OPTIMAL SPECTRAL ESTIMATION AND SYSTEM TRADE-OFF IN LONG-DISTANCE FREQUENCY-MODULATED CONTINUOUS-WAVE LIDAR

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ABSTRACT

Frequency-modulated continuous-wave (FMCW) LIDAR is a promising technology for next-generation integrated 3D imaging systems. However, it has been considered difficult to apply FMCW LIDAR for long-distance (>100m) targets, such as those in automotive and airborne applications. Maintaining coherence between the reflected beam from the target and locally forwarded beam becomes a significant challenge for tunable laser design. This paper demonstrates the possibility of extending the detection range of FMCW LIDAR beyond the coherence range of its laser by improving the spectral estimation algorithm. By exploiting the Lorentzian prior of the received signal in the spectral domain, >10x improvement in ranging accuracy is achieved compared to traditional algorithms that do not consider phase noise in the signal model. In light of this finding, the end-to-end modeling framework is presented to examine true system-level trade-offs of FMCW LIDAR and the feasibility of long-distance measurement.

Index Terms— Frequency-modulated continuous-wave (FMCW) LIDAR, remote sensing, 3D imaging, spectral estimation

1. INTRODUCTION

Using light as the sensing medium, LIDAR (light detection and ranging) [1] can achieve orders-of-magnitude superior lateral resolution compared to ultrasound or RF wave-based 3D imaging systems of similar form-factor and is thus considered as a crucial building block for autonomous vehicles [2] and smart robots [3]. In particular, frequency-modulated continuous-wave (FMCW) LIDAR has a strong advantage over time-of-flight (TOF) LIDAR for high-volume, low-cost implementation largely due to its compatibility with existing integrated electro-optics platform [4], and robustness to background effects.

In FMCW LIDAR, the depth information is captured by the frequency of the beating tone at the coherent receiver. This implies that the quality of the received signal is a function of the spectral purity of the laser source [5]. Beyond the coherence range of the laser, which is inversely proportional to the laser linewidth, the power spectral density (PSD) of the receiver signal is closer to Lorentzian shape rather than a clean harmonic beat tone. As a result, it is commonly assumed that FMCW measurement is not possible for the range beyond the coherence range of the laser [6]. Under that constraint, producing frequency-modulated laser signal with the phase noise low enough for long-distance (>100m) FMCW LIDAR, relevant to automotive or airborne applications, becomes highly challenging.

It is still possible to extract the target distance from an incoherent measurement even though the accuracy is worse. Many works on FMCW RADAR, the same ranging method using RF wave instead of laser, analyzed the general impact of the phase noise on the ranging performance. [7] measured the sensitivity of frequency estimation accuracy with respect to the sinusoidal phase noise so that the relationship between the given phase noise profile and the ranging performance can be quantified. [8] studied the impact of phase noise in a more general system context and verified their analysis with measurement results. However, the frequency estimation algorithm itself has not been a primary focus in these studies. Even though there are a few frequency estimation algorithms proposed for improving ranging accuracy of FMCW measurements [9][10], none of them attempted to modify the signal model to explicitly address the phase noise. As a result, a system-level study including the impact of detection algorithm choice is lacking.

In this work, we first propose a spectral estimation algorithm tailored for long-distance FMCW LIDAR measurement, corrupted both by phase noise and additive white noise. By leveraging known Lorentzian prior of the received signal in the spectral domain, the accuracy was improved by >10x, making it possible to achieve reasonable performance for targets well beyond the coherence range. Based on this algorithm, we also present our end-to-end model for FMCW LIDAR to map the system specifications to the ranging accuracy so that one can estimate required laser power and receiver bandwidth for given system parameters and accuracy/range targets.

The remainder of the paper is organized as follows. Section 2 provides an overview of the operating principle of FMCW LIDAR. In Section 3, the impact of phase noise in the laser on FMCW measurement is studied. Section 4 introduces proposed spectral estimation scheme optimized for targets beyond the laser coherence range. Section 5 finally presents the end-to-end model for FMCW LIDAR system including the spectral estimation algorithm and provides the system design guideline based on this model. Section 6 concludes the paper.

2. FMCW LIDAR CONCEPT

Overview of the FMCW LIDAR system is shown in Fig. 1. At the core of FMCW LIDAR is continuous-wave tunable laser whose frequency is modulated by a certain waveform shape. Sawtooth wave is one of the most frequently used pattern in practice and we apply it to the analysis in this paper. The results are extendable to other modulation formats. Over the observation time T, frequency of the laser is linearly increased by the chirp bandwidth f_{BW} . The resulting modulated laser with chirping rate $\gamma = f_{BW}/T$ is split into two beams. One beam goes into the free space directed towards the target of interest, and gets reflected by the target. The other beam goes directly into the coherent receiver as a local oscillator signal (E_{LO}) where it combines with the beam collected from the reflection (E_{RX}).

Due to the group delay mismatch between two beams, there is an instantaneous frequency difference, as shown in Fig. 1. The coherent receiver acts as an analog mixer and produces the photocurrent whose frequency f_{beat} is equal to the frequency difference of the two

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Fig. 1. FMCW LIDAR overview

beams. It is clear from Fig. 1 that f_{beat} is directly proportional to both γ and the delay mismatch τ . Since the delay mismatch is a linear function of the round-trip time of the reflected beam, we can measure the distance of the remote target d from this beat frequency. Assuming on-chip propagation delay is negligible compared to the free-space propagation, we can simplify the relationship between d and f_{beat} with speed of light c as follows.

$$f_{\text{beat}} = \gamma \tau = \frac{f_{\text{BW}}}{T} \frac{2d}{c} \tag{1}$$

The FMCW LIDAR is particularly attractive for monolithic implementation since its building blocks are readily available in silicon photonics technology. For example, coherent detection achieves shot-noise limited performance without avalanche photodiodes unlike time-of-flight (TOF) LIDAR [4]. In addition, it is easily extended to velocity detection through Doppler measurement, and the detection is relatively insensitive to the interference from other sensors or ambient light. Recent demonstrations [4][11][12] of the FMCW LIDAR in integrated platforms show its potential as a baseline technology for low-cost, chip-scale 3D imaging systems.

3. COHERENCE IN FMCW LIDAR MEASUREMENT

The photocurrent signal at the coherent receiver is a pure sinusoidal tone only when the phase offset of the laser is constant. In reality, the phase and frequency of any practical laser are random processes, and their randomness is quantified by full-width at half-maximum (FWHM) of the baseband spectrum of the electric field, often referred to as linewidth $\Delta \nu$. Assuming spontaneous emission as dominanat phase noise source, the laser frequency noise is white and its spectral density ϕ_n is equal to its linewidth.

$$S_{\dot{\phi}_n}(\omega) = \Delta\omega = 2\pi\Delta\nu \tag{2}$$

Given non-zero linewidth, the phase of the photocurrent at the receiver is also a random process and its noise process is directly related to the laser phase noise by the following relationship:

$$\phi_{n,\text{photocurrent}}(t) = \phi_{n,\text{LO}}(t) - \phi_{n,\text{RX}}(t) = \phi_n(t) - \phi_n(t-\tau).$$
(3)

The resulting power spectral density $S_i(\omega)$ of the photocurrent signal for observation time T and delay mismatch τ in shot-noise limited receiver is expressed as follows [5]

$$S_i(\omega) = S_i^{\circ}(\omega - 2\pi\gamma\tau) + S_i^{\circ}(\omega + 2\pi\gamma\tau) + qP_{\rm LO}R_{\rm PD} \quad (4)$$



Fig. 2. Signal peak magnitude of the FMCW LIDAR signal for different target distance (left) and corresponding PSD in the frequency domain. Power density was normalized to the peak at zero distance and shot noise was ignored ($T = 10\mu$ s, $\Delta \nu = 1$ MHz, $d_c \approx 82$ m).

$$S_{i}^{\circ}(\omega) = R_{\rm PD}^{2} P_{\rm RX} P_{\rm LO} \left[T \operatorname{sinc}^{2} \left(\frac{T\omega}{2} \right) e^{-\frac{2\tau}{\tau_{c}}} + \frac{\tau_{c}}{1 + \left(\frac{\omega\tau_{c}}{2} \right)^{2}} \\ \cdot \left\{ 1 - e^{-\frac{2\tau}{\tau_{c}}} \left[\cos(\omega\tau) + \frac{2}{\omega\tau_{c}} \sin(\omega\tau) \right] \right\} \right],$$
(5)

where $R_{\rm PD}$ is the responsivity of photodetector (PD), q is the electron charge, and $P_{\rm RX/LO}$ is the power of the RX/LO beam. The first term in (5) is the main beating tone convolved with sinc-squared function due to finite observation time, and the second term forms a pedestallike Lorentizan distribution around the main beat tone. The last term in (4) represents shot-noise floor, which becomes more pronounced when $P_{\rm RX}$ is weak. The coherence time of the laser, a commonly used metric in the literature is tied to the linewidth as $\tau_c = 2/\Delta\omega$. This expression assumes $T \gg \tau, \tau_c$, which is normally the case for FMCW measurements.

When the target is close to the sensor (i.e. within laser's coherent range where $\tau \ll \tau_c$), the phase noise processes of the RX and LO beams are highly correlated. In this *coherent* regime, the two processes tend to cancel each other in the photocurrent phase, as shown in (3). Indeed, when $\Delta\omega\tau \ll 1$, the second term in (5) vanishes and the receiver yields clean beating tone. On the other hand, if the target is outside of the coherent range ($\Delta\omega\tau \gg 1$), (5) converges into the following expression

$$S_i^{\circ}(\omega) = R_{\rm PD}^2 P_{\rm RX} P_{\rm LO} \left[\frac{\tau_c}{1 + \left(\frac{\omega \tau_c}{2}\right)^2} \right].$$
(6)

The receiver signal PSD now exhibits the Lorentzian shape around f_{beat} with linewidth $2\Delta\nu$, two times the original linewidth, since the phase noise power simply adds up when two beams are *incoherent*.

Figure 2 shows the relationship between the distance of the target and the PSD peak power density. Shot noise floor is neglected to emphasize the impact of phase noise and the power is normalized by the beating tone at zero distance. For short distance, peak power follows the first term in (5) and decays exponentially as distance increases. This trend gradually diminishes as the second term takes over, and eventually the peak becomes independent of the target distance as in (6). This crossing point between the beating tone and the pedestal induced by the Lorentzian term, or the coherence range d_c relevant to the FMCW measurement, is expressed as follows.

$$d_c = \frac{c\tau_c}{4} \ln\left(\frac{T}{\tau_c}\right) \tag{7}$$

Note that the coherence range is also a function of T (i.e. the measurement enters incoherent regime more quickly for shorter observations).

Previous works assumed that the detection range of the FMCW LIDAR is fundamentally limited by d_c [6]. This is very challenging for the laser design in the context of long-distance, fast-scanning LI-DAR relevant to automotive, airborne and other autonomous system applications. For example, if the range is 100m with $10\mu s$ observation time, the laser linewidth is required to be a few hundreds of kHz or less, which is difficult to guarantee especially when the laser is required to be equipped with fast wideband wavelength tuning. Any technique enabling fast, continuous tuning over wide wavelength range within compact footprint, such as MEMS mirror [13] or DBR [14], comes with significant additive phase noise. Alternatively, a chirped laser can be generated using continuous-wave laser followed by an external I/Q modulator [15]. However, this approach requires fast drivers and chirp generator in the electrical domain which can also add phase noise. Largely due to this coherence issue, the detection range of FMCW LIDAR demonstrations in the literature has been limited to only a few meters [1].

However, note that even though the signal spectrum in the incoherent regime has a different shape and is relatively weak, the PSD of the photocurrent is still a function of the target distance as it is evident in (4) and (5). Moreover, we have the prior information from (6) that the PSD corresponding to a target is Lorentzian in the baseband. Motivated by these observations, we leverage such knowledge in the spectral estimation algorithm and evaluate the FMCW LIDAR performance in the incoherent regime in the next section.

4. SPECTRAL ESTIMATION ALGORITHM FOR INCOHERENT FMCW MEASUREMENTS

In the coherent regime, the phase offset of the beat tone corresponding to each target may be unknown but can be modeled as constant within single observation. The role of the receiver backend is to solve a classical problem of line spectra estimation. It is one of the most well-studied topics in signal processing [16], and there are numerous algorithms one can choose depending on the nature of additive noise and affordable complexity. In general, it is possible to achieve arbitrary accuracy as long as the signal-to-noise ratio is sufficiently high.

On the other hand, the constant phase offset assumption is invalid for the FMCW measurement in the incoherent regime. In [17], it was shown that the fundamental lower bound of the frequency estimation variance becomes a function of the amount of the phase noise. Especially, it was also shown that any algorithm designed assuming constant phase offset performs poorly in presence of the phase noise. In other words, it may not be possible to achieve desired accuracy with classic frequency estimation methods even if the sensor can afford infinite power from the reflection (P_{RX}).

In order to improve the performance of distance estimation in the incoherent regime, we devised a new frequency estimation algorithm that exploits prior knowledge about the received signal PSD shape. From (4) and (6), it is evident that each target shows up in the spectral domain as a Lorentzian function shifted by the beating frequency. Therefore, we can define the signal model of the single-sided power spectral density of any incoherent FMCW measurement as

$$\tilde{S}(\omega; \alpha, \omega_{\text{beat}}) = \sum_{i=1}^{n} \frac{\alpha_i \Delta \omega}{(\omega - \omega_{\text{beat},i})^2 + \Delta \omega^2} + 2qR_{\text{PD}}P_{\text{LO}}.$$
 (8)

If the number of possible targets is assumed to be n, there are 2n parameters: $\omega_{\text{beat},i}$ and α_i are the center frequency and relative power of the *i*th Lorentzian, respectively. Given this model and the periodogram estimate of the PSD from the measurement, we can simply

perform nonlinear least-squares to estimate those parameters.

$$\alpha^*, \omega_{\text{beat}}^* = \underset{\alpha, \omega_{\text{beat}}}{\operatorname{arg\,min}} \left| S(\omega) - \tilde{S}(\omega; \alpha, \omega_{\text{beat}}) \right|^2 \tag{9}$$

Note that the Lorentzian-shape PSD itself is deterministic, but its estimate using periodogram adds uncertainty. It can also have small bias if the length of the measurement is too small [18], but this is generally negligible considering realistic sample rates and observation times. In addition, it is possible that the lineshape of the practical laser deviates from Lorentzian shape, depending on its dominant phase noise mechanism [12]. However, it is possible to measure the lineshape of the laser under test and improve the parametric model using different prior (for example such as the Voigt function [19]).

In order to test the performance of the proposed Lorentzian least squares estimation (LLSE) and compare it to the standard frequency estimation schemes with constant phase offset model, we built a behavioral model of the FMCW LIDAR using Simulink and ran transient simulations to generate realistic data. We have assumed that the $E_{\rm LO}$ beam is strong enough for the receiver to be shot-noise limited, and only one target exists in the measurement. Baseline laser parameters were $f_{\rm BW} = 10$ GHz, $T = 10\mu$ s, $\Delta\nu = 1$ MHz which correspond to $\gamma = 1$ GHz/ μ s, $d_c = 82.3$ m, $\tau_c = 0.318\mu$ s. $R_{\rm PD}$ was 1A/W. With simulated time-domain data for target distance up to 100m, we applied different algorithms including the proposed LLSE method to estimate the distance and recorded estimation variance from 100 Monte Carlo simulations per each distance. Among a number of constant-phase frequency estimation methods, Rife and Boorstyn's [20] and MUSIC [21] were used for comparison.

Figure 3(a) shows the result when P_{RX} is 1mW. Estimated PSD shows that the shot noise floor is almost negligible compared to the Lorentzian noise pedestal. Such high SNR is expected from shortdistance targets with high reflectivity. In this case, the performance of MUSIC algorithm is the best, and proposed LLSE shows similar, but slightly worse performance. The accuracy of R&B algorithm is much worse than other algorithms except for deeply coherent regime. In Fig. 3(b), P_{RX} is reduced to 1nW so that the SNR level is more relevant for long-distance LIDAR where the reflected beam undergoes significant loss during free-space propagation. Under this setting, the performance of the LLSE and R&B was almost unchanged, but the MUSIC algorithm performed very poorly compared to the high SNR case. This is not surprising since frequency estimation algorithms based on eigendecomposition of autocorrelation matrix, including MUSIC, rely heavily on the model for additive noise and are known to be unstable in the general case [22]. For both high SNR and low SNR case, the proposed LLSE algorithm showed consistently excellent performance.

Finally, linewidth of the laser is increased to 10MHz in Fig. 3(c). Coherence range is only 10m in this case, and the measured PSD clearly shows that $E_{\rm LO}$ and $E_{\rm RX}$ are completely incoherent. With this noisy laser and low SNR, the proposed LLSE is the only algorithm that can yield acceptable performance. For a 100m target, the variance of the LLSE estimator was 4.82cm in contrast to 56cm of the R&B estimation, showing >10x improvement. From this result, we can clearly see that the impact of the frequency estimation algorithm choice for the system is critical, especially for low SNR, incoherent measurements.

5. END-TO-END FMCW LIDAR MODEL

Based on the observation so far, we created an end-to-end modeling framework including the frequency estimation algorithm to reveal the system level trade-offs of the FMCW LIDAR. Table 1 summarizes the constraints and baseline design variables for our model.



Fig. 3. Impact of frequency estimation algorithm on FMCW measurement. Distance estimation variance for different algorithms and periodogram PSD estimates with Least squares fit are shown for (a) $(P_{RX}, \Delta \nu) = (1 \text{mW}, 1 \text{MHz})$ (b) $(P_{RX}, \Delta \nu) = (1 \text{nW}, 1 \text{MHz})$ (c) $(P_{\text{RX}}, \Delta \nu) = (1 \text{nW}, 10 \text{MHz})$

Note that the maximum chirping bandwidth is limited by the receiver bandwidth here, as the fastest tone corresponding to the maximum target distance should still be within the receiver bandwidth. Even though it is also possible that the maximum chirping bandwidth is limited by the tuning range of the laser itself, for long-distance LI-DAR, it is usually the receiver that limits the chirping bandwidth.

Using a behavioral model with these parameters, we evaluated the worst-case measurement accuracy at the maximum distance while varying the linewidth and received power. The range of received optical power is picked so that it corresponds with typical long range FMCW LIDAR use cases. For example, the LIDAR receives 0.5nW of power with a 60mm diameter aperture when an 80% reflectivity target at 150m is illuminated with an 8mW laser, assuming the most conservative case in which the reflected power radiates isotropically. The result for the baseline parameters is shown on the left plot in Fig. 4. We clipped the worst-case accuracy to the accuracy target (5cm) so that the failing region is easily recognized. It is clear that the larger linewidth requires higher $P_{\rm RX}$ to meet the accuracy target. Also, for linewidth close to 10MHz, it can be seen that no matter how much power the sensor collects from the target, it is not possible to achieve desired accuracy due to fundamental limit from the phase noise [17].

Table 1. Baseline system specification and device parameters

Ranging accuracy <5cm	
Scanning time 10µs	
Frequency estimation Lorentzian L	LSE
Photodetector responsivity 1A/W	
Receiver bandwidth 500MHz	
Detection range 150m	
Max. Chirping bandwidth 5GHz	
$ \underbrace{\underbrace{}_{0}^{10^{-8}}}_{10^{-10}} \underbrace{\underbrace{}_{0}^{10^{-10}}}_{10^{0} 10^{0.5} 10^{1} 10^{0} 10^{0.5} 10^{0.5} 10^{0} 10^{0.5} 10^{0.5} 10^{0} 10^{0.5} 10^{0.5} 10^{0} 10^{0.5} 10^{0.5} 10^{0} 10^{0.5} 10^{0.5} 10^{0} 10^{0.5} 10^{0.5} 10^{0} 10^{0.5} 10^{0$	Hz $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 $
Linewidth (MHz) Linewidth (MHz) Linewidth (M	AHz)

Fig. 4. FMCW LIDAR performance for three sets of system constraints.

Doubling the receiver bandwidth to 1GHz directly doubles maximum chirping bandwidth, which can now be 10GHz, and also the chirping rate. The simulation result is shown in the middle plot in Fig. 4, and it can be seen that the measurement with desired accuracy is now possible for a wider range of linewidths beyond 10MHz, as long as enough power is received from the target. Increasing the chirping rate means that the same difference in distance is mapped to a bigger difference in beating frequency. In other words, the Lorentzian in the spectral domain is less quantized by discrete frequency bins in the periodogram estimate. Therefore we can expect better accuracy for the same noise level.

Finally, we reduced the target detection range to 75m, and the result is shown in the right plot of Fig. 4. By reducing the target range the worst-case SNR itself is improved as the largest τ is now two times smaller. More importantly, as the fastest beating frequency is also two times smaller, we can increase the laser chirp bandwidth for the same receiver bandwidth. As a result, a similar performance boost as in the previous case is observed.

We would like to emphasize that the proposed LLSE algorithm was used for all results in Fig. 4. The operation of the FMCW LIDAR with parameters in this example was completely impossible with both the R&B and the MUSIC algorithm. This demonstrates that ranging with the FMCW LIDAR beyond its coherence range is possible by properly optimizing the frequency estimation algorithm based on the phase noise characteristic of the laser in the system.

6. CONCLUSION

In this paper, we demonstrated the FMCW LIDAR ranging beyond the coherence range of the laser. This is enabled by a newly developed frequency estimation algorithm with a signal model that properly reflects phase noise in the beating tone from incoherent measurement. Using this algorithm, we demonstrated dramatic improvement in ranging accuracy. This in turn reveals system level trade-offs between the laser linewidth, SNR, and receiver bandwidth. As such significant performance gain is difficult to achieve through device or circuit level optimization alone, this finding clearly highlights the importance of cross-layered approach in integrated system design.

7. REFERENCES

- B. Behroozpour, P. A. M. Sandborn, M. C. Wu, and B. E. Boser, "Lidar System Architectures and Circuits," *IEEE Communications Magazine*, vol. 55, no. 10, pp. 135–142, Oct. 2017.
- [2] A. Teichman and S. Thrun, "Practical object recognition in autonomous driving and beyond," in *Advanced Robotics and its Social Impacts*, Oct. 2011, pp. 35–38.
- [3] J.-F. Lalonde, N. Vandapel, D. F. Huber, and M. Hebert, "Natural terrain classification using three-dimensional ladar data for ground robot mobility," *Journal of Field Robotics*, vol. 23, no. 10, pp. 839–861, Oct. 2006.
- [4] C. V. Poulton, A. Yaacobi, D. B. Cole, M. J. Byrd, M. Raval, D. Vermeulen, and M. R. Watts, "Coherent solid-state LIDAR with silicon photonic optical phased arrays," *Optics Letters*, vol. 42, no. 20, pp. 4091, Oct. 2017.
- [5] A. Vasilyev, The optoelectronic swept-frequency laser and its applications in ranging, three-dimensional imaging, and coherent beam combining of chirped-seed amplifiers, Ph.D. thesis, 2013.
- [6] J. Zheng, "Coherence analysis of optical frequency-modulated continuous-wave interference," *Applied Optics*, vol. 45, no. 16, pp. 3681, 2006.
- [7] M. El-Shennawy, B. Al-Qudsi, N. Joram, and F. Ellinger, "Fundamental limitations of phase noise on FMCW radar precision," in *Electronics, Circuits and Systems (ICECS), 2016 IEEE International Conference on*, 2016, pp. 444–447.
- [8] F. Herzel, D. Kissinger, and H. J. Ng, "Analysis of Ranging Precision in an FMCW Radar Measurement Using a Phase-Locked Loop," *IEEE Transactions on Circuits and Systems I: Regular Papers*, pp. 1–10, 2017.
- [9] A. Wojtkiewicz and R. Rytel-Andrianik, "Optimal detection and estimation in FMCW radar," in 14th International Conference on Microwaves, Radar and Wireless Communications. MIKON - 2002. Conference Proceedings (IEEE Cat.No.02EX562), 2002, vol. 3, pp. 778–781.
- [10] S. Ayhan, P. Pahl, T. Kayser, M. Pauli, and T. Zwick, "Frequency estimation algorithm for an extended FMCW radar system with additional phase evaluation," in 2011 German Microwave Conference, Mar. 2011, pp. 1–4.
- [11] F. Aflatouni, B. Abiri, A. Rekhi, and A. Hajimiri, "Nanophotonic coherent imager," *Optics Express*, vol. 23, no. 4, pp. 5117, Feb. 2015.
- [12] B. Behroozpour, P. A. M. Sandborn, N. Quack, T. J. Seok, Y. Matsui, M. C. Wu, and B. E. Boser, "Electronic-Photonic Integrated Circuit for 3d Microimaging," *IEEE Journal of Solid-State Circuits*, vol. 52, no. 1, pp. 161–172, Jan. 2017.
- [13] Y. Rao, W. Yang, C. Chase, M. C. Y. Huang, D. P. Worland, S. Khaleghi, M. R. Chitgarha, M. Ziyadi, A. E. Willner, and C. J. Chang-Hasnain, "Long-Wavelength VCSEL Using High-Contrast Grating," *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 19, no. 4, pp. 1701311–1701311, July 2013.
- [14] H. Arimoto, T. Kitatani, T. Tsuchiya, K. Shinoda, A. Takei, H. Uchiyama, M. Aoki, and S. Tsuji, "Wavelength-Tunable Short-Cavity DBR Laser Array With Active Distributed Bragg Reflector," *Journal of Lightwave Technology*, vol. 24, no. 11, pp. 4366–4371, Nov. 2006.

- [15] P. A. M. Sandborn, N. Kaneda, Y.-K. Chen, and M. C. Wu, "Dual-Sideband Linear FMCW Lidar with Homodyne Detection for Application in 3d Imaging," in *Conference on Lasers* and Electro-Optics (2016), paper STu4H.8, June 2016.
- [16] P. Stoica and R. L. Moses, Spectral analysis of signals.
- [17] A. Barbieri, D. Bolletta, and G. Colavolpe, "On the Cramer-Rao bound for carrier frequency estimation in the presence of phase noise," in *GLOBECOM '05. IEEE Global Telecommunications Conference*, 2005., Nov. 2005, vol. 3.
- [18] M. H. Hayes, Statistical Digital Signal Processing and Modeling, John Wiley & Sons, Inc., New York, NY, USA, 1st edition, 1996.
- [19] G. M. Stéphan, T. T. Tam, S. Blin, P. Besnard, and M. Têtu, "Laser line shape and spectral density of frequency noise," *Physical Review A*, vol. 71, no. 4, Apr. 2005.
- [20] D. Rife and R. Boorstyn, "Single tone parameter estimation from discrete-time observations," *IEEE Transactions on Information Theory*, vol. 20, no. 5, pp. 591–598, Sept. 1974.
- [21] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276–280, Mar. 1986.
- [22] V. I. Morgenshtern and E. J. Candès, "Super-Resolution of Positive Sources: The Discrete Setup," *SIAM Journal on Imaging Sciences*, vol. 9, no. 1, pp. 412–444, Jan. 2016.