications, for which a significant gain in terms of the direction of arrival (DOA) estimation performance can be achieved by taking into consideration both the covariance matrix and conjugate covariance matrix of noncircular signals.

To the best of our knowledge, no work for noncircularity-based localization of mixed NF and FF signals in the presence of unknown mutual coupling (UMC) has been reported thus far. Therefore, in this paper, based on a ULA, we propose a localization method for mixed NF and FF sources by exploiting the noncircularity of the signals under UMC. By using the principle of rank reduction (RARE) [19, 20], it is shown that only several one-dimensional (1-D) spectral searches are required to successively estimate the parameters of mixed NF and FF rectilinear sources including DOA, range and mutual coupling coefficient (MCC). Meanwhile, distinguishing the types of sources is also solved without any extra processing.

Notations: $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^{-1}$ represent operations of conjugation, transpose, conjugate transpose, and inverse, respectively; $E[\cdot]$ is the expectation operation; $diag\{\cdot\}$ stands for the diagonalization operation; $\mathbf{I}_p$ denotes the p-dimensional identity matrix; $\mathbf{\Pi}_p$ is a $p \times p$ exchange matrix with ones on its anti-diagonal and zeros elsewhere; $blkdiag\{\mathbf{Z}_1, \mathbf{Z}_2\}$ represents a block diagonal matrix with diagonal entries $\mathbf{Z}_1$ and $\mathbf{Z}_2$.

This work is sponsored by the National Natural Science Foundation of China under Grant (No. 61571250 and 61628101), and by K.C.Wong Magna Fund in Ningbo University.

2. ARRAY SIGNAL MODEL

Similar to the scenario considered in [9], we suppose that K uncorrelated narrowband strictly noncircular sources $s_k(l)$ $(k = 1, 2, \ldots, K)$ located in either NF or FF region, impinge upon a symmetric uniform linear array (ULA) with $M = 2N + 1$ sensors. Without loss of generality, we assume the first K_1 incoming sources $s_{N,k}(l)$ are NF parameterized by (θ_k, r_k), where θ_k and r_k are the DOA of range and NF sources, $(k = 1, 2, \ldots, K_1)$, while the remaining $K_2 = K - K_1$ sources $s_{F,k}(l)$ are FF parameterized by (θ_k, ∞) $(k = K_1 + 1, K_1 + 2, \ldots, K)$, and K_1 and K_2 are known in advance. Clearly, an FF source can be considered as a special NF one where the range r_k approaches to ∞. With the array center indexed by 0 being the phase reference point, the L snapshots of the array observed signal $\mathbf{x}(l) = [x_{-N}(l), \cdots, x_0(l), \cdots, x_N(l)]^T$ can be expressed as

$$\mathbf{x}(l) = \mathbf{A}_N \mathbf{s}_N(l) + \mathbf{A}_F \mathbf{s}_F(l) + \mathbf{n}(l) \tag{1}$$

where $\mathbf{n}(l) = [n_{-N}(l), \cdots, n_0(l), \cdots, n_N(l)]^T$ is the circular Gaussian noise vector, with zero mean and variance σ_n^2 for each sensor, which is uncorrelated with the impinging signal, $\mathbf{s}_N(l)$ and $\mathbf{s}_F(l)$ are the signal vectors of NF and FF sources, respectively, and $\mathbf{A}_N$ and $\mathbf{A}_F$ are the array steering matrices of NF and FF signals with $\mathbf{a}_N(\theta_k, r_k)$ and $\mathbf{a}_F(\theta_k)$ representing, respectively, the NF and FF steering vectors, i.e.,

$$\mathbf{A}_N = [\mathbf{a}_N(\theta_1, r_1), \cdots, \mathbf{a}_N(\theta_{K_1}, r_{K_1})] \tag{2}$$

$$\mathbf{a}_N(\theta_k, r_k) = [e^{j(-N\gamma_k + N^2\chi_k)}, \cdots, 1, \cdots, e^{j(N\gamma_k + N^2\chi_k)}]^T \tag{3}$$

$$\begin{aligned} \mathbf{A}_F &= [\mathbf{a}_N(\theta_{K_1+1}, \infty), \cdots, \mathbf{a}_N(\theta_K, \infty)] \\ &= [\mathbf{a}_F(\theta_{K_1+1}), \cdots, \mathbf{a}_F(\theta_K)] \end{aligned} \tag{4}$$

$$\mathbf{a}_F(\theta_k) = [e^{j(-N\gamma_k)}, \cdots, 1, \cdots, e^{j(N\gamma_k)}]^T \tag{5}$$

where $\gamma_k = -2\pi d sin\theta_k/\lambda$ and $\chi_k = \pi d^2 cos^2\theta_k/(\lambda r_k)$ are called electric angles with λ being the wavelength of the incoming signal, $\theta_k \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], (k = 1, \cdots K)$, the DOA of the kth NF or FF signal, d the spacing between adjacent sensors satisfying $d \leq \lambda/4$ to avoid estimation ambiguity and r_k the range of the kth NF signal which is within the Fresnel region and satisfies $r_k \in \left[0.62(D^3/\lambda)^{1/2}, 2D^2/\lambda\right]$, $k = 1, \cdots K_1$, with D being the array aperture.

Due to the strictly noncircularity, the signal vectors $\mathbf{s}_N(l)$ and $\mathbf{s}_F(l)$ can be expressed as $\mathbf{s}_N(l) = \boldsymbol{\psi}_{No}^{1/2}\mathbf{s}_{No}(l)$ and $\mathbf{s}_F(l) = \boldsymbol{\psi}_{Fo}^{1/2}\mathbf{s}_{Fo}(l)$, respectively, where $\mathbf{s}_{No}(l) = [s_{o,1}(l), ..., s_{o,K_1}(l)]^T$ and $\mathbf{s}_{Fo}(l) = [s_{o,K_1+1}(l), ..., s_{o,K}(l)]^T$ are the NF and FF real-valued signals, respectively. The diagonal matrices $\boldsymbol{\psi}_{No}^{1/2} = diag(e^{j\psi_1/2}, \cdots, e^{j\psi_{K_1}/2})$ and $\boldsymbol{\psi}_{Fo}^{1/2} = diag(e^{j\psi_{K_1+1}/2}, \cdots, e^{j\psi_K/2})$ are the arbitrary phase shifts corresponding to the NF and FF strictly non-circular sources $\mathbf{s}_N(l)$ and $\mathbf{s}_F(l)$, respectively.

In order to avoid the phase ambiguities, the inter-element spacing d should be within a quarter wavelength, which will greatly increase the mutual coupling effect between neighboring sensors. In the presence of mutual coupling, (1) should be modified as

$$\mathbf{x}(l) = \mathbf{C}\mathbf{A}_N \mathbf{s}_N(l) + \mathbf{C}\mathbf{A}_F \mathbf{s}_F(l) + \mathbf{n}(l) \tag{6}$$

where $\mathbf{C}$ denotes the $M \times M$ mutual coupling coefficient (MCC) matrix of the ULA, which is a banded symmetric Toeplitz matrix with $P + 1$ nonzero MCCs [6]

$$\mathbf{C} = toeplitz(\mathbf{c}, \mathbf{c}) \tag{7}$$

where $\mathbf{c} = [1, \mathbf{c}_0^T]^T, \mathbf{c}_0 = [c_1, c_2, \ldots, c_P]^T$, $toeplitz(\cdot, \cdot)$ is the toeplitzation operation. Here P is the maximum range for which the mutual coupling effect is considered.

3. THE PROPOSED METHOD

The DOA estimation performance of a traditional subspace method based on the array signal model (6) would degrade without compensating for mutual coupling. Here, we develop a two-stage RARE-based method to determine the DOAs and ranges of the mixed NF and FF strictly noncircular sources under UMC.

To exploit the noncircularity of incident signals, a new vector $\mathbf{z}(l)$ is constructed by stacking the observed data vector $\mathbf{x}(l)$ and its conjugate counterpart $\mathbf{x}^*(l)$ as follows

$$\begin{aligned} \mathbf{z}(l) &= \begin{bmatrix} \mathbf{x}(l) \\ \mathbf{x}^*(l) \end{bmatrix} \\ &= \mathbf{C}_e \mathbf{A}_{eN} \mathbf{s}_N(l) + \mathbf{C}_e \mathbf{A}_{eF} \mathbf{s}_F(l) + \mathbf{n}_e(l) \end{aligned} \tag{8}$$

where

$$\mathbf{C}_e = blkdiag\{\mathbf{C}, \mathbf{C}^*\} \tag{9}$$

$$\begin{aligned} \mathbf{A}_{eN} &= \begin{bmatrix} \mathbf{A}_N \\ \mathbf{A}_N{}^* \boldsymbol{\psi}_N{}^* \end{bmatrix} \\ &= [\mathbf{a}_{eN}(\theta_1, r_1, \psi_1), \cdots, \mathbf{a}_{eN}(\theta_{K_1}, r_{K_1}, \psi_{K_1})] \end{aligned} \tag{10}$$

with

$$\mathbf{a}_{eN}(\theta_k, r_k, \psi_k) = \begin{bmatrix} \mathbf{a}_N(\theta_k, r_k) \\ \mathbf{a}_N{}^*(\theta_k, r_k) e^{-j\psi_k} \end{bmatrix} \tag{11}$$

$$\begin{aligned} \mathbf{A}_{eF} &= \begin{bmatrix} \mathbf{A}_F \\ \mathbf{A}_F{}^* \boldsymbol{\psi}_F{}^* \end{bmatrix} \\ &= [\mathbf{a}_{eF}(\theta_{K_1+1}, \psi_{K_1+1}), \cdots, \mathbf{a}_{eF}(\theta_K, \psi_K)] \end{aligned} \tag{12}$$

with

$$\mathbf{a}_{eF}(\theta_k, \psi_k) = \begin{bmatrix} \mathbf{a}_F(\theta_k) \\ \mathbf{a}_F{}^*(\theta_k) e^{-j\psi_k} \end{bmatrix} \tag{13}$$

$$\mathbf{n}_e(l) = \begin{bmatrix} \mathbf{n}(l) \\ \mathbf{n}^*(l) \end{bmatrix} \tag{14}$$

The covariance matrix of $\mathbf{z}(l)$ is then given by

$$\begin{aligned} \mathbf{R} &= E[\mathbf{z}(l)\mathbf{z}^H(l)] \\ &= \mathbf{C}_e \mathbf{A}_{eN} \mathbf{R}_{sN} \mathbf{A}_{eN}^H \mathbf{C}_e^H + \mathbf{C}_e \mathbf{A}_{eF} \mathbf{R}_{sF} \mathbf{A}_{eF}^H \mathbf{C}_e^H + \sigma_n^2 \mathbf{I}_{2M} \end{aligned} \tag{15}$$

where $\mathbf{R}_{sN} = E[\mathbf{s}_N(l)\mathbf{s}_N^H(l)]$, $\mathbf{R}_{sF} = E[\mathbf{s}_F(l)\mathbf{s}_F^H(l)]$ and $\mathbf{R}_s = E[\mathbf{s}(l)\mathbf{s}^H(l)]$ are the covariance matrices of the NF, FF and their mixed signals, respectively. The eigenvalue decomposition of $\mathbf{R}$ is can be written as

$$\mathbf{R} = \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{U}_s^H + \mathbf{U}_n\mathbf{\Lambda}_n\mathbf{U}_n^H \tag{16}$$

where the $2M \times K$ matrix $\mathbf{U}_s$ and the $2M \times (2M-K)$ matrix $\mathbf{U}_n$ are the signal subspace and noise subspace, respectively. The $K \times K$ matrix $\mathbf{\Lambda}_s = diag\{\lambda_1, \dots, \lambda_K\}$ and the $(2M-K)\times(2M-K)$ matrix $\mathbf{\Lambda}_n = diag\{\lambda_{K+1}, \dots, \lambda_{2M}\}$ are diagonal matrices, where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K > \lambda_{K+1} = \cdots = \lambda_{2M} = \sigma_n^2$ are the eigenvalues of $\mathbf{R}$.

3.1. DOA Estimation of FF Sources and MCC Estimation

According to [7, 8], $\mathbf{C}\mathbf{a}_F(\theta)$ has the alternative expression as

$$\mathbf{C}\mathbf{a}_F(\theta) = \mathbf{T}_x(\theta)\mathbf{c} \tag{17}$$

where

$$\mathbf{T}_x(\theta) = \mathbf{T}_{x1}(\theta) + \mathbf{T}_{x2}(\theta) \tag{18}$$

$$[\mathbf{T}_{x1}(\theta)]_{i,j} = \begin{cases} [\mathbf{a}_F(\theta)]_{i+j-1} & i+j \leq M+1 \\ 0 & otherwise \end{cases} \tag{19}$$

$$[\mathbf{T}_{x2}(\theta)]_{i,j} = \begin{cases} [\mathbf{a}_F(\theta)]_{i-j+1} & i \geq j \geq 2 \\ 0 & otherwise \end{cases} \tag{20}$$

Obviously, based on the orthogonality between the noise subspace spanned by $\mathbf{U}_n$ and the signal subspace spanned by $\mathbf{U}_s$, and the fact that the signal subspace can be also spanned by $\mathbf{C}_e\mathbf{A}_{eN}$ and $\mathbf{C}_e\mathbf{A}_{eF}$ jointly, we have

$$\mathbf{U}_n^H\mathbf{C}_e\mathbf{a}_{eN}(\theta_k, r_k, \psi_k) = \mathbf{0}, k = 1, 2, \dots, K^1. \tag{21}$$

$$\mathbf{U}_n^H\mathbf{C}_e\mathbf{a}_{eF}(\theta_k, \psi_k) = \mathbf{0}, k = K_1+1, K_1+2, \dots, K. \tag{22}$$

To avoid multi-dimensional spectral search for estimating the DOA-range pairs, we have to decouple the multiple-parameters to decrease the computational load. First, the FF DOA parameters are decoupled from the other parameters. Substituting (9) and (13) into (22) and using (17), we have

$$\mathbf{U}_n^H\mathbf{T}(\theta_k)\mathbf{\Gamma}(\mathbf{c}, \psi_k) = \mathbf{0} \tag{23}$$

where $\mathbf{T}(\theta_k) = blkdiag\{\mathbf{T}_x(\theta_k), \mathbf{T}_x^*(\theta_k)\}$, $\mathbf{\Gamma}(\mathbf{c}, \psi_k) = \begin{bmatrix} \mathbf{c} \\ \mathbf{c}^* e^{-j\psi_k} \end{bmatrix}$. Now define a function $p_F(\theta)$ that is related to the DOA parameter as follows

$$p_F(\theta) = \{\det[\mathbf{Q}_F(\theta)]\}^{-1}. \tag{24}$$

where $\mathbf{Q}_F(\theta) = \mathbf{T}^H(\theta)\mathbf{U}_n\mathbf{U}_n^H\mathbf{T}(\theta)$.

Based on the RARE principle, if and only if $\theta = \theta_k, (k = K_1+1, \dots, K)$, the matrix $\mathbf{Q}_F(\theta)$ is rank deficient or equivalently $\det[\mathbf{Q}_F(\theta)] = 0$. If searched over the confined region $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the DOA estimates of all FF sources could be obtained from the K_2 highest peaks.

Then, with the estimated DOA of FF sources, $\{\hat{\mathbf{c}}, \hat{\psi}_k\}$ can be obtained by finding the minima of the following function

$$\{\hat{\mathbf{c}}, \hat{\psi}_k\} = \min_{\mathbf{c},\psi} \mathbf{\Gamma}(\mathbf{c}, \psi)^H\mathbf{Q}_F(\hat{\theta})\mathbf{\Gamma}(\mathbf{c}, \psi), \tag{25}$$

which implies that $\mathbf{\Gamma}(\hat{\mathbf{c}}, \hat{\psi}_k)$ is just the unique eigenvector corresponding to the smallest eigenvalue of $\mathbf{Q}_F(\hat{\theta}_k)$, namely

$$\hat{\mathbf{\Gamma}}_k = \mathbf{\Gamma}(\hat{\mathbf{c}}, \hat{\psi}_k) = \mathbf{e}_{min}[\mathbf{Q}_F(\hat{\theta}_k)], \mathbf{e}_{min}(1) = 1. \tag{26}$$

And we obtain the MCCs $\hat{\mathbf{c}}_0$ as

$$\hat{\mathbf{c}}_0 = \sum_{k=K_1+1}^{K} \hat{\mathbf{\Gamma}}_k(2:P+1)/(K-K_1) \tag{27}$$

The mutual coupling matrix $\hat{\mathbf{C}}$ can then be reconstructed according to its banded symmetric Toeplitz structure in (7).

3.2. DOA and Range Estimation of NF Sources

With estimated MCC $\hat{\mathbf{C}}$, DOA and range estimation of NF sources can be obtained from (21). Since the array is symmetric about the center sensor, (3) can be rewritten as

$$\mathbf{a}_N(\theta_k, r_k) = \boldsymbol{\kappa}_N(\theta_k)\boldsymbol{\varsigma}_N(\theta_k, r_k) \tag{28}$$

where $\boldsymbol{\kappa}_N(\theta_k)$ is a $(2N+1)\times(N+1)$ matrix, whose elements depend only on the DOA parameter, i.e.,

$$\boldsymbol{\kappa}_N(\theta_k) = \begin{bmatrix} \boldsymbol{\vartheta}_1^T(\theta_k) & \boldsymbol{\vartheta}_2^T(\theta_k) & \boldsymbol{\vartheta}_3^T(\theta_k) \end{bmatrix}^T \tag{29}$$

where the matrices $\boldsymbol{\vartheta}_i^T(\theta_k), (i = 1, 2, 3)$ are given by

$$\boldsymbol{\vartheta}_1(\theta_k) = [\mathbf{p}_1(\theta_k), \mathbf{0}_{N\times 1}]_{N\times(N+1)} \tag{30}$$

with

$$\mathbf{p}_1(\theta_k) = diag\{e^{j(-N)\gamma_k}, e^{j(-N+1)\gamma_k}, \dots, e^{j[-N+(N-1)]\gamma_k}\} \tag{31}$$

$$\boldsymbol{\vartheta}_2(\theta_k) = [\mathbf{0}_{1\times N}, 1]_{1\times(N+1)} \tag{32}$$

$$\boldsymbol{\vartheta}_3(\theta_k) = [\mathbf{\Pi}_N\mathbf{p}_3(\theta_k), \mathbf{0}_{N\times 1}]_{N\times(N+1)} \tag{33}$$

with

$$\mathbf{p}_3(\theta_k) = diag\{e^{j(N)\gamma_k}, e^{j(N-1)\gamma_k}, \dots, e^{j[N-(N-1)]\gamma_k}\} \tag{34}$$

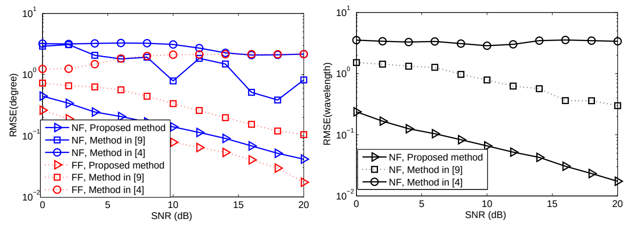
Meanwhile, $\boldsymbol{\varsigma}_N(\theta_k, r_k)$ is dependent on both the DOA and range parameters as given by

$$\boldsymbol{\varsigma}_N(\theta_k, r_k) = [e^{j(-N)^2\chi_k}, e^{j(-N+1)^2\chi_k}, \dots, e^{j(-N+N)^2\chi_k}]^T \tag{35}$$

Then, with (28), (21) can be rewritten as

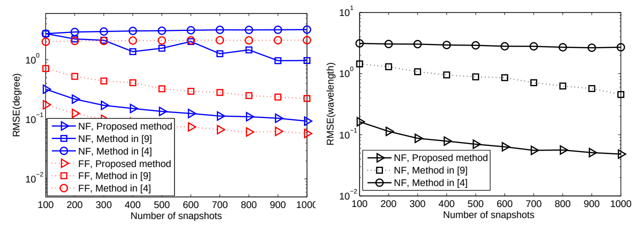
$$\begin{aligned} \mathbf{0} &= \mathbf{U}_n^H\mathbf{C}_e\mathbf{a}_{eN}(\theta_k, r_k, \psi_k) \\ &= \mathbf{U}_n^H\mathbf{C}_e\boldsymbol{\upsilon}_N(\theta_k)\boldsymbol{\zeta}_N(\theta_k, r_k)\boldsymbol{\iota}_N(\psi_k) \end{aligned} \tag{36}$$

[1]Becasuse an FF source can be considered as a special NF one where the range r_k approaches to ∞, (21) holds for both NF and FF sources.



(a) DOA estimation (b) range estimation

Fig. 1: RMSE versus SNR.



(a) DOA estimation (b) range estimation

Fig. 2: RMSE versus snapshots.

where

$$\boldsymbol{\upsilon}_N(\theta_k)=blkdiag\{\boldsymbol{\kappa}_N(\theta_k),\boldsymbol{\kappa}_N^*(\theta_k)\} \quad (37)$$

$$\boldsymbol{\zeta}_N(\theta_k,r_k)=blkdiag\{\boldsymbol{\varsigma}_N(\theta_k,r_k),\boldsymbol{\varsigma}_N^*(\theta_k,r_k)\} \quad (38)$$

$$\boldsymbol{\iota}_N(\psi_k)=\begin{bmatrix} 1 \\ e^{-j\psi_k} \end{bmatrix} \quad (39)$$

We now define a function $p_N(\theta)$ that is related only to the DOA parameter as follows

$$p_N(\theta)=\{\det[\mathbf{Q}_{N1}(\theta)]\}^{-1}. \quad (40)$$

where $\mathbf{Q}_{N1}(\theta)=\boldsymbol{\upsilon}_N^H(\theta)\hat{\mathbf{C}}_e^H\mathbf{U}_n\mathbf{U}_n^H\hat{\mathbf{C}}_e\boldsymbol{\upsilon}_N(\theta)$. Similarly, we noticed that $\boldsymbol{\zeta}_N(\theta_k,r_k)\neq\mathbf{0}$, $\boldsymbol{\iota}_N(\psi_k)\neq\mathbf{0}$, and $\mathbf{U}_n^H\hat{\mathbf{C}}_e\boldsymbol{\upsilon}_N(\theta_k)\boldsymbol{\zeta}_N(\theta_k,r_k)\boldsymbol{\iota}_N(\psi_k)=\mathbf{0}, k=1,2\ldots K',(K'\leq K)$.[2] Based on the RARE principle, the DOAs θ_k of all mixed signals can be obtained from the highest peaks of $p_N(\theta)$ searched over the confined region $\theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. Next, we substitute the estimate $\hat{\theta}_k$ from (40) into (37) and obtain the following function of the range parameter r:

$$p_N(r)=\{\det[\mathbf{Q}_{N2}(\hat{\theta}_k,r)]\}^{-1}. \quad (41)$$

where

$$\begin{aligned}\mathbf{Q}_{N2}(\hat{\theta}_k,r)=&\boldsymbol{\zeta}_N^H(\hat{\theta}_k,r)\boldsymbol{\upsilon}_N^H(\hat{\theta}_k)\hat{\mathbf{C}}_e^H\mathbf{U}_n\mathbf{U}_n^H\hat{\mathbf{C}}_e\\&\times\boldsymbol{\upsilon}_N(\hat{\theta}_k)\boldsymbol{\zeta}_N(\hat{\theta}_k,r).\end{aligned}$$

Again, by searching $p_N(r)$ over the region $r\in[0.62(D^3/\lambda)^{1/2}, 2D^2/\lambda]$, the corresponding range of the NF signals can be obtained from the peaks of $p_N(r)$.

4. SIMULATION RESULTS

In this section, the performance of the proposed method is compared with the existing methods in [4] and [9] using a ULA consisting of $M=9$ (or N = 4) sensors with $d=\lambda/4$. The impinging sources are equal power BPSK signals, and the additive noise is assumed to be spatially white complex Gaussian, and the SNR is defined relative to each signal. Further, we assume two NF signals are located at $(5°,1.9\lambda)$, $(30°,2.6\lambda)$, and two FF signals are located at $(5°,+\infty)$, $(-25°,+\infty)$, respectively. The nonzero MCCs are [1, 0.3515+0.4656i, 0.0916-0.1218i]. We carry out two set of simulations, each compromising 500 independent Monte Carlo trials. In the first simulation, we investigate the RMSE of DOA and range estimates when the SNR varies from 0 dB to 20 dB, with the number of snapshots fixed at 500. In Fig. 1, we can see that the proposed method outperforms the methods in [4] and [9] for DOA and range estimation of both NF and FF sources. This is because the proposed method has exploited the noncircular information of mixed signals, which effectively increases the array aperture to some extent. In addition, auto-calibration is applied for the FF sources, while mutual coupling is compensated for the NF sources in the proposed method.

In the second simulation, the SNR is set at 10dB, and the number of snapshots varies from 100 to 1000. It is observed from Fig. 2, as the number of snapshots increases, the proposed method is always superior to the existing methods in DOA and range estimation.

[2]Here, it should be pointed out that when some of the NF signals have the same DOAs as the FF signals do, we only get K' DOAs which are no more than K true DOAs, namely $K'\leq K$.

5. CONCLUSIONS

We have presented an effective method for the localization of mixed FF and NF sources using the noncircular information of the impinging signals under UMC. Compared with existing mixed source localization methods, the proposed one has its superiority in the case of UMC and is capable of identifying source types reasonably. Moreover, the usual multi-dimensional spectral search has been avoided by decoupling the array steering matrix of NF and FF signals. As our future work, the Cramer-Rao bound (CRB) of the problem under consideration will be studied.

6. REFERENCES

[1] J. Liang and D. Liu, "Passive localization of mixed near-field and far-field sources using two-stage MUSIC algo-

rithm," *IEEE Trans. Signal Process.*, vol. 58, no. 1, pp. 108-120, Jan. 2010.

[2] B. Wang, J. Liu and X. Sun, "Mixed sources localization based on sparse signal reconstruction," *IEEE Signal Process. Lett.*, vol. 19, no. 8, pp. 487-490, Aug. 2012.

[3] B. Wang, Y. Zhao and J. Liu, "Mixed-order MUSIC algorithm for localization of far-field and near-field sources," *IEEE Signal Process. Lett.*, vol. 20, no. 4, pp. 311-314, Apr. 2013.

[4] J. He, M. N. S. Swany, and M. O. Ahmad, "Efficient application of music algorithm under the coexistence of far-field and near-field sources," *IEEE Trans. Signal Process.*, vol. 60, no. 4, pp. 2066-2070, Apr. 2012.

[5] W. Zuo, J. Xin, J. Wang, N. Zheng and A. Sano, "A computationally efficient source localization method for a mixture of near-field and far-field narrowband signals," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, Florence, 2014, pp. 2257-2261.

[6] G. Liu and X. Sun, "Spatial differencing method for mixed far-field and near-field sources localization," *IEEE Signal Process. Lett.*, vol.21, pp. 1331-1335, 2014.

[7] B. Friedlander and A. J. Weiss, "Direction finding in the presence of mutual coupling," *IEEE Trans. Antenna Propag.*, vol. 39, no. 3, pp. 273-284, Mar. 1991.

[8] S. Cao, Z. Ye and X. Xu, "A Khatri-Rao based method for DOA estimation in the presence of mutual coupling,"in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, Kyoto, 2012, pp. 2549-2552.

[9] J. Xie, H. Tao, X. Rao and J. Su, "Localization of mixed far-field and near-field sources under unknown mutual coupling," *Digital Signal Process.*, vol. 50, pp. 229-239, 2016.

[10] H. Abeida, J.P. Delmas, "Statistical performance of MUSIC-like algorithms in resolving noncircular sources," *IEEE Trans. Signal Process.*, vol. 56, no. 9, pp. 4317-4329, Sep. 2008.

[11] H. Chen, C. P. Hou, W. P. Zhu, W. Liu, Y. Y. Dong, Z.J. Peng and Q. Wang. "ESPRIT-like two-dimensional direction finding for mixed circular and strictly noncircular sources based on joint diagonalization," *Signal Process.*, vol.141, pp.48-56, Dec.2017.

[12] J. Steinwandt, F. Roemer, M. Haardt, and G. Del Galdo, "Performance analysis of multi-dimensional ESPRIT-type algorithms for arbitrary and strictly non-circular sources with spatial smoothing," *IEEE Trans. Signal Process.*, vol. 65, no. 9, pp. 2262-2276, May 2017.

[13] H. Abeida and J. P. Delmas, "Direct derivation of the stochastic CRB of DOA estimation for rectilinear sources," *IEEE Signal Process. Lett.*, vol. 24, no. 10, pp. 1522-1526, Oct. 2017.

[14] J. Steinwandt, F. Roemer, M. Haardt, and G. Del Galdo, "R-dimensional ESPRIT-type algorithms for strictly second-order non-circular sources and their performance analysis," *IEEE Trans. Signal Process.*, vol. 62, no. 18, pp. 4824-4838, Sept. 2014.

[15] Y. Shi, X. Mao, M. Cao and Y. Liu, "Deterministic maximum likelihood method for direction-of-arrival estimation of strictly noncircular signals," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, Shanghai, 2016, pp. 3061-3065.

[16] H. Chen, C. P. Hou, W. Liu, W. P. Zhu, and M.N.S. Swamy. "Efficient two-dimensional direction of arrival estimation for a mixture of circular and noncircular sources," *IEEE Sensors J.*, vol. 16, no. 8, pp. 2527-2536, Apr. 2016.

[17] J. Steinwandt, F. Roemer, and M. Haardt, "Performance analysis of ESPRIT-type algorithms for non-circular sources," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, Vancouver, 2013, pp. 3986-3990.

[18] A. Liu, G. Liao, Q. Xu and C. Zeng, "A circularity-based DOA estimation method under coexistence of noncircular and circular signals," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, Kyoto, 2012, pp. 2561-2564.

[19] A. Ferreol, E. Boyer and P. Larzabal, "Low-cost algorithm for some bearing estimation methods in presence of separable nuisance parameters," *Electron. Lett.*, vol. 40, no. 15, pp. 966-967, Jul. 2004.

[20] M. Pesavento, A. B. Gershman and K. M. Wong, "Direction finding in partly calibrated sensor arrays composed of multiple subarrays," *IEEE Trans. Signal Process.*, vol. 50, no. 9, pp. 2103-2115, Sep. 2002.