COMPUTATIONALLY EFFICIENT IV-BASED BIAS REDUCTION FOR CLOSED-FORM TDOA LOCALIZATION

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ABSTRACT

This paper develops a new computationally efficient bias reduction method for the well-known algebraic closed-form solution for time-difference-of-arrival (TDOA) localization developed by Chan and Ho. The noise correlation between the regressor and regressand in the formulation of the linearized least-squares computation is the main cause of bias problems associated with this TDOA localization method. The bias reduction method proposed in this paper, which we call IV-BiasRed, exploits the use of instrumental variables (IV) to eliminate the troublesome noise correlation between the regressor and regressand. The IV-BiasRed method is demonstrated by way of simulations to achieve a significant bias reduction and mean-squared error performance close to the Cramér-Rao lower bound. While producing an estimation performance on par with the maximum likelihood estimator and a recently proposed bias reduction method, the proposed IV-BiasRed method is computationally much more efficient than existing bias reduction methods.

Index Terms— Source localization, time difference of arrival, least squares, instrumental variables, bias compensation

1. INTRODUCTION

Source localization by time difference of arrival (TDOA) has received considerable attention in passive source localization applications thanks to its capability to locate non-cooperative emitters [1–13]. The objective of TDOA source localization is to estimate the unknown location of a source by exploiting the TDOA between signals received at spatially separated sensors. In particular, each TDOA defines a hyperbolic surface as possible locations of the source. The intersection of the hyperbolic surfaces from a number of sensor pairs yields the source position estimate. Numerous TDOA localization techniques are available in the literature and can be classified into three broad categories: (i) the maximum likelihood estimator (MLE) [6], (ii) the hyperbolic asymptote intersection solutions [7–9], and (iii) the linearized least-squares solutions with parameter constraints [1–3, 10–13].

The MLE, a widely used solution for nonlinear estimation problems, enjoys the desirable properties of aymptotic unbiasedness and efficiency. However, the MLE does not admit a closed-form solution and must be implemented via computationally expensive numerical search algorithms. In addition, the MLE is prone to divergence due to the nonconvex nature of the MLE cost function for TDOA localization. On the other hand, the hyperbolic asymptote intersection solutions aim to exploit the intersection of the asymptotes of the TDOA hyperbolae for the source position estimate. Specifically, using the hyperbolic asymptotes, the TDOA localization problem is converted into a bearings-only localization problem which can be solved effectively via pseudolinear least-squares techniques with closed-form solutions. However, this technique was mainly considered in the two-dimensional (2D) scenario and its extension to the three-dimensional (3D) case is not straightforward. In addition to the hyperbolic asymptote intersection solutions, the linearized least-squares solutions with parameter constraint are attractive alternatives that also provide closed-form solutions and overcome the complexity and divergence problems of the MLE. This approach can be applied to both 2D and 3D geometries.

The closed-form solution developed in [1] is one of the most popular TDOA constrained least-squares solutions in the literature. It first transforms the nonlinear TDOA measurement equations into a set of equations that are linear in the unknown source position by introducing a nuisance parameter, thus allowing the use of linear least-squares to estimate the solution. The dependence between the nuisance parameter and the source location is also taken into account to further improve the performance of the linearized least-squares solution. However, the resulting estimate suffers from a severe bias problem due to the noise correlation between the regressor and regressand in the formulation of the linearized least-squares estimate [2]. To alleviate the bias problem of [1], the work in [12] applied a constraint to the unknowns directly when solving the least-squares minimization problem. However, this approach requires numerical search. On the other hand, the approach in [13] utilized a total least-squares (TLS) method to tackle the noise correlation between the regressor and regressand. Unfortunately, the variance of the TLS estimate is larger than that of the solution in [1]. Recently, the work in [2] proposed two bias compensation techniques for the solution in [2], namely the BiasSub and BiasRed methods. In particular, the BiasSub method estimates and subtracts the bias of the solution, while the BiasRed method resolves the bias problem by introducing an augmented solution equation and imposing a quadratic constraint. The BiasSub method unfortunately requires the prior knowledge of the exact TDOA noise power which may not be available in practice. In constrast, the BiasRed method only requires the prior knowledge of the structure of the TDOA noise covariance matrix. However, a drawback of the BiasRed method is that it is computationally almost twice as expensive as the original solution in [1].

The main contribution of this paper is to propose a new computationally efficient bias reduction method for the closed-form solution [1] by exploiting the use of instrumental variables. In this paper we are particularly interested in scenarios where the exact TDOA noise covariance matrix is not available and only its structure is known *a priori*, which is often the case in practice. The proposed method, called the IV-BiasRed method, aims to eliminate the noise correlation between the regressor matrix and regressand vector, which is the root cause of bias in [1], by replacing the transpose of the regressor matrix in the normal equations with the transpose of an instrumental variable matrix that is approximately uncorrelated with the regressand vector. The advantage of the proposed IV-BiasRed method is that it is computationally much simpler than the BiasRed method [2] while producing an estimation performance on par with the BiasRed method [2] and the MLE. Numerical Monte-Carlo simulations are presented at the end of the paper to corroborate the performance of the proposed IV-BiasRed method.

2. OVERVIEW OF TDOA LOCALIZATION

2.1. Problem Formulation

The objective of TDOA source localization is to estimate the position of a source at $\boldsymbol{u}^o$ using TDOA measurements obtained from M spatially separated sensors located at $\boldsymbol{s}_i$, $i = 1, 2, \ldots, M$. Here, $\boldsymbol{s}_i$ and $\boldsymbol{u}_o$ are $N \times 1$ column vectors of Cartesian coordinates, where $N = 2$ or 3 is the space dimension. The TDOA measurement at the sensor $\boldsymbol{s}_i$ with respect to the reference sensor $\boldsymbol{s}_1$ is given by

$$r_{i1} = r_{i1}^o + n_{i1}, \quad i = 2, 3, \ldots, M \tag{1}$$

where r_{i1}^o is the true TDOA defined by

$$r_{i1}^o = \|\boldsymbol{u}^o - \boldsymbol{s}_i\| - \|\boldsymbol{u}^o - \boldsymbol{s}_1\|. \tag{2}$$

Here, n_{i1} is the additive measurement noise and $\|\cdot\|$ denotes the Euclidean norm. Note that the TDOA measurement model given in (1) and (2) is already normalized by the speed of signal propagation c. In addition, it is a common practice to exploit one of the sensors as the reference sensor (e.g., the sensor $\boldsymbol{s}_1$ in this case) although any non-redundant set of $M-1$ sensor pairs can be used. In this paper, $(\cdot)^o$ with the superscript ^{o} denotes the true noise-free version of the matrix, vector or scalar in the argument.

Stacking $M-1$ TDOA measurements for $i = 2, 3, \ldots, M$ produces

$$\boldsymbol{r} = \boldsymbol{r}^o + \boldsymbol{n} \tag{3}$$

where $\boldsymbol{r} = [r_{21}, r_{31}, \ldots, r_{M1}]^T$, $\boldsymbol{r}^o = [r_{21}^o, r_{31}^o, \ldots, r_{M1}^o]^T$, and $\boldsymbol{n} = [n_{21}, n_{31}, \ldots, n_{M1}]^T$ is zero-mean Gaussian with covariance matrix $\boldsymbol{Q}$. In this paper, we assume that the covariance matrix $\boldsymbol{Q}$ is not known exactly while its structure is available *a priori*, which is often the case in practice. The TDOA localization problem is stated as estimating $\boldsymbol{u}^o$ from $\boldsymbol{r}$.

2.2. The Closed-Form Solution [1] and Its Bias

The closed-form TDOA solution in [1] consists of three stages. The first stage uses the square of the measurement equation and introduces the nuisance variable $r_1^o = \|\boldsymbol{u}^o - \boldsymbol{s}_1\|$ to formulate a system of linear equations with respect to $\boldsymbol{u}^o$ and r_1^o. A preliminary estimate of $\boldsymbol{u}^o$ and r_1^o is then obtained via linear least-squares estimation. The second stage aims to enhance the estimation accuracy of the first stage by exploiting the constraint relationship between $\boldsymbol{u}^o$ and r_1^o. The second stage solution is mapped onto the final position estimate in the third stage. The details of this closed-form solution can be found in [1]. Here is a summary of the computational steps:

Stage 1:

$$\boldsymbol{\varphi}_1 = (\boldsymbol{G}_1^T \boldsymbol{W}_1 \boldsymbol{G}_1)^{-1} \boldsymbol{G}_1^T \boldsymbol{W}_1 \boldsymbol{h}_1 \tag{4}$$

where

$$\boldsymbol{\varphi}_1 = \begin{bmatrix} \boldsymbol{u} \\ r_1 \end{bmatrix}, \; \boldsymbol{G}_1 = -2 \begin{bmatrix} (\boldsymbol{s}_2 - \boldsymbol{s}_1)^T & r_{21} \\ (\boldsymbol{s}_3 - \boldsymbol{s}_1)^T & r_{31} \\ \vdots & \vdots \\ (\boldsymbol{s}_M - \boldsymbol{s}_1)^T & r_{M1} \end{bmatrix}, \tag{5}$$

$$\boldsymbol{h}_1 = \begin{bmatrix} r_{21}^2 - \boldsymbol{s}_2^T \boldsymbol{s}_2 + \boldsymbol{s}_1^T \boldsymbol{s}_1 \\ r_{31}^2 - \boldsymbol{s}_3^T \boldsymbol{s}_3 + \boldsymbol{s}_1^T \boldsymbol{s}_1 \\ \vdots \\ r_{M1}^2 - \boldsymbol{s}_M^T \boldsymbol{s}_M + \boldsymbol{s}_1^T \boldsymbol{s}_1 \end{bmatrix}, \; \boldsymbol{W}_1 = (\boldsymbol{B}_1 \boldsymbol{Q} \boldsymbol{B}_1)^{-1}, \tag{6}$$

$$\boldsymbol{B}_1 = 2\,\mathrm{diag}\{[r_2^o, r_3^o, \ldots, r_M^o]\}, \; r_i^o = \|\boldsymbol{u}^o - \boldsymbol{s}_i\|. \tag{7}$$

Here, the superscript ^{T} denotes the matrix transpose operator. Note that $\boldsymbol{B}_1$ is a function of the true source location $\boldsymbol{u}^o$, which is not available. Therefore, $\boldsymbol{B}_1$ is computed using the initial estimate of $\boldsymbol{\varphi}_1$, denoted as $\hat{\boldsymbol{\varphi}}_1$, which is first obtained by setting $\boldsymbol{B}_1$ to identity. Specifically, r_i^o in (7) is replaced by $\|\hat{\boldsymbol{\varphi}}_1(1:N) - \boldsymbol{s}_i\|$.

Stage 2:

$$\boldsymbol{\varphi}_2 = (\boldsymbol{G}_2^T \boldsymbol{W}_2 \boldsymbol{G}_2)^{-1} \boldsymbol{G}_2^T \boldsymbol{W}_2 \boldsymbol{h}_2 \tag{8}$$

where

$$\boldsymbol{\varphi}_2 = (\boldsymbol{u} - \boldsymbol{s}_1) \odot (\boldsymbol{u} - \boldsymbol{s}_1), \; \boldsymbol{G}_2 = \begin{bmatrix} \boldsymbol{I}_{N \times N} \\ \boldsymbol{1}_{N \times 1}^T \end{bmatrix}, \tag{9}$$

$$\boldsymbol{h}_2 = \left(\boldsymbol{\varphi}_1 - \begin{bmatrix} \boldsymbol{s}_1 \\ 0 \end{bmatrix} \right) \odot \left(\boldsymbol{\varphi}_1 - \begin{bmatrix} \boldsymbol{s}_1 \\ 0 \end{bmatrix} \right), \tag{10}$$

$$\boldsymbol{W}_2 = \boldsymbol{B}_2^{-1} (\boldsymbol{G}_1^T \boldsymbol{W}_1 \boldsymbol{G}_1) \boldsymbol{B}_2^{-1}, \; \boldsymbol{B}_2 = 2\,\mathrm{diag} \left\{ \boldsymbol{\varphi}_1 - \begin{bmatrix} \boldsymbol{s}_1 \\ 0 \end{bmatrix} \right\}. \tag{11}$$

Here, $\odot$ denotes the Schur product (i.e., the element-by-element product).

Stage 3: The final position estimate is obtained by

$$\boldsymbol{u} = \boldsymbol{\Pi} \sqrt{\boldsymbol{\varphi}_2} + \boldsymbol{s}_1 \tag{12}$$

where $\boldsymbol{\Pi} = \mathrm{diag}\{\mathrm{sgn}(\boldsymbol{\varphi}_1(1:N) - \boldsymbol{s}_1)\}$ with $\mathrm{sgn}(\cdot)$ denoting the signum function.

Note that this solution assumes that there are at least $N+2$ sensors and the sensors are not located on a straight line for the 2D scenario or on a plane for the 3D scenario.

Unfortunately, the closed-form solution summarized above suffers severely from a bias problem due to the correlation between the noise in the regressor $\boldsymbol{G}_1$ and the regressand $\boldsymbol{h}_1$ of the weighted least-squares estimation in the first stage [2]. Specifically, the noise correlation between $\boldsymbol{G}_1$ and $\boldsymbol{h}_1$ results in an estimation bias in $\boldsymbol{\varphi}_1$. This estimation bias subsequently propagates to the second and third stages, thus leading to a bias in the final position estimate $\boldsymbol{u}$ of the third stage. Exact expressions for the estimation bias in each stage of the closed-form solution were derived in [2] by ignoring the third and higher-order noise terms. The bias of the closed-form solution can be compensated by estimating and subtracting the bias in the final position estimate $\boldsymbol{u}$ using the source position estimate and the noisy TDOA measurements [2]. However, this requires exact knowledge of the TDOA noise power and therefore is not applicable when only the structure of the TDOA noise covariance matrix is available as assumed in this paper.

2.3. The Bias-Reduced Closed-Form Solution [2]

To overcome the bias problem of the closed-form solution, the work in [2] has proposed a bias-reduced closed-form solution, namely the BiasRed method, by introducing an augmented solution equation and imposing a quadratic constraint. Specifically, the BiasRed method only modifies the estimation in the first stage while the second and third stages remain the same. Importantly, the BiasRed method only requires the prior knowledge of the structure of the TDOA noise covariance matrix. The BiasRed method [2] is summarized as below.

By introducing the augmented matrix $\boldsymbol{A}$ and vector $\boldsymbol{v}$, (i.e., $\boldsymbol{A} = [-\boldsymbol{G}_1, \boldsymbol{h}_1]$ and $\boldsymbol{v}^o = \left[\boldsymbol{\varphi}_1^{oT}, 1\right]^T$), the minimization objective function of the first stage of the closed-form solution [1] of $\epsilon = (\boldsymbol{h}_1 - \boldsymbol{G}_1\boldsymbol{\varphi}_1)^T \boldsymbol{W}_1(\boldsymbol{h}_1 - \boldsymbol{G}_1\boldsymbol{\varphi}_1)$ becomes

$$\epsilon = \boldsymbol{v}^T \boldsymbol{A}^T \boldsymbol{W}_1 \boldsymbol{A}\boldsymbol{v}. \tag{13}$$

Now decomposing $\boldsymbol{A}$ into the true noise-free matrix $\boldsymbol{A}^o$ and the noise term $\Delta\boldsymbol{A}$ yields $\boldsymbol{A} = \boldsymbol{A}^o + \Delta\boldsymbol{A}$, where $\Delta\boldsymbol{A} = 2\left[\boldsymbol{0}_{(M-1)\times N}, \boldsymbol{n}, \tilde{\boldsymbol{B}}_1\boldsymbol{n}\right]$ and $\tilde{\boldsymbol{B}}_1 = \operatorname{diag}\{[r_{21}^o, r_{31}^o, \ldots, r_{M1}^o]\}$. Substituting this into (13) and taking the expectation of $\boldsymbol{\epsilon}$ produces the objective function on the average as

$$E\{\epsilon\} = \boldsymbol{v}^T \boldsymbol{A}^{oT} \boldsymbol{W}_1 \boldsymbol{A}^o \boldsymbol{v} + \boldsymbol{v}^T E\{\Delta\boldsymbol{A}^T \boldsymbol{W}_1 \Delta\boldsymbol{A}\}\boldsymbol{v}. \tag{14}$$

where both terms are nonnegative functions of $\boldsymbol{v}$. If $E\{\epsilon\}$ is minimized with respect to $\boldsymbol{v}$, the second term $\boldsymbol{v}^T E\{\Delta\boldsymbol{A}^T \boldsymbol{W}_1 \Delta\boldsymbol{A}\}\boldsymbol{v}$ steers the solution away from $\boldsymbol{v}^o$, thus leading to an estimation bias.

To overcome such a problem, the BiasRed method aims to minmize ϵ subject to the constraint that the second term of (14) is constant. In addition, $E\{\epsilon\} = 0$ at the true solution $\boldsymbol{v} = \boldsymbol{v}^o$. As a result, the problem becomes

$$\min_{\boldsymbol{v}} \left\{\boldsymbol{v}^T \boldsymbol{A}^T \boldsymbol{W}_1 \boldsymbol{A}\boldsymbol{v}\right\} \quad \text{subject to} \quad \boldsymbol{v}^T \boldsymbol{\Omega}\boldsymbol{v} = k \tag{15}$$

where $\boldsymbol{\Omega} = E\{\Delta\boldsymbol{A}^T \boldsymbol{W}_1 \Delta\boldsymbol{A}\}$. Note that any value can be used for the constant k because it only affects the scaling of $\boldsymbol{v}$.

The Lagrange multiplier method can be used to solve the constrained minimization problem in (15), i.e.,

$$\min_{\boldsymbol{v}} \left\{\boldsymbol{v}^T \boldsymbol{A}^T \boldsymbol{W}_1 \boldsymbol{A}\boldsymbol{v} + \lambda(k - \boldsymbol{v}^T \boldsymbol{\Omega}\boldsymbol{v})\right\} \tag{16}$$

where λ is the Lagrange multiplier. Setting the derivative of the argument inside the minimization in (16) with respect to $\boldsymbol{v}$ to zero yields

$$(\boldsymbol{A}^T \boldsymbol{W}_1 \boldsymbol{A})\boldsymbol{v} = \lambda\boldsymbol{\Omega}\boldsymbol{v} \tag{17}$$

By noting that $k\lambda$ is the cost to be minimized if premultiplying both sides of (17) by $\boldsymbol{v}^T$, the solution $\boldsymbol{v}$ is the generalized eigenvector of the pair $(\boldsymbol{A}^T \boldsymbol{W}_1 \boldsymbol{A}, \boldsymbol{\Omega})$ which yields the smallest generalized eigenvalue. An explicit solution for $\boldsymbol{v}$ was proposed in [2] without the use of generalized eigen-decomposition. The solution $\boldsymbol{\varphi}_1$ is then obtained from $\boldsymbol{v}$ by dividing the first $(N+1)$ elements of $\boldsymbol{v}$ by its last element.

3. PROPOSED BIAS REDUCTION SOLUTION UTILIZING INSTRUMENTAL VARIABLES

In this section, we propose a new alternative bias reduction method, namely the IV-BiasRed method, based on the use of instrumental variables. Compared to the BiasRed method [2], the proposed IV-BiasRed method has low computational complexity while producing a performance almost identical to that of the BiasRed method [2] both in terms of bias and mean-squared error.

Recall from Section 2.2 that the bias of the closed-form solution arises from the noise correlation between the regressor $\boldsymbol{G}_1$ and the regressand $\boldsymbol{h}_1$. Motivated by this fact, the proposed IV-BiasRed method aims to resolve the bias problem of the closed-form solution by eliminating the noise correlation between $\boldsymbol{G}_1$ and $\boldsymbol{h}_1$ using the idea of instrumental variables. Specifically, the normal equations of the weighted least-squares estimation in (4)

$$(\boldsymbol{G}_1^T \boldsymbol{W}_1 \boldsymbol{G}_1)\boldsymbol{\varphi}_1 = \boldsymbol{G}_1^T \boldsymbol{W}_1 \boldsymbol{h}_1 \tag{18}$$

are modified to

$$(\boldsymbol{F}_1^T \boldsymbol{W}_1 \boldsymbol{G}_1)\boldsymbol{\varphi}_1^{\mathrm{IV}} = \boldsymbol{F}_1^T \boldsymbol{W}_1 \boldsymbol{h}_1, \tag{19}$$

where $\boldsymbol{F}_1$ is the instrumental variable matrix [14, 15]. As a result, the IV solution $\boldsymbol{\varphi}_1^{\mathrm{IV}}$ is given by

$$\boldsymbol{\varphi}_1^{\mathrm{IV}} = (\boldsymbol{F}_1^T \boldsymbol{W}_1 \boldsymbol{G}_1)^{-1} \boldsymbol{F}_1^T \boldsymbol{W}_1 \boldsymbol{h}_1. \tag{20}$$

In theory, if the IV matrix $\boldsymbol{F}_1$ is selected so that $E\left\{\frac{\boldsymbol{F}_1^T \boldsymbol{W}_1 \boldsymbol{G}_1}{M-1}\right\}$ is nonsingular and $E\{\frac{\boldsymbol{F}_1^T \boldsymbol{W}_1 \boldsymbol{h}_1}{M-1}\} = \boldsymbol{0}$ as $M \to \infty$, the IV solution $\boldsymbol{\varphi}_1^{\mathrm{IV}}$ in (20) becomes asymptotically unbiased, i.e., $E\{\boldsymbol{\varphi}_1^{\mathrm{IV}} - \boldsymbol{\varphi}_1^o\} \to \boldsymbol{0}$ as $M \to \infty$.

The optimal choice for the IV matrix $\boldsymbol{F}_1$ is the noise-free version of the matrix $\boldsymbol{G}_1$, i.e., $\boldsymbol{G}_1^o$. However, being a function of the unknown source position $\boldsymbol{u}^o$, $\boldsymbol{G}_1^o$ is not available. Instead the solution of Stage 1 can be used to construct a suboptimal IV matrix:

$$\boldsymbol{F}_1 = -2\begin{bmatrix} (\boldsymbol{s}_2 - \boldsymbol{s}_1)^T & r_{21}^{\mathrm{IV}} \\ (\boldsymbol{s}_3 - \boldsymbol{s}_1)^T & r_{31}^{\mathrm{IV}} \\ \vdots & \vdots \\ (\boldsymbol{s}_M - \boldsymbol{s}_1)^T & r_{M1}^{\mathrm{IV}} \end{bmatrix} \tag{21}$$

where $r_{i1}^{\mathrm{IV}} = \|\boldsymbol{\varphi}_1(1:N) - \boldsymbol{s}_i\| - \|\boldsymbol{\varphi}_1(1:N) - \boldsymbol{s}_1\|$.

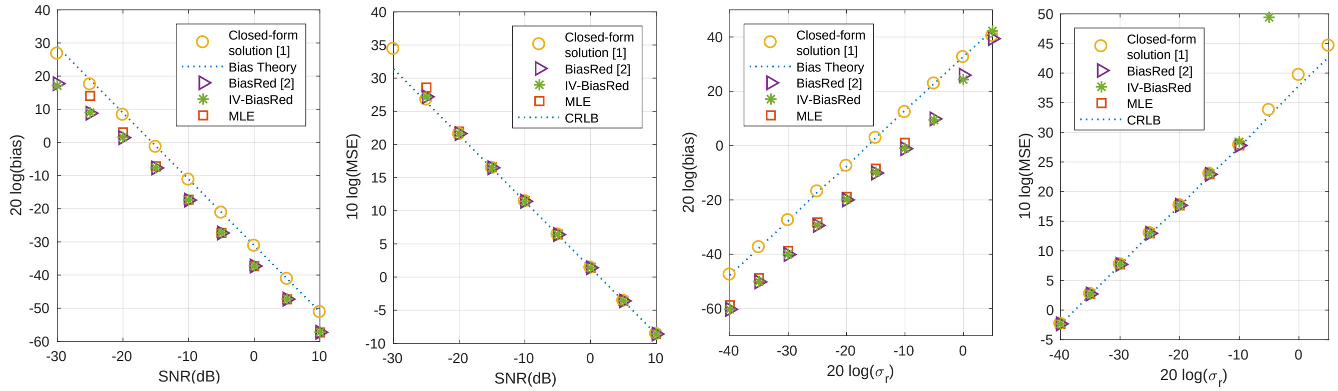
Since the bias of $\boldsymbol{\varphi}_1$ in the first stage the closed-form solution is already resolved by the new IV solution $\boldsymbol{\varphi}_1^{\mathrm{IV}}$, the second and third stages do not require any further bias compensation procedure. As a result, after the IV solution $\boldsymbol{\varphi}_1^{\mathrm{IV}}$ is calculated from (20) using the IV matrix $\boldsymbol{F}_1$ in (21), (8) and (12) are used to compute the final position estimate $\boldsymbol{u}$ with $\boldsymbol{\varphi}_1$ replaced by $\boldsymbol{\varphi}_1^{\mathrm{IV}}$.

For a finite number of sensors M, the IV-BiasRed method presented here is not strictly unbiased. Nevertheless, the proposed IV-BiasRed method is capable of achieving significant bias reduction compared to the original closed-form solution [1]. In Section 4, it is numerically demonstrated that the performance of the proposed IV-BiasRed method is on par with the BiasRed method proposed in [2] by achieving the same bias as the MLE and getting very close to the Cramér-Rao lower bound (CRLB). More importantly, the proposed IV-BiasRed method requires much less computation than the BiasRed method [2] as analytically shown in Table 1 and numerically demonstrated in Section 4.

4. SIMULATIONS

In this section, we present a performance evaluation of the proposed IV-BiasRed method in comparison with the closed-form solution [1], the BiasRed method [2] and the MLE using Monte Carlo simulations in terms of bias and mean-squared error. The mean-squared error (MSE) and bias are defined by $\mathrm{MSE} = \sum_{i=1}^{L} \|\boldsymbol{u}^{(i)} - \boldsymbol{u}^o\|^2/L$ and $\text{bias} = \|\sum_{i=1}^{L}(\boldsymbol{u}^{(i)} - \boldsymbol{u}^o)\|/L$, respectively, where L is the number of Monte Carlo runs. The MLE is implemented using 10 Gauss-Newton iterations and initialized using the true source position. The expression of the CRLB is given in [1]. Two simulation scenarios from [2], as described below, are considered.

Scenario 1 (two-dimensional): A sensor array with five sensors is considered, where the first sensor (the reference sensor) is located at the origin $[0, 0]^T$ m while the other four sensors are uniformly placed along a circle of radius 10 m, i.e., $\boldsymbol{s}_i = [10\cos(\frac{2\pi}{M-1}(i-1)), 10\sin(\frac{2\pi}{M-1}(i-1))]^T$ m for $i = 2, 3, 4$ and 5. The source is located at $\boldsymbol{u}^o = [100\cos(\frac{2\pi}{32}), 10\sin(\frac{2\pi}{32})]^T$ m. The TDOA noise



(a) Bias (Scenario 1) (b) MSE (Scenario 1) (c) Bias (Scenario 2) (d) MSE (Scenario 2)

Fig. 1. Performance comparison between the proposed IV-BiasRed method versus the solution [1], BiasRed method [2] and MLE.

Table 1. Comparison of computational complexities of the Stage 1†

Algorithm	Solution [1]	BiasRed [2]	IV-BiasRed
Multiplication	$(2N+6)(M-1)^2$	$4(M-1)^3+(2N+10)(M-1)^2$	$(3N+8)(M-1)^2$
	$+(2N^2+7N+6)(M-1)$	$+(2N^2+9N+9)(M-1)$	$+(3N^2+11N+8)(M-1)$
	$+2(N+1)^2$	$+(4N^2+8N+40)$	$+(3N^2+7N+3)$
Division	$M-1$	$M+2N+3$	$M-1$
Square Root	$M-1$	$M+1$	$2M-1$
Matrix Inversion	$2\,\mathrm{Inv}_{(N+1)\times(N+1)}$	$4\,\mathrm{Inv}_{N\times N}+2\,\mathrm{Inv}_{2\times 2}$	$3\,\mathrm{Inv}_{(N+1)\times(N+1)}$

†Only the complexity of the first stage is considered as the last two stages of the three methods are identical. Here, $\mathrm{Inv}_{K\times K}$ stands for an inversion of $K\times K$ matrix. The analysis excludes the computation for the terms which can be precalculated such as $\boldsymbol{Q}^{-1}$ and $\boldsymbol{s}_i^T\boldsymbol{s}$.

Table 2. Averaged runtime in MATLAB

Algorithm	MLE	Solution [1]	BiasRed [2]	IV-BiasRed
Runtime*	3.61	1	1.94	1.24

*normalized by the runtime of the closed-form solution [1].

covariance matrix is set to $\boldsymbol{Q} = \varrho(\boldsymbol{I}+\boldsymbol{1}^T)/2$ where ϱ is the noise power. Here, $\boldsymbol{I}$ is the identity matrix and $\boldsymbol{1}$ is the matrix of unity. At a given signal-to-noise ratio (SNR) value, the noise power ϱ is computed according to (31) of [13] multiplied with c^2, where $c = 3\times 10^8$ m/s. Here, 10,000 Monte Carlo runs are carried out.

Scenario 2 (three-dimensional): A total of 2000 localization geometries are randomly generated according to the uniform distribution. Eight sensors are randomly allocated within 100 m from the origin, while the source is randomly placed with a distance from the origin in between 100 m and 600 m. The MSE and bias are first computed over $L = 1{,}000$ Monte Carlo run for each geometry and then averaged over all 2000 geometries to obtain the final MSE and bias. The TDOA noise covariance matrix is set to $\boldsymbol{Q} = \sigma_r^2(\boldsymbol{I}+\boldsymbol{1}^T)/2$.

Fig. 1 compares the bias and MSE performance of the proposed IV-BiasRed method versus those of the closed-form solution [1], the BiasRed method [2] and the MLE for the two simulated scenarios. It is apparent the closed-form solution [1] exhibits a bias much larger than that of the MLE as expected. Moreover, the bias of the closed-form solution [1] agrees with its theoretical bias value derived in [2]. On the other hand, the proposed IV-BiasRed method and the BiasRed method produces an almost identical bias performance which is much lower than the bias of the closed-form solution [1]. In addition, the bias performance of the IV-BiasRed and BiasRed methods is very close to that of the MLE. This observation demonstrates the capability of the proposed IV-BiasRed method to achieve the bias as governed by the nonlinear nature of the TDOA localization problem.

In addition, Fig. 1 also demonstrates the optimum performance of the proposed IV-BiasRed method in term of the MSE performance (i.e., exhibiting a MSE almost identical to those of the BiasRed and MLE and closely achieving the theoretical CRLB) before the threshold effect occurs at SNR $= -25$ dB in Scenario 1 and at $20\log(\sigma_r) = -5$ in Scenario 2.

It is important to note that, although the proposed IV-BiasRed method exhibits a comparable performance with the BiasRed method [2] and the MLE, it requires much less computation than the BiasRed method and the MLE. It is observed from Table 2 that the proposed IV-BiasRed method is only 24% slower than the closed-form solution [1] due to the additional IV estimation, while the BiasRed method [2] and the MLE require 94% and 261% longer runtimes compared to the closed-form solution [1] respectively.

5. CONCLUSION

This paper has presented a computationally efficient bias reduction technique, namely the IV-BiasRed method, for the closed-form TDOA source localization solution [1]. The proposed IV-BiasRed method exploits the use of instrumental variables to eliminate the noise correlation between the regressor matrix and regresand vector, which is the main cause of bias for the closed-form solution in [1]. It was demonstrated via Monte Carlo simulations that the proposed IV-BiasRed method is capable of lowering the bias of the closed-form solution [1] to a level as governed by the nonlinear nature of the TDOA localization problem. While achieving bias and MSE performance comparable with those of the BiasRed method [2] and the MLE, the main advantage of the proposed IV-BiasRed method lies in its simplicity and low computational complexity.

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