MOBILE BAYESIAN SPECTRUM LEARNING FOR HETEROGENEOUS NETWORKS

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ABSTRACT

Spectrum sensing in heterogeneous networks is very challenging as it usually requires a large number of static secondary users (SUs) to capture the global spectrum states. In this paper, we tackle the spectrum sensing in heterogeneous networks from a new perspective. We exploit the mobility of multiple SUs to simultaneously collect spatial-temporal spectrum sensing data. Then, we propose a new non-parametric Bayesian learning model, referred to as beta process hidden Markov model to capture the spatio-temporal correlation in the collected spectrum data. Finally, Bayesian inference is carried out to establish the global spectrum picture. Simulation results show that the proposed algorithm can achieve a significant spectrum sensing performance improvement in terms of receiver operating characteristic curve and detection accuracy compared with other existing spectrum sensing algorithm.

1. INTRODUCTION

Cognitive radio (CR) is a promising paradigm to increase the spectrum usage efficiency and alleviate the spectrum scarcity problem for wireless networks [4]. In a CR network (CRN), spectrum sensing is widely recognised as the most critical function, which enables secondary users (SUs) to identify the spectrum holes [4] and access the idle spectrum opportunistically with no or minimal interference to primary users (PUs).

With the emerging new wireless networks and applications, a CRN could cover a large area, sometimes involving multiple PUs. Due to many factors such as path loss, shadowing, and fading, at any given time moment, the spectrum status observed by different SUs at different locations within the CRN may vary significantly, depending on SUs being within or out of the transmission ranges of PUs. A typical heterogeneous CRN is shown in Fig. 1. This renders the heterogeneous property, which entails great difficulty in sensing and predicting the spectrum availability at any given location. To acquire heterogeneous spectrum states, cooperative spectrum sensing (CSS) methods were proposed to exchange sensing information among SUs such as [5–7, 13, 14]. In essence, CSS exploits the spatial diversity to enhance sensing performance, taking advantages of spatially located SUs. In general, these methods assume SUs to be static, and require a large number of SUs, which is either infeasible or very costly in implementations.

However, it is worth noted that mobility is one of the most important characteristics inherent to the SU in a wireless network. Some initial studies in [9] verified that SU's mobility can significantly increase spatial-temporal diversity of the received signal by the SU in various wireless environments. In this paper, we propose a novel mobile CSS scheme for large-scale CRNs, drawing upon the

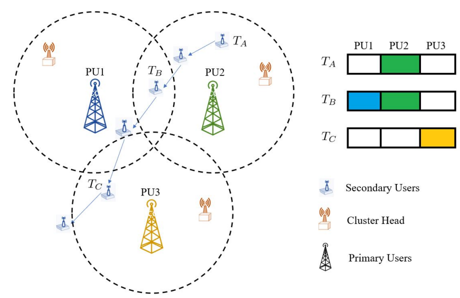

Fig. 1. A typical heterogeneous CRN. Black dash-line circle is the transmission range of each PU. At different sensing instants (T_A, T_B and T_C), a mobile SU experiences different spectrum states. Channels filled with colours indicate being occupied by corresponding PU.

recent development in Bayesian machine learning. Our idea is to utilise a small number of mobile SUs to collect spatio-temporal spectrum data, and derive the global spectrum states from these data. In specific, we propose a new non-parametric Bayesian learning model, referred to as beta process hidden Markov model (BP-HMM), to capture the spatio-temporal correlation in the collected spectrum data. Then Bayesian inference is carried out to automatically group sensing data into different classes in an unsupervised manner, where spectrum data in each class shares a common spectrum state. With the classification results, we can predict the accurate spectrum state for a new SU by assigning it to one of the existing classes.

Our proposed scheme features spectrum learning intelligence and high detection accuracy. Unlike the existing CSS schemes such as [12] which needs the priori information on the number of possible spectrum states, in the proposed scheme, such information can be automatically learnt from the sensing data. Meanwhile, the proposed scheme achieves better decision performance compared with traditional approaches such as energy detection [15] and Gaussian mixture model (GMM)-based machine learning algorithm [12]. Simulation results verify the supremacy of the proposed scheme in terms of receiver operating characteristics (ROC) curve and detection accuracy.

2. SYSTEM MODEL AND ASSUMPTION

We consider a large-scale CR network consisting of N PUs and M mobile SUs. The spectrum range of interest consists of L sub-bands, in one of which each PU operates and remains for a certain period if active. All the M SUs move within the network with low speed so that Doppler effect could be ignored. In the same time, they will collect spectrum data with certain time interval and store it locally. The entire network is organised into a cluster topology with a cluster head (CH) within each cluster. Every time when SUs finish their sensing task, depending on the location, SUs will transmit their spectrum sensing results to the nearby CH for processing, assuming error-free transmission. In the mean time, sensing information exchange among all CHs also take place.

At sensing time index i, the received sample for the l-th ($1 \leq l \leq L$) sub-band of the m-th ($1 \leq m \leq M$) SU can be expresses as:

$$x_l^m[i] = \begin{cases} n_l^m[i] & \mathcal{H}_0^{(l)}. \\ \sqrt{\gamma_l^m} s_l^m[i] + n_l^m[i] & \mathcal{H}_1^{(l)}. \end{cases} \tag{1}$$

where $\mathcal{H}_0^{(l)}$ denotes the hypothesis that in the l-th sub-band no PU is detected, while $\mathcal{H}_1^{(l)}$ indicates a PU is transmitting at sub-band l; $\sqrt{\gamma_l^m} s_l^m[i]$ is the received primary signal with average power $\gamma_l^{(m)}$; $n_l^{(m)}[i]$ is the additive white Gaussian noise denoted as $\mathcal{N}(0, \sigma_n^2)$ for all cases; following [8], $s_l^m[i]$ is assumed to follow complex Gaussian distribution with zero mean and unit variance, i.e., $s_l^m[i] \sim \mathcal{CN}(0, 1)$; Combined absent case with normal situation, then a unified distribution of $x_l^m[i]$ can be expressed by a compact complex Gaussian distribution with zero mean and covariance written as

$$x_l^m[i] \sim \mathcal{CN}(0, \gamma_l^m + \sigma_n^2). \tag{2}$$

We decompose $x_l^m[i]$ into its real and imaginary parts written as

$$x_l^m[i] = x_{l,re}^m[i] + j x_{l,im}^m[i]. \tag{3}$$

Both parts are independent and identically distributed (i.i.d) Gaussian variables with zero mean and equal variance $(x_{l,re}^m[i], x_{l,im}^m[i] \sim \mathcal{N}(0, (\gamma_l^m + \sigma_n^2/2))$. Note that since SUs are sensing L sub-bands simultaneously, the final sensing sample $\boldsymbol{x}^m[i]$ is L dimension observation written as

$$\boldsymbol{x}^m[i] = \begin{pmatrix} \boldsymbol{x}_{1,re}^m[i], ..., \boldsymbol{x}_{L,im}^m[i] \\ \boldsymbol{x}_{1,re}^m[i], ..., \boldsymbol{x}_{L,im}^m[i] \end{pmatrix}. \tag{4}$$

3. MODELLING OF COLLECTED SPECTRUM SENSING DATA

Given spectrum sensing data $\boldsymbol{x}^m[i]$, our goal is to find i) *which spectrum states each data point belongs to* and ii) *how many different spectrum states exist in the CRN*. To achieve the above goals, we first propose non-parametric Bayesian learning algorithm to capture the spatial-temporal correlation in spectrum sensing data. The graphical model is shown in Fig. 2, where HMM discovers the latent statistical correlation within single SU's sequential data $\boldsymbol{x}^m[i]$, while beta process finds common and unique spectrum states among multiple SUs.

In Fig. 2, the discrete hidden states $z_i^m = 1, 2, ..., k, (1 \leq k \leq K)$ is the index of spectrum states to indicate observations at sensing index i should belong to which spectrum states. For observation $\boldsymbol{x}^m[i]$ and hidden states z_i^m, the HMM assumes

$$\begin{gathered} z_{i+1}^m | z_i^m \sim \pi_{z_i}^m, \\ \boldsymbol{x}^m[i] | z_i^m \sim F(\theta_{z_i^m}), \end{gathered} \tag{5}$$

where $\pi_{z_i}^m$ is the state transition distribution which captures the latent correlation between two consecutive sensing samples; $F(\cdot)$ is the indexed distribution and here we use multivariate Gaussian distribution

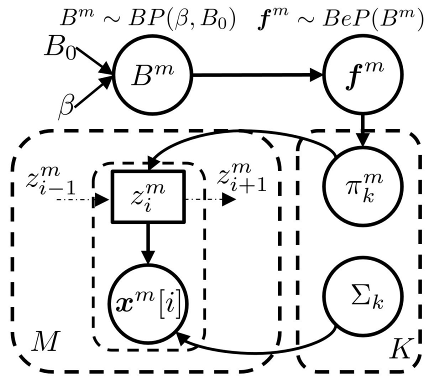

Fig. 2. Graphic model representation for the proposed BP-HMM.

$\mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_{z_i})$ in order to match the characteristics of our sensing sample in (2). Each state $z_i = k$ would have its own unique covariance $\boldsymbol{\Sigma}_k$. The covariance is further modelled by inverse Wishart (IW) distribution given by $\boldsymbol{\Sigma}|\boldsymbol{\Phi}, \nu \sim \mathcal{W}^{-1}(\boldsymbol{\Phi}, \nu)$ with degrees of freedom ν and scale matrix $\boldsymbol{\Phi}$. Note that IW distribution is the conjugate prior for the covariance matrix of a multivariate Gaussian distribution [1].

Since the traditional HMM tends to produce redundant states and quickly switch among them, however, In practice, the spectrum states that mobile SUs experience normally do not change rapidly. Therefore, we adopt sticky Hidden Markov Models (SHMM) [3] which enhances the self-transition probability by introducing an extra hyper-parameter κ. In other words, κ increases the probability of the spectrum state being the same for two consecutive samples. With the introduction of κ, the transition distribution out of state k is defined by a modified Dirichlet distribution. Given by

$$\boldsymbol{\pi}_k^m \sim Dir([\gamma, ..., \gamma, \gamma + \kappa, \gamma, \gamma]), \tag{6}$$

where γ is the global transition hyperparameter and κ is placed on self-transition probability to implement sticky function.

On the other hand, the 'cooperation' between multiple SUs' time series data are related by the overlap in the set of spectrum states each exhibits. Therefore, we define a globally shared set of possible spectrum states. Our goal is to discover which states are shared amongst the time series and which are unique. Therefore, we define a binary state indicator f_k^m, where $f_k^m = 1, (1 \leq k \leq K)$ implies that time series sensing data at the m-th SU exhibits k-th spectrum state. Consequently, transition probability is naturally restrained by its corresponding state indicator. For example, from current state s to new state s', its transition probability $\pi_{ss'}^m$ can be written as

$$\begin{cases} \pi_{ss'}^m = 0 & \text{if } f_{s'}^m = 0, \\ \pi_{ss'}^m > 0 & \text{if } f_{s'}^m = 1. \end{cases} \tag{7}$$

By utilising element-wise vector product function $\odot$ with the vector of binary indicator variables $\boldsymbol{f}^m = [f_1^m, ..., f_K^m]$, the transition distribution (6) is modified as

$$\boldsymbol{\pi}_k^m \sim Dir([\gamma, ..., \gamma, \gamma + \kappa, \gamma, \gamma]) \odot \boldsymbol{f}^m, \tag{8}$$

In Fig. 2, the function of beta process is to generate prior distribution on the vector of state binary indicator $\boldsymbol{f}^m$. Beta process can automatically determine the number of possible features in the object with an unbounded set of possible features [11]. Motivated by this ability, we refer to our time series data and spectrum states as objects and features respectively. Specifically, beta process results in a random measure B denoted as $B \sim BP(\beta, B_0)$. B is constructed by a collection of points $\{p_k, \omega_k\}$ as $B = \sum_{k=1}^{\infty} p_k \delta_{\omega_k}$. Each atom ω_k can be considered as one possible spectrum state with its associated probability p_k to be present. p_k is defined by beta distribution given as

$$p_k \sim Beta(\beta q_k, \beta(1 - q_k)), \tag{9}$$

where $q_k = m_k/(\beta + M - 1), q_k \in (0, 1)$ denotes the mass of the k-th atom in B_0, and m_k is the number of time series possessing state k.

Then we determine the value of binary state indicator f_k^m independently by Bernoulli process sampling its associated probability as $f_k^m \sim Bernoulli(p_k)$. We summarise our BP-HMM in Fig . 2 as follows

$$B^m | B_0, \beta \sim BP(\beta, B_0), \tag{10}$$

$$\boldsymbol{f}^m | B^m \sim BeP(B^m), \tag{11}$$

$$\boldsymbol{\pi}^m \sim Dir([\gamma, ..., \gamma, \gamma + \kappa, \gamma, \gamma]) \odot \boldsymbol{f}^m, \tag{12}$$

$$z_i^m | z_{i-1}^m, \boldsymbol{\pi}^m \sim Mult(\boldsymbol{\pi}_{z_{i-1}}^m), \tag{13}$$

$$\boldsymbol{x}^m[i] | z_i^m, \boldsymbol{\Sigma} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_{z_i^m}). \tag{14}$$

4. BAYESIAN INFERENCE

Given BP-HMM model and time series data $\boldsymbol{x}^m$ measured by SUs, next step is to infer latent spectrum state indicator z_i^m for every sensing sample index i. Note that the direct inference of the graphic model in Fig. 2 is intractable. We adopt Markov chain Monte Carlo (MCMC) sampling as our inference model. Since the graphic model is formulated hierarchically based on the conjugacy of distributions. Therefore, we update parameters one at a time with the other parameters assumed as known by the following updating rule in each iteration.

4.1. Updating Rule of Shared Features $\boldsymbol{f}^m$

Due to the conjugacy property in beta-Bernoulli process[11], the posterior distribution of binary state indicator f_k^m in m-th SU's time series data is given by

$$\begin{aligned} &p(f_k^m = 1 | \boldsymbol{F}_{-(mk)}, \boldsymbol{x}^m, \boldsymbol{\Sigma}, \boldsymbol{\pi}^m) \\ &\propto p(f_k^m = 1 | \boldsymbol{F}_{-(mk)}) p(\boldsymbol{x}^m | \boldsymbol{f}^m, \boldsymbol{\Sigma}), \end{aligned} \tag{15}$$

where the set of all feature vectors $\boldsymbol{f}^m$ is defined as a compact feature matrices $\boldsymbol{F}$, and $\boldsymbol{F}_{-(mk)}$ are the entries of $\boldsymbol{F}$ excluding f_k^m. According to the exchangeability of beta process [11], the first term in (15) is calculated as

$$P(f_k^m = 1 | \boldsymbol{F}_{-(mk)}) = n_k^{-m}/M, \tag{16}$$

where n_k^{-m} is the number of time series other than m possessing state k. This information is required to be exchanged among CHs. The second term can be derived by applying forward algorithm in HMMs [10]. The forward message satisfies the recursion

$$\tilde{\alpha}_{i+1}^m(z_{i+1}^m) = p(\boldsymbol{x}^m[i+1] | z_{i+1}^m) \sum_{z_i} \boldsymbol{\pi}_{z_i}^m \tilde{\alpha}_i^m(z_i^m), \tag{17}$$

where $\tilde{\alpha}_i^m(z_i^m)$ is the forward message defined as $\tilde{\alpha}_i(z_i^m) = p(\boldsymbol{x}^m[1], ..., \boldsymbol{x}^m[T], z_i^m)$. The initialisation is as follows,

$$\alpha_1^m(z_1) = \mathcal{N}(\boldsymbol{x}^m[1]; \mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{z_1}}) \pi_0^m(z_1^m). \tag{18}$$

Then we run the recursion from $i = 1, ..., T$ to obtain forward message $\alpha_2, ..., \alpha_T$, the desired likelihood is simply computed by summing over the components of the forward message at the last time index T as

$$P(\boldsymbol{x}^m[1], ..., \boldsymbol{x}^m[T] | \boldsymbol{f}^m, \boldsymbol{\Sigma}) = \sum_{z_T} \alpha_T(z_T). \tag{19}$$

4.2. Updating Rule of Transition distribution $\boldsymbol{\pi}^m$

Due to the conjugacy of Dirichlet prior $\boldsymbol{\pi}^m$ and multinomial likelihood $p(\boldsymbol{x}^m | \boldsymbol{\pi}^m)$, the posterior of transition distribution will result in a Dirichlet distribution. Hence, the update equation for the transition probabilities $\boldsymbol{\pi}_k^m$ out of current state k can be derived as

$$\pi_k^m | \boldsymbol{z}^m \sim Dir([..., \gamma + n_{kk'}^m + \kappa\delta(k, k'), ...] \odot \boldsymbol{f}^m), \tag{20}$$

where $n_{kk'}^m$ counts the transitions from state k to new state k' in the m-th SU's data, $\delta(k, k')$ is an indicator whose value will be one only when $k = k'$, γ is the global transition hyperparameter, and κ is the self-transition bias weight.

4.3. Updating Rule of Gaussian Parameters $\boldsymbol{\Sigma}_c$

Since the emission distribution lies in the exponential family, given IW prior on unknown covariance $\boldsymbol{\Sigma}_k \sim \mathcal{W}^{-1}(\boldsymbol{\Phi}_k, \nu_k)$, its degree of freedom $\nu_{k(new)}$ and scale matrix $\boldsymbol{\Phi}_{k(new)}$ can be updated by simply aggregating data from any time step in any time series assigned to state k written as

$$\nu_k^{new} = n_k^{old} + \nu_s, \tag{21}$$

and

$$\boldsymbol{\Phi}_k^{new} = \boldsymbol{\Phi}_k^{old} + \sum_{i=1}^{n} \boldsymbol{x}^m[i] \boldsymbol{x}^m[i]^T, \tag{22}$$

with corresponding hidden state indicator $z_i^m = k$. n_c is the total number of samples assigned to state k.

4.4. Updating Rule of Latent State Indicator $\boldsymbol{z}^{(j)}$

Given transition probabilities $\boldsymbol{\pi}^m$ and the information of binary states indicator $\boldsymbol{f}^m$, following [2], we can block-sample $z[1:T]^m$ in one coherent move by backward messages $\beta_i^m(z_i^m) = p(\boldsymbol{x}[i+1]^m, ..., \boldsymbol{x}[T]^m | z_i^m)$. It satisfies the backward recursion as

$$\beta_t^m(z_i^m) = \sum_{z_{i+1}} \beta(z_{i+1}^m) \mathcal{N}(\boldsymbol{x}^m[i+1]; \mathbf{0}, \boldsymbol{\Sigma}_{z_{i+1}^m}) p(z_{i+1}^m | z_i^m), \tag{23}$$

Finally, each z_i^m can be sampled recursively by

$$z_i^m | z_{i-1}^m, \boldsymbol{x}[1:T]^m, \boldsymbol{\pi}^m, \boldsymbol{\Sigma} \sim \beta_{i+1,i} \boldsymbol{\pi}_{z_{i-1}}^m N(\boldsymbol{x}^m[i]; \boldsymbol{\Sigma}_{z_i^m}). \tag{24}$$

In summary, this MCMC inference model iteratively samples these parameters conditionally on the others based on the updating rules given in Subsection 4.1 to 4.4 in one iteration. After reaching the pre-set upper bound of iteration time, we can obtain the final state assignment result z_i^m in every time series at any time step. In the same time, the common spectrum state $z_i^m = k$ for group k is represented by its corresponding multivariate Gaussian distribution $\mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_k)$. With this information, given a new SU and its sensing sample, we can calculate the probability of belonging to each group and assign it to the group with maximum probability. Due to page limitation, this part will be presented in our future work.

5. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed spectrum sensing algorithm and compare it with two popular spectrum sensing algorithms (energy detection [15] and GMM-based algorithm [12]) in static scheme. Note that GMM-based algorithm needs the prior information of the number of spectrum states in the CRN.

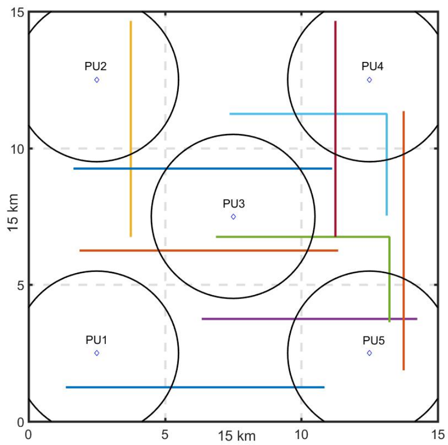

Fig. 3. The CRN configuration for simulation ($N_p = 5, N_s = 9$).

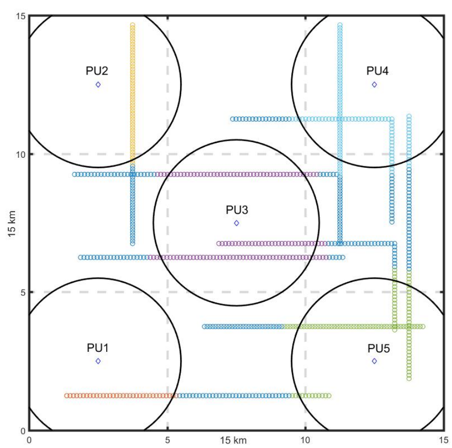

Fig. 4. Classification result of Fig. 3 based on our proposed algorithm, a different colour representing a different class.

We consider a typical CRN as a 15 km x 15 km area as shown in Fig. 3. The area is divided into nine (3x3) small grids, each corresponding to a cluster. We have $N = 5$ PUs deployed in one of the nine grids. Each PU operates independently in one of $L = 9$ sub-bands, each of which is assumed to have similar channel parameters. The transmission power of each PU is set to 20mW (black circle is the PU transmission range). Transmission signals attenuate according to a free-space propagation model. $M = 9$ mobile SUs are assumed, and their tracks are represented by different colours. Each track consists of eighty data points where SU performs sensing every 10 seconds and the overall sensing period is 800 seconds. Note that both PUs' position information and transmission power are unknown to SUs. The pre-set upper bound of iteration times is 100. The hyperparameter in BP-HMM model is set to be $\beta = 1$, $\kappa = 10$, $\gamma = 2$ and the initial variance in Gaussian distribution $\mathbf{\Sigma}$ is unity.

Fig. 4 presents the segmentation result of the CRN setup in Fig. 3. It clearly shows that the classification result is consistent with its actual results in terms of the spectrum states that each data point belongs to and the total number of spectrum states. Here, six different colours indicate six unique spectrum states in the CRN.

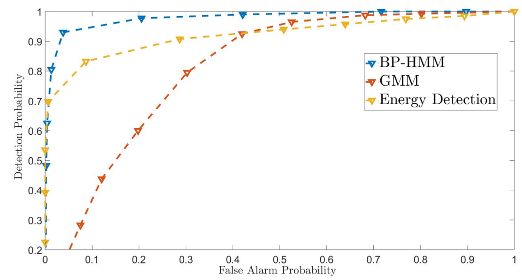

Fig. 5. ROC curve performance comparison with other algorithm under the same CRN setup in Fig. 3.

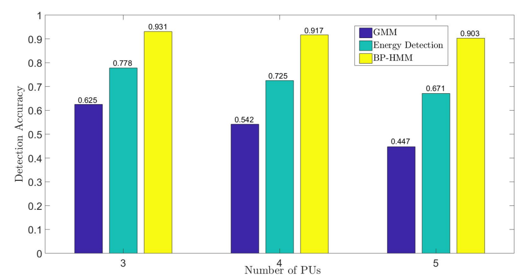

Fig. 6. Detection accuracy comparison with other algorithm under different PU number.

Fig. 5 and 6 depict the performance of different CSS scheme in terms of receiver operating characteristics (ROC) curve and detection accuracy respectively. As we can see, our BP-HMM based algorithm, even without prior knowledge of the number of heterogeneous spectrum states, significantly outperforms energy detection and GMM-based sensing schemes. It is because both energy detection and GMM-based algorithm only exploit the spatial correlation in essential while our algorithm takes full advantages of spatial-temporal characteristics in the data. In Fig. 6, with the increase of the number of PUs, the spectrum sensing performance degrades for all algorithms. However, our algorithm only suffers negligible detection accuracy degradation. This clearly demonstrates the robustness of the proposed algorithm against the increasing number of PUs.

6. CONCLUSIONS

A non-parametric machine learning based mobile CSS scheme was proposed for spectrum sensing in large-scale heterogeneous CRNs. By exploiting the mathematical correlation between multiple spectrum sensing time series, the proposed method jointly groups data points with common spectrum states. Furthermore, the performance evaluation based on ROC and detection accuracy showed that the proposed scheme significantly outperforms energy detection and GMM-based schemes.

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