AN EUCLIDEAN ELLIPSE COMPARISON METRIC FOR QUANTITATIVE EVALUATION

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ABSTRACT

Ellipse detection is a popular problem in image processing and is utilized in a broad set of image processing applications. Similar to object detection studies, ellipse detection algorithms need to be evaluated quantitatively via the use of datasets. These datasets include ground truth annotations which enable objective assessment to rate the algorithms. However, in contrast to the variety of ellipse datasets in the literature, there is only a few number of ellipse comparison methods to be utilized in matching of annotated and detected ellipses. Moreover, these methods are more like similarity measures and have certain deficiencies which prohibit accurate evaluation of the algorithms. In this study, we propose an ellipse comparison method defined in Euclidean space which accurately compares two ellipses and provides a single quantifiable scalar. Thus, proximity of two ellipses can precisely be estimated without aforementioned flaws and a robust assessment can be performed.

Index Terms— Ellipse detection, fitting error, quantitative evaluation, comparison metric, ellipse distance.

1. INTRODUCTION

Extraction of ellipses from point clusters or real images is a critical step in many computer vision applications ranging from object detection to pose estimation [1, 2, 3, 4, 5, 6]. Due to the fact that circular shapes are very common in real life and their projection onto a camera image plane is in elliptic form, ellipse detection studies constitute a crowded set in the literature [7, 8, 9, 10, 11, 12, 13].

There are some difficulties that distinguish ellipse detection from other basic geometric feature detection methods. Ellipse is a special conic whose discriminant provides $B^2 - 4AC < 0$ for Eq.1 and has 5 degrees of freedom (DoF) which corresponds to center coordinates, semi-major and semi-minor axes, and rotation angle, respectively.

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \qquad (1)$$

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Because of its high DoF, different geometric shapes such as a rectangle or a line can be represented by an ellipse with a reasonable accuracy that aggravates the problem. Moreover, ellipse has some odd properties which makes even harder to work on it. For instance, perimeter computation of an ellipse does not have an exact formulation and requires summation of infinite number of series, therefore it is approximated [14]. Similarly, there is no straight way to compute the distance between an ellipse and a point and requires gradient based estimations [15]. Therefore calculation of the fitting error when an ellipse fit is applied onto a point set also becomes a non-trivial problem.

Besides all, there is no proper option if comparison of two ellipses with a quantifiable metric is the case. Due to these reasons, evaluation of ellipse detection algorithms usually employ approximations and similarity measures which may prevent to perform accurate assessments. In this study, we propose an ellipse comparison metric which is quantifiable in Euclidean space. Unlike conventional methods, two ellipses can be compared in a robust manner regardless their size and positions with the proposed metric. The rest of the paper is organized as follows. In Section 2 we give a brief overview of commonly utilized methods in assessment of ellipse detection studies. Next, we explain the proposed method in detail in Section 3 and present several examples indicating pros and cons of the methods in the literature in Section 4. Finally, we conclude the paper in Section 5.

2. RELATED WORK

In ellipse detection studies, comparison between ground truth and detected ellipses is an important step in quantitative assessment of two algorithms. However, comparison metric can affect the evaluation of algorithms and may result inaccurate rankings. There are a few number of methods such as shape overlapping and distance calculation methods to compare ellipses and provide a score.

Shape overlapping methods produce a ratio by using intersection of ellipses. Mai et al. [11] calculated the ratio α between the non-overlapping area (S_n) and area of ground truth ellipse (S_g) as shown in Eq. 2. According to this formula, an ellipse is counted to be a true positive (TP) when $\alpha < 0.1$ is provided.

$$\alpha = S_n/S_g \tag{2}$$

Cooke [12] proposes representing ground truth ellipses in form of bounding boxes. Similarity between detected ellipse (E) and bounding box (B) is calculated by a formula which provides a binary result as 0 or 1:

$$Sim(B, E) = \sqrt{\frac{Area(B \cap E)}{max(Area(B), Area(E))}} \tag{3}$$

One of the most common method to decide whether the detected ellipse matches the ground truth is using ratio of intersecting area of two ellipses. Prasad et al. [7] propose a comparison metric (D) which is known as overlap ratio as seen in Eq. 4. They define $f(x, y) = 0$ as a quadratic equation which represents an ellipse and $P(x', y')$ as a pixel in the same image. If $f(x', y')$ is less than or equal to 0 then $P(x', y')$ is said to be inside the ellipse or on the ellipse contour. As a result, a matrix (I) is constructed with logical $1s$ and $0s$. They use 1 to represent a pixel inside or on the ellipse contour and 0 to represent a pixel outside the ellipse.

$$D = 1 - \frac{count(XOR(I_1, I_2))}{count(OR(I_1, I_2))} \tag{4}$$

Even though this method provides useful information, it has some serious flaws in practice that we will provide more detail in Sec. 4.

In distance calculation methods, matching of two ellipses is calculated in metric space. Swirski et al. [16] use Hausdorff distance [17] as the comparison metric in their pupil detection study. They choose uniformly located 100 points on the contour of each ellipse. The closest distance between each point and the contour of the other ellipse is calculated. Maximum distance is determined for both ellipses by a brute force search. Finally, the larger distance is selected as the Haussdorff distance of two ellipses as indicated in the equation:

$$d_H(A, B) = max\{\sup_{a \in A} \inf_{b \in B} \mathrm{d}(a, b), \sup_{b \in B} \inf_{a \in A} \mathrm{d}(a, b)\} \tag{5}$$

Although Hausdorff distance is capable to provide a quantifiable metric, it is obviously based on only the distance of two points along two ellipses. Cuevas et al. [13] compare two ellipses with error score which is formulated by using parameters of two ellipses as follows:

$$\begin{aligned} E_s =& P_1(|{x_0}^{GT} - {x_0}^{D}| + |{y_0}^{GT} - {y_0}^{D}|) + \\ & P_2(|{r_{max}}^{GT} - {r_{max}}^{D}| + \\ & |{r_{min}}^{GT} - {r_{min}}^{D}|) \; + P_3(\theta_{GT} - \theta_D) \end{aligned} \tag{6}$$

Superscripts D and GT are used to represent parameters of detected and ground truth ellipses, respectively. As seen in Eq. 6, differences of parameters are multiplied by weights. These weights are determined as $P_1 = 0.05$, $P_2 = 0.1$ and $P_3 = 0.2$. Weights are set to different values since each parameter has not the same impact on error score. Prasad and Leung [18] normalize ellipse parameters and suggest a boolean metric as a result (See Eq. 7, 8, 9 and 10).

$$D_x = \frac{|x_1 - x_2|}{X}, \; D_y = \frac{|y_1 - y_2|}{Y} \tag{7}$$

$$D_a = \frac{|a_1 - a_2|}{max(a_1, a_2)}, \; D_b = \frac{|b_1 - b_2|}{min(b_1, b_2)} \tag{8}$$

$$D_\theta = \begin{cases} 0, & b_1/a_1 \geq 0.9 \text{ AND } b_2/a_2 \geq 0.9 \\ 1, & b_1/a_1 \geq 0.9 \text{ AND } b_2/a_2 < 0.9 \\ 1, & b_1/a_1 < 0.9 \text{ AND } b_2/a_2 \geq 0.9 \\ \frac{\angle(\theta_1, \theta_2)}{\pi}, & b_1/a_1 < 0.9 \text{ AND } b_2/a_2 < 0.9 \end{cases} \tag{9}$$

X and Y define number of pixels in input image as width and height, respectively. Smaller angle between major axes of ellipses is denoted by $\angle(\theta_1, \theta_2)$. Similarity between two ellipses is formulated in Eq. 10:

$$\begin{aligned} D = AND\{&(D_x < \widetilde{D}_x), (D_y < \widetilde{D}_y), (D_a < \widetilde{D}_a), \\ &(D_b < \widetilde{D}_b), (D_\theta < \widetilde{D}_\theta)\} \end{aligned} \tag{10}$$

Thresholds $\widetilde{D}_x$, $\widetilde{D}_y$, $\widetilde{D}_a$, $\widetilde{D}_b$ and $\widetilde{D}_\theta$ are set to 0.1 for final decision.

3. PROPOSED METHOD

We propose an Euclidean ellipse distance metric which enables accurate quantification of ellipse matches in experimental evaluation. Our method starts by uniformly computing points along the ellipses' contours. Then calculates the distance between each point to the contour of other ellipse. Finally we compute the average Euclidean distance.

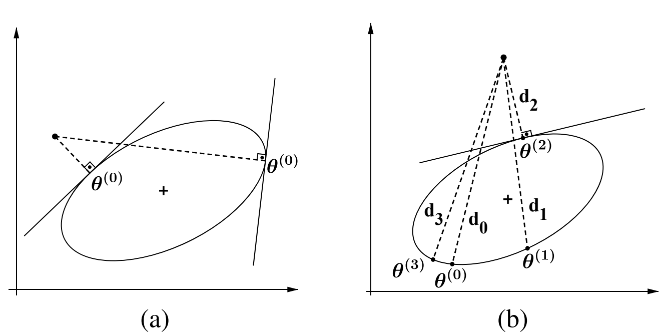
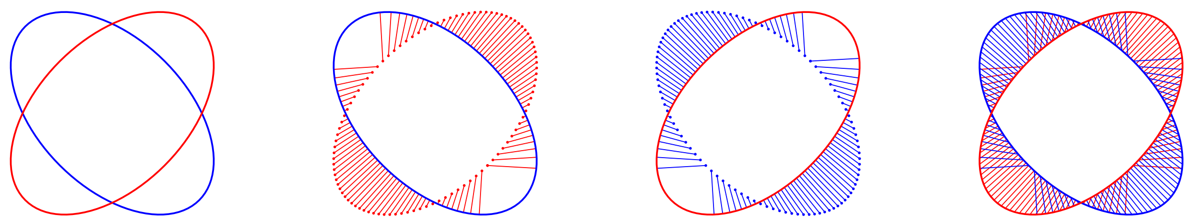


Fig. 1: (a) Successful and unsuccessful convergence of $\theta^{(0)}$, (b) Determination of true convergence.



(a) (b) (c) (d)

Fig. 2: (a) Two ellipses are drawn by computing ellipse points, (b) Distances from red ellipse points to blue ellipse contour, (c) Distances from blue ellipse points to red ellipse contour, (d) Final representation of Euclidean ellipse distance.

The fundamental task for estimating the distance between two ellipses (or ellipse fitting error) is computing the Euclidean distance between a point and an ellipse. This task has no straightforward method and needs to be estimated by a gradient based iterative method [15, 19]. Therefore, several approximations for ellipse are employed in majority of the studies in the literature [20, 21, 22]. Although these approximations usually works faster, they cannot provide a quantifiable magnitude.

To calculate the distance between a point and an ellipse we develop a method based on [15] to accurately compute ellipse fitting error [9, 10, 23]. According to [15], ellipse equation in conic form can be formulated in parametric form and expressed as univariate in angular representation:

$$\left(\frac{x}{\alpha}\right)^2 + \left(\frac{y}{\beta}\right)^2 = r^2 \tag{11}$$

In Eq. 12, θ denotes the angle of closest point on ellipse $\vec{q}$ to point $\vec{p} = (x, y)$.

$$\vec{q} = r(\alpha\ cos(\theta),\ \beta\ sin(\theta))^T \tag{12}$$

Tangent of $\vec{q}$ and line $\vec{p} - \vec{q}(\theta)$ are orthogonal to each other, so θ is calculated by solving Eq. 13 iteratively.

$$(\vec{p} - \vec{q}(\theta)).\vec{q}\,'(\theta) = 0 \tag{13}$$

Initial value of θ is recommended as $tan^{-1}(\alpha y/\beta x)$ in [15]. We implemented and tested this method using Newton-Raphson estimation. In our experiments, it usually converges in three iterations as seen in Fig. 1(a). However, we noticed that it sometimes fails to converge to the true correct point, especially when the point is in the proximity of major axis (see Fig. 1(b)).

One of the reasons for this failure might be the existence of two reciprocal points verifying Eq. 13. To overcome this problem we modify the method [15] and start to iterate from four different quadrants separately with the following initial values of θ:

$$\theta^{i=\{0,3\}} = tan^{-1}(\alpha y/\beta x) + i(\pi/2) \tag{14}$$

At the end of the iterations, the method ends up with four different angular values each of which addresses a point locations on the ellipse contour. Finally, we select the closest point to $\vec{p}$ among all results. In this way, Euclidean distance from point to ellipse contour is precisely computed.

3.1. Euclidean Ellipse Distance Metric

E_1 and E_2 are ellipses with parameters $(xc_1, yc_1, a_1, b_1, \alpha_1)$ and $(xc_2, yc_2, a_2, b_2, \alpha_2)$, respectively. First, we need to compute contour points of ellipses. Number of points sampled from each ellipse contour is denoted by n. Points are sampled step by step iteratively as seen in Fig. 2(a). Each step is determined as $360/n$ and iteration starts from 0 to 360:

$$\begin{aligned} x_i &= a.sin(\theta_i) \\ y_i &= b.cos(\theta_i) \end{aligned} \tag{15}$$

This operation is performed with the assumption of the ellipse is at the origin of Cartesian coordinate system with no

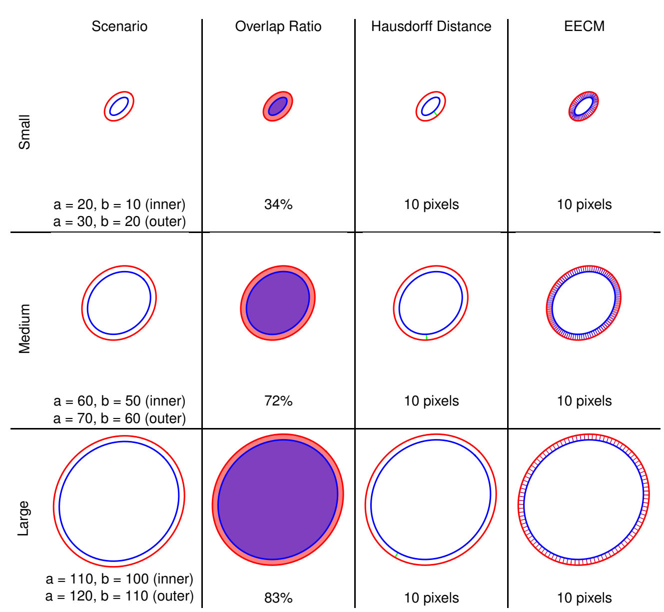


Fig. 3: Comparison of same ellipse pairs in different sizes.

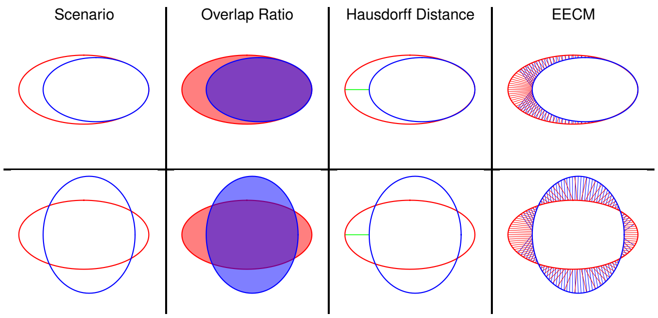


Fig. 4: Metric results when distance between contours of ellipses is not constant.

rotation. Otherwise, the ellipse is centered and rotated by converting the conic equation (Eq. 1) to the parametric one (Eq. 11) [24].

Distances from contour points of each ellipse to another ellipse are calculated as discussed in Sec. 3 and is depicted in Fig. 2(b) and 2(c). Finally, average distance between two ellipses is calculated as follows:

$$\Phi = \frac{1}{2n} \sum_{i=1}^{n} (|p_i^{E_1} - E_2| + |p_i^{E_2} - E_1|) \tag{16}$$

Fig. 2(d) gives an outline of our Euclidean comparison metric (EECM) for two ellipses. We provide the proposed metric downloadable for interested researches to be utilized[1]. Codes for both ellipse fit error and ellipse distance computations are available.

4. EXPERIMENTAL RESULTS

In this section we analyse ellipse comparison metrics by providing counter examples for several cases where they fail. In the first example seen in Fig. 3, we show that the overlapping ratio cannot provide robust results for the same ellipse pairs in different sizes. Although the distances between the contours of inner and outer ellipses are identical, overlapping ratio gives different results as seen in Fig. 3. For the same ellipse pairs, Hausdorff distance and the proposed EECM metrics can provide the correct distance whereas overlapping ratio have bias range up to 250% due to the scale difference of ellipses.

Another example is about the scenario where the distance between contours of ellipses is not constant. In the two examples presented in Fig 4 there is a significant difference in the ellipse pairs in the first and the second row. Although overlapping error is able to represent the variation between ellipse pairs (75% and 56%, respectively), Hausdorff gives the same amount as the distance between two ellipses. For EECM, calculated values are 16 pixels and 34 pixels for the ellipse pairs, respectively.

[1]http://c-viz.anadolu.edu.tr/eecm

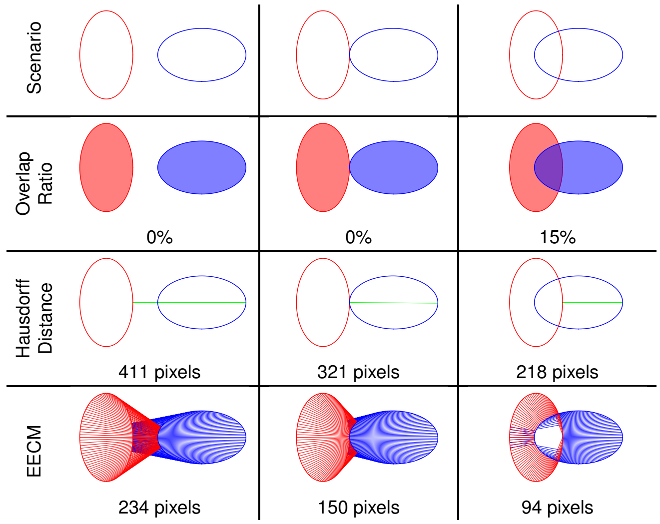


Fig. 5: Scenarios for ellipse pairs which have different distance between them and intersection is the case or not.

In the last example we examine ellipse pairs which have and have not intersection. For this scenario we present ellipse pairs which have different distance between them as seen in Fig. 5. Results of overlap ratio for this example are 0%, 0% and 24%, respectively, while the closer ellipses should have higher similarity. It is obvious that the overlap ratio remains irrelevant if two ellipses do not intersect. On the other hand, results for Hausdorff and EECM metrics are more reasonable as they increase or decrease for the closer ellipse pairs.

5. CONCLUSION

Ellipse detection is one of the mostly studied topics in geometric feature extraction literature. To provide the methods evolve in time, quantitative evaluation has substantial importance. For this reason, there are various ellipse detection datasets with ground truth annotations. Every up and coming algorithms can be ranked by the utilization of these datasets in a quantitative evaluation manner. During the evaluation process, several measures are employed to estimate the match score of two ellipses. In this study we examine these measures and propose a novel metric to estimate that match score in a more accurate manner. EECM aims to estimate how close two ellipses are in Euclidean space instead of using approximations for calculation of distance from a point to ellipse contour. A more robust and quantitative way to evaluate ellipse detection algorithms is emerged as a result. In experimental results, we provide examples which indicate pros and cons of the existing methods and the proposed one.

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