FAST POISSONIAN-GAUSSIAN NOISE ESTIMATION

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ABSTRACT

We propose a fast and straightforward noise level function estimation method for signal-dependent noise from single-image raw-data. The noise is modeled as Poissonian-Gaussian as it is naturally suited for the raw-data of modern digital imaging sensors. We believe that hard segmentation approach used in the conventional noise estimation schemes degrades the accuracy of the estimation by reducing the sample size for estimation. Increasing the sample size, by using soft segmentation approach, improves the accuracy of the estimation. We propose to estimate the noise variance directly in the image intensity domain via weighted summation. Experiments on synthetic as well as real raw-images show that proposed method provides reliable noise estimation 20 times faster than the state-of-the-art.

Index Terms— *signal-dependent noise, Poisson noise, noise estimation, noise reduction*

1. INTRODUCTION

Images are contaminated mostly during acquisition from the image sensor. The raw-data acquired by sensors undergoes many image processing stages. Estimating the noise characteristics is vital for improving the image quality in the subsequent image processing stages. Most of the denoising methods assume additive white Gaussian noise (AWGN), which can successfully model thermal and electrical noise. The variance of noise under AWGN is fixed and signal-independent. However, due to the significant size reduction of pixel units, modern high-resolution imaging sensors are more sensitive to the photon noise [1]. Therefore, images acquired with modern imaging sensors are better modeled as signal-dependent, where the variance of noise is a function of signal intensity. The objective of the signal-dependent noise estimation methods is to predict the dependency of the noise variance on signal intensity, which is formally known as the *noise level function*.

Generally, the existing signal-dependent noise estimation methods consist of four steps: 1) image segmentation, 2) smooth region detection, 3) noise variance estimation and 4) fitting obtained data samples into a global parametric model. To estimate the noise variance – intensity pairs, most common practice is to cluster a noisy image into several segments with nearly constant intensities [1, 4]. Any intensity variations within the homogenous regions of these segments are considered to be caused by the noise of constant variance. To detect such image regions some methods utilize high pass filters [5, 6], edge and texture extractors or local statistics [7, 8]. In this manner, for each intensity segment, a set of homogeneous patches are obtained by means of hard segmentation. Generally, an input image is transformed into other domains, such as discrete cosine transform (DCT) [3, 9], and wavelet transform [1], where the noise can be distinguished from the original image information in high-frequency subbands. For high textured images, the noise level estimates of aforementioned methods are overestimated. In addition, according to the law of large number, the estimation accuracy is naturally dependent on the size of a sample. Due to the hard segmentation, the number of valid candidates on some intensity segments are low. Therefore, the noise variance estimates on these intensity segments do not converge towards the expected value. In [13] authors propose a noise estimation approach, based on the principal component analysis (PCA), which does not require the existence of homogeneous regions. In [17] authors demonstrate the possibility of processing together large heterogeneous image regions containing different intensity levels and noise variances without compromising the accuracy of estimation.

In the last step the obtained noise variance – intensity pairs are fitted into a global parametric model. Foi et al. [1] proposed to fit the estimated data into Poissonian-Gaussian noise model using iterative maximum likelihood fitting. Liu et al. [10] extended the method by using generalized signal-dependent noise model. Li et al. [3] proposed an iterative re-weighted least squares method, by taking the texture strengths of patches into account. Although these methods improve the accuracy of the noise estimation, they are computationally complex due to their iterative framework.

In this paper, we propose a fast and accurate method for image signal-dependent noise estimation. We believe that increasing the sample size for estimation by using soft segmentation approach would improve the accuracy of the estimation. Based on the noisy image histogram we initialize a pre-fixed number of *intensity centroids*, whose occurrence is large enough [3], and directly compute the noise variance for each intensity centroid via weighted summation. Specifically, we assign the pixels, which have similar intensity with the intensity centroid and correspond to the low gradient region, with high weights and vice versa. Unlike other conventional approaches, we estimate the noise variance directly in the image intensity domain. To fit the estimated noise variance – intensity pairs into a noise model we utilize a least mean square error approach.

The rest of the paper is organized as follows. In Section 2 we briefly review the image sensor noise model. The proposed method is then presented in Section 3. The accuracy and applications of the proposed method are discussed in Section 4. Finally, we draw conclusions in Section 5.

2. NOISE MODEL

We consider a generalized signal-dependent noise model [11] of the form

$$z(x) = y(x) + y(x)^{\gamma} \cdot u(x) + w(x) \tag{1}$$

where $x \in X$ is the pixel position, $z(x)$ is the observed noisy pixel value, $y(x)$ is the original noise-free pixel value, γ is a parameter that controls the dependence of noise on signal, and $u(x)$ and $w(x)$

$$\sigma'(k)^2 = \frac{1}{2}\left[\frac{\sum_{x\in X} W(x,k)\, G_h(x)^2}{\sum_{x\in X} W(x,k)} - \left(\frac{\sum_{x\in X} W(x,k)\, G_h(x)}{\sum_{x\in X} W(x,k)}\right)^2\right] \quad (9), \qquad y'(k) = \frac{\sum_{x\in X} z(x)\cdot W(x,k)}{\sum_{x\in X} W(x,k)} \quad (10).$$

are two mutually independent zero-mean Gaussian variables with variances σ_u^2 and σ_w^2 respectively. The noise level function $\sigma(y(x))$ of generalized signal-dependent noise model can be derived from (1), and has the affine form

$$\sigma(y(x)) = \sqrt{y(x)^{2\gamma}\cdot\sigma_u^2 + \sigma_w^2} \tag{2}$$

The Poissonian-Gaussian noise model is a special type of generalized signal-dependent noise model with $\gamma = 0.5$. In this paper, we consider this noise model as it is naturally suited for the raw-data of digital imaging sensors [1]. Our objective is to estimate the noise level function parameters (σ_u^2 and σ_w^2) from a single raw image.

3. PROPOSED METHOD

Ideally, one can obtain the true noise level function parameters if the noise variances corresponding to every intensity are known. Since we have a single raw-image at our disposal, it is impossible to estimate the noise variance for every intensity. In order to obtain the robust estimate of the noise variance, the size of the sample has to be large enough. Conventionally, the noise estimation algorithms segment an image into a collection of non-overlapping level sets, where each set is characterized by its mean value and allowed deviation. The noise variance on each segment is assumed to be constant and is computed only on the homogeneous regions. Therefore, for highly textured images the sample size (set of homogeneous patches in a given intensity level) is small. Our approach focuses on increasing the sample size for estimation. We proceed with the detailed explanation of our proposed approach.

3.1. Intensity Centroids

The objective of the first step is to initialize a pre-fixed number of intensity levels for which the noise variances are to be estimated in the subsequent steps. Hereafter, we refer to these intensity levels as the *intensity centroids*. Similar to [3], instead of using all intensity levels we choose K intensity levels whose occurrence are larger than the *p-quantile* ε ($p = 0.5$) of the noisy image histogram h

$$Y = \{\, y \mid h[y] \geq \varepsilon \} \tag{3}$$

where $h[y]$ stands for the occurrence of an intensity level y. In our experiments, we consider 8-bit (i.e., 256 intensity levels) grayscale images. To avoid outliers due to the *clipping problem* [1], we systematically discard the minimum and maximum of the dynamic range [0, 255] [2]. From the obtained set of intensity levels, we choose a pre-fixed number K of elements with fixed step-size $\Delta = \frac{|Y|}{K}$. In this manner, we form a vector of intensity centroids

$$I(k) = Y(k\Delta);\; k = 1,2,\dots K \tag{4}$$

3.2. Weighted Noise Level Estimation

To ease the explanation of our approach we first consider the noise estimation of an image contaminated with AWGN of variance σ_w^2

$$z(x) = y(x) + w(x) \tag{5}$$

Now consider the horizontal gradient G_h of the noisy image

$$G_h(x) = z(x+1) - z(x-1) \tag{6}$$

Then, on the homogeneous region the variance of (6) can be derived as follows

$$\sigma(G_h(x))^2 = 2\sigma_w^2 \tag{7}$$

In other words, the noise variance can be directly estimated by computing the variance of the gradient of homogeneous patches.

$$\sigma_w'^2 = \frac{1}{2}\left[\frac{\sum_{x\in X^{smo}} G_h(x)^2}{|X^{smo}|} - \left(\frac{\sum_{x\in X^{smo}} G_h(x)}{|X^{smo}|}\right)^2\right] \tag{8}$$

where X^{smo} is a set of pixels that correspond to a homogeneous region. The application of equation (8) for signal-dependent noise model of the form (1) is straightforward. The image is segmented into a collection of non-overlapping level sets and the noise variance of each segment is computed using equation (8). The accuracy of the estimation, however, will depend on the accuracy of the homogeneous region detection and the sample size ($|X^{smo}|$).

Backed by a Gaussian-mixture modeling, in [17] authors show that individual noise variance – intensity pairs estimates computed from large heterogeneous samples still follows (2), and the variance of a sample is a weighted summation of individual variances of homogeneous patches within the sample. Extending this observation, we propose a weighted noise variance estimation approach, where we directly compute the noise variance – intensity pairs for each intensity centroid via weighted summation as shown in (9) and (10), where X is a set of all pixels in the image, $W(x,k)$ is the weight of a pixel $x \in X$ for the estimation of the noise variance for an intensity centroid k, and $\sigma'(k)$ and $y'(k)$ correspond to the estimated noise variance – intensity pair. We assign the pixels, which have similar intensity with the intensity centroid and correspond to the low gradient region, with high weights and vice versa:

$$W(x,k) = exp\left(\frac{-\left(I(k) - z(x)\right)^2}{\delta_1}\right)\cdot exp\left(\frac{-G(x)^2}{\delta_2}\right) \tag{11}$$

where $I(k)$ is the intensity centroid, $G(x)$ is the gradient of a pixel, and δ_1 and δ_2 are sensitivity parameters. The first term of (11) takes into account the intensity similarity of the pixel with the intensity centroid. In our experiments, the sensitivity parameter δ_1 is set to 80. The second term takes into account the flatness of the neighborhood around the pixel to reduce the effect of outliers. To discard the effect of the noise on weighting, in our implementation we employ smoothed horizontal, vertical and average gradients of the image as follows

$$G(x) = max\left(G'_h, G'_v, G_{avg}\right) \tag{12}$$

The smoothed horizontal G_h' and vertical G_v' gradients are obtained by convolving the noisy image with [3×3] average operator and [3×1] gradient operators (6). The average gradient is obtained by convolving the noisy image with following operator:

$$g_{avg} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{13}$$

To further improve the accuracy of our estimation we set the sensitivity parameter δ_2 as the *average noise variance*. The average noise variance of the image is obtained by using the method presented in [12], where the variance of additive white Gaussian noise was estimated by using the difference of two Laplacians.

Table 1: Comparison of estimation accuracy in terms of RMSE ($\times 10^{-2}$) against theoretical noise level function (2) for various noise le vels (Low: $\sigma_u = 1.0, \sigma_w = 2.0$, Medium: $\sigma_u = 1.5, \sigma_w = 3.0$, and Heavy: $\sigma_u = 2.0, \sigma_w = 4.0$)

Image	Low contamination			Medium contamination			Heavy contamination		
	Pyatykh et al. [13]	Foi et al. [1]	Proposed	Pyatykh et al. [13]	Foi et al. [1]	Proposed	Pyatykh et al. [13]	Foi et al. [1]	Proposed
Street1	**0.13**	0.31	0.16	**0.23**	0.31	0.28	**0.29**	0.34	0.33
Street2	**0.08**	0.30	0.30	**0.23**	0.40	0.33	**0.26**	0.44	0.36
Yard	0.23	0.39	**0.07**	0.29	0.38	**0.11**	0.27	0.48	**0.03**
Building1	0.09	0.29	**0.06**	0.21	0.39	**0.04**	0.57	0.53	**0.01**
Building2	**0.11**	0.29	0.16	0.24	0.35	**0.11**	0.45	0.54	**0.31**
Hallway	**0.04**	0.22	0.10	**0.05**	0.27	0.37	**0.15**	0.34	0.47
Plaza	**0.19**	0.22	0.31	0.33	0.36	**0.33**	0.42	0.39	**0.35**
Traffic	**0.17**	1.12	0.32	0.41	0.86	**0.27**	0.68	0.79	**0.27**
Average	**0.13**	0.39	0.18	0.25	0.41	**0.23**	0.39	0.48	**0.27**

3.3. Noise Level Function Estimation

Let us denote the estimated noise variance – intensity pairs and noise level function parameters as vectors of the form $Y = [y_1', y_2', \dots, y_K; 1,1,\dots,1]^T$, $v = [\sigma_1'^2, \sigma_2'^2, \dots, \sigma_K'^2]^T$ and $p = [\sigma_u^2, \sigma_w^2]^T$, where $[\cdot]^T$ denotes the transpose operator. Then, one can formulate the global parameter estimation problem as the least square optimization problem of the form

$$p' = \arg\min_p \|Yp - v\|_2^2 \tag{14}$$

The problem (14) can be solved iteratively or directly by means of the closed form solution

$$p' = (Y^T Y)^{-1} Y^T v \tag{15}$$

Considering the non-linearity of the curve due to the clipping problem, in our implementation we utilized the iterative solution, as it provided more robust estimates with negligible additional complexity.

4. EXPERIMENTAL RESULTS

We evaluate the performance of the proposed method against the conventional approaches [1, 13] in terms of computational complexity, accuracy of the noise level estimation, and their corresponding effect on the noise reduction. The algorithms were implemented in MATLAB on a PC with Intel Core i5-3770K 3.5 GHz CPU and 8 GB RAM. Throughout the experiment, the number of intensity centroids was fixed ($K = 15$).

4.1. Estimation accuracy and complexity

To evaluate the accuracy of the estimation we test algorithms on the noise free image set shown in Fig. 1(a). The synthetic noisy images are generated by using three parameter settings ($\sigma_u = 1.0, \sigma_w = 2.0$; $\sigma_u = 1.5, \sigma_w = 3.0$; $\sigma_u = 2.0, \sigma_w = 4.0$) corresponding to low, medium and heavy contaminations, respectively. We measure the accuracy of the estimation in terms of the root mean squared error (RMSE) against the theoretical curve (2). Table 1 shows the detailed comparison of estimation accuracy of the algorithms. The presented RMSE results are the average of ten realizations. The average RMSE indicates that proposed method outperforms the state-of-the-art for medium and heavy noise levels. In addition, while the estimation accuracy of all competing algorithms degrade with increasing the noise level, the proposed method is comparatively robust against noise level variations. The estimated noise level functions for 'Traffic' test image are depicted in Fig. 2. For high noise level, conventional approaches overestimate the noise standard deviation, while the estimates of proposed approach almost coincides with the theoretical curve. Foi et al. approach [1] fails when the noise level is low, while Pyatykh et al. approach [13] performs well (best RMSE). In Table 2, we summarized the average RMSE results and the average runtimes of corresponding algorithms. The proposed method on average has the best accuracy and is 20 times faster than [1], and 700 times faster than [13].

Table 2: Comparison of the average estimation accuracy (RMSE) and computational complexity (runtime).

	Pyatykh et al.		Foi et al.		Proposed	
	RMSE ($\times 10^{-2}$)	Runtime (sec.)	RMSE ($\times 10^{-2}$)	Runtime (sec.)	RMSE ($\times 10^{-2}$)	Runtime (sec.)
Low	**0.13**	328.2	0.39	7.51	0.18	**0.42**
Medium	0.25	321.8	0.41	8.84	**0.23**	**0.44**
Heavy	0.39	287.7	0.48	9.92	**0.27**	**0.46**
Average	0.26	312.6	0.43	8.75	**0.23**	**0.44**

4.2. Application in Noise Reduction

To highlight the effectiveness of the noise level function estimation, we utilize the obtained noise parameters on the noise reduction. The

Table 3: Comparison of BM3D based noise reduction performance in terms of PSNR in dB.

Image	Noise Level	BM3D [14]		
		Original	Foi et al.	Proposed
Shoes	35.11	40.96	40.83	**41.09**
Kitchen	32.03	36.39	39.16	**39.20**
Desk	26.12	33.42	**33.50**	33.40
Sink	33.96	41.26	42.62	**43.17**
Painting	33.39	36.53	37.19	**37.53**
Papers	34.66	36.48	37.16	**37.21**
Wall	27.06	33.14	34.79	**35.08**
Flowers	35.38	37.01	**39.93**	39.65
Average	32.21	36.90	38.15	**38.29**
Runtime (in seconds)		13.17	35.14	**17.09**

estimated noise parameters can be applied to any AWGN noise reduction algorithm to consider signal-dependent noise. In this paper, we consider BM3D [14], the state-of-the-art AWGN noise reduction algorithm. Similar to other approaches [1, 2, 16], in our implementation we utilized the variance stabilizing transformation (VST) [15]. The objective of this transformation is to transform an input signal-dependent noisy image into a domain where the noise has fixed variance (AWGN). The accuracy of VST and, consequently, the performance of the noise reduction are dependent on the accuracy of the noise parameter estimation. We evaluated the performance of only fast and practical noise reduction algorithms in terms of PSNR on real noisy image set shown in Fig. 1(b). The base for comparison is the original BM3D results with default parameters. We compared the performance of our scheme with the performance of VST-BM3D based method often referred to as state-of-the-art [1]. As shown in Table 3, the incorporation of the proposed fast estimation approach has the highest PSNR among compared algorithms. Additionally, it is two times faster than [1], making it suitable for practical signal-dependent noise reduction applications.

5. CONCLUSION

We presented a fast and practical noise level function estimation method for Poissonian-Gaussian noise from a single image. The hard segmentation approach used in the conventional noise estimation schemes reduces the sample size of the estimation, consequently degrading the estimation accuracy. We showed that increasing the sample size for estimation, by using the proposed soft segmentation approach, is not only computationally efficient, but also improves the accuracy of the estimation. We initialize a pre-fixed number of intensity centroids based on the noisy image histogram, and compute the noise variances for each intensity centroid in the image intensity domain via weighted summation. The pixels, which have similar intensity with the intensity centroid and correspond to the low gradient region, are assigned with high weights and vice versa. According to our experiments on synthetic and real noisy images, our method provides fast and reliable noise estimates.

Fig. 1. Image sets used in experiments (a) noise free 704×469 8-bit grayscale image set [18], and (b) noisy 2599×1733 8-bit grayscale image set [19].

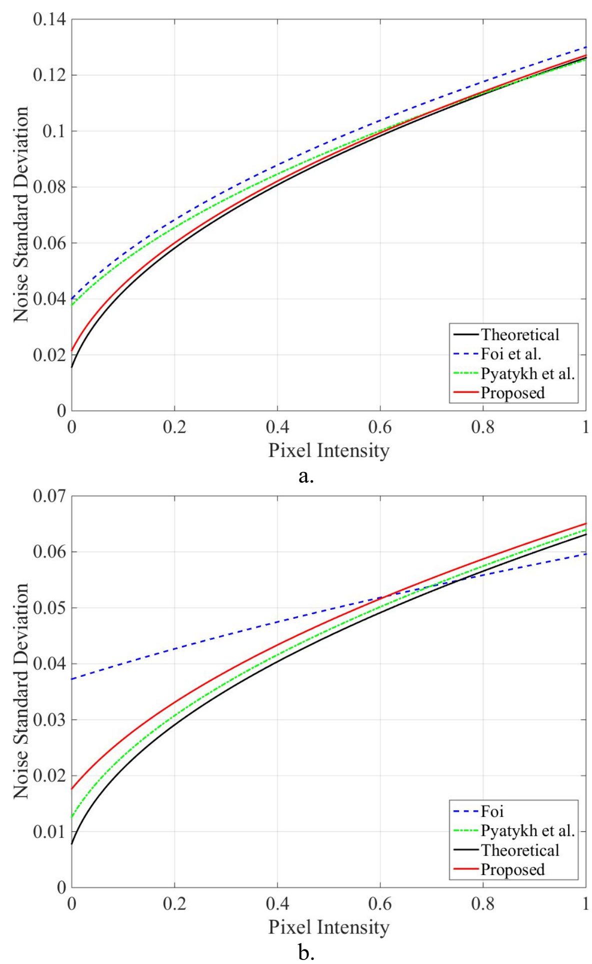

Fig. 2. Comparison of noise level function estimation with Foi et al. [1], and Pyatykh et al. [13] against theoretical noise level f unction (2). The estimation is performed on 'Traffic' test image for (a) heavy, and (b) low noise levels.

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