Adaptive Sparse Array Reconfiguration based on Machine Learning Algorithms

Xiangrong Wang*, Pengcheng Wang* and Xianghua Wang†
*School of Electronic and Information Engineering, Beihang University, Beijing, China; xrwang@buaa.edu.cn
†College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao, China.

***Abstract*—The sparse array design for adaptive beamforming has been recently formulated into combinatorial antenna selection problems, which belong to notorious NP-hard problems. As the commonly deployed convex relaxation algorithms are susceptible to local optima, several trials with different initial points are conducted for the global optima. Moreover, the high computational load of optimization techniques prohibits the real-time adaptive array reconfiguration. In this work, we propose to utilize machine learning algorithms, specifically support vector machine (SVM) and artificial neural network (ANN), for solving combinatorial antenna selection problems. Numerical examples are presented to validate the effectiveness and efficiency of machine learning algorithms for sparse array design. Moreover, the SVM based antenna selection is robust against DOA estimate uncertainties.**

***Keywords*—Capon, Sparse array, Machine learning**

I. Introduction

Adaptive beamforming is capable of spatial filtering and finds exensive applications in radar, sonar, wireless communications, radio astronomy, and satellite navigation, to list a few [1]–[5]. Although the nominal array configuration is uniform, sparse arrays have recently emerged to play a fundamental role in various sensing systems involving multi-antenna transmitters and receivers [6]–[8]. It has been shown that sparse arrays with a given number of antennas placed at an optimum subset of grid locations, connecting with the Radio-Frequency (RF) front-end receivers, can preserve a large aperture while reducing system complexity [9]–[13]. The main task of sparse array design is, in essence, to decide on where to place a given number of sensors to deliver optimum performance. Different optimization metrics lead to differed sparse array configurations [14]–[23]. From signal enhancement and interference suppression perspective, an optimum array configuration is the one that yields maximum signal-to-interference-plus-noise ratio (MaxSINR).

Whether the problem is cast as optimum placement of a given number of antennas or equivalently selecting a subset of antennas to connect with RF front-end receivers, the underlying problem falls into the framework of antenna selection. The antenna selection perspective of the problem relies on low-complexity RF switches [13], [24], with the fundamental goal of reducing hardware cost associated with expensive RF chains. The antenna selection was formulated into combinatorial optimization problems in terms of the MaxSINR and solved by convex relaxation algorithms [25]–[28]. The high computational complexity of optimization techniques may not be suitable for adaptive array reconfiguration applications. Moreover, relaxation techniques of solving non-convex optimization problems are susceptiable to local optima. Several runs with different initial points are conducted for search of the global optima, which siginificantly increases the requried computational time. In this paper, we propose to utilize machine learning (ML) algorithms, specifically support vector machine (SVM) [29], [30] and Artificial Neural Network (ANN) [31], [32], to achieve optimum sparse array reconfiguration. The flowchart of ML based antenna selection strategy is illustrated in Fig. 1. The SVM and ANN are completely trained using a large set of training data from all possible scenarios. After training, a Capon beamformer is first utilized to sense the operating environment and extract features, such as the number and directions of arrival (DOAs) of interferences. The SVM/ANN then decides the status of each antenna (either selected or discarded) according to the features provided by Capon beamformer. It is shown that the ML based selection algorithms are capable of quickly obtaining the global optimum solution, making them most suitable for rapidly changing environments. Moreover, simulation results also demonstrate the robustness of the SVM algorithm against DOA estimate uncertainties.

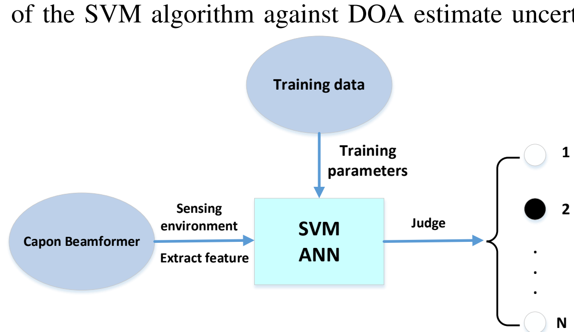

Fig. 1. Flowchart of machine learning based antenna selection.

The rest of this paper is organized as follows: We formulate the mathematical model in section II. The optimization and ML based sparse array design methods are examined in sections III and IV, respectively. Simulation results in section V validate the utility of ML algorithms for sparse array design. Finally, conclusions are provided in section VI.

II. Mathematical Model

Consider a linear array of N isotropic antennas with positions specified by multiple integer of unit inter-element spacing $p_n d$, $p_n \in \mathbb{N}, n = 1, \ldots, N$. The symbol $\mathbb{N}$ denotes the set of integer numbers. Suppose that a single source is impinging on

The work by X Wang is supported by National Natural Science Foundation of China under Grant No. 61701016.

the array from directions ϕ_s and q strong interfering signals are arriving from directions $\{\phi_1, \ldots, \phi_q\}$. The corresponding steering vectors are,

$$\begin{aligned} \mathbf{s} &= [e^{jk_0 p_1 d\cos\phi_s}, \ldots, e^{jk_0 p_N d\cos\phi_s}]^T, \\ \mathbf{v}_l &= [e^{jk_0 p_1 d\cos\phi_l}, \ldots, e^{jk_0 p_N d\cos\phi_l}]^T, l = 1, \ldots, q, \end{aligned} \tag{1}$$

where the wavenumber is defined as $k_0 = 2\pi/\lambda$ with λ being the wavelength and ^{T} denotes the transpose operation. The received signal at time instant t is given by,

$$\mathbf{x}(t) = s(t)\mathbf{s} + \mathbf{V}\mathbf{v}(t) + \mathbf{n}(t), \tag{2}$$

where $\mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_q]$ is the interference array manifold matrix with the full column rank. In the above equation, $s(t) \in \mathbb{C}$ and $\mathbf{v}(t) \in \mathbb{C}^q$ are, respectively, the statistically independent source and interfering signals, $\mathbf{n}(t) \in \mathbb{C}^N$ denotes the received Gaussian noise vector. The symbol $\mathbb{C}$ denotes the set of complex numbers. The Capon beamformer, which minimizes the output variance while keeping the desired signal distortionless [33], is $\mathbf{w}_c = \mathbf{R}_n^{-1}\mathbf{s}$, where $\mathbf{R}_n = \mathbf{V}\mathbf{C}_v\mathbf{V}^H + \sigma_n^2\mathbf{I}$ is the interference plus noise covariance matrix with $\mathbf{C}_v = E\{\mathbf{v}(t)\mathbf{v}^H(t)\}$ denoting the interference correlation matrix, σ_n^2 the noise power level and ^{H} stands for the Hermitian operation. The output SINR of the Capon beamformer can be expressed as [25], [26],

$$\begin{aligned} \text{SINR} &= \sigma_s^2\mathbf{s}^H\mathbf{R}_n^{-1}\mathbf{s}, \\ &\approx \text{SNR}\{\mathbf{s}^H[\mathbf{I} - \mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H]\mathbf{s}\}, \\ &= N\text{SNR}(1 - |\alpha|^2), \end{aligned} \tag{3}$$

where σ_s^2 denotes the power of the source signal, $\text{SNR} = \sigma_s^2/\sigma_n^2$ is the input signal-to-noise ratio. In the second line of Eq. (3), we make the assumption that the interferences are much stronger than white noise, which occurs frequently in satellite navigation and radio astronomy. The spatial correlation coefficient (SCC) α is defined as $|\alpha|^2 = (1/N)\mathbf{s}^H\mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H\mathbf{s}$. We can observe from Eq. (3) that the output SINR depends only on the squared SCC value for a given number of antennas. The sparse array design for maximizing the output SINR is tantamount to that for minimizing the SCC value.

III. Optimization based Sparse Array Design

The sparse array design can be cast as selecting K out of N candidate antennas placed on uniform grid points with half wavelength spacing. The positions of the K antennas are freely determined by the optimization technique, which in this case is the Capon beamformer. Denote an antenna selection vector $\mathbf{z} = [z_i, i = 1, \ldots, N] \in \{0, 1\}^N$ with "zero" entry for a discarded antenna and "one" entry for a selected one. As mentioned in section II, the optimum sparse array can be configured through minimizing the SCC value, that is, [26]

$$\begin{aligned} \min_{\mathbf{z}} \quad & \log|\mathbf{V}^H\text{diag}(\mathbf{z})\mathbf{V}| - \log|\mathbf{V}_s^H\text{diag}(\mathbf{z})\mathbf{V}_s|, \\ \text{s.t.} \quad & \mathbf{1}^T\mathbf{z} = K, \ \mathbf{z} \in \{0, 1\}^N, \end{aligned} \tag{4}$$

where s.t. stands for "subject to", diag($\mathbf{z}$) is a diagonal matrix with the vector $\mathbf{z}$ populating along the diagonal, $\mathbf{V}_s = [\mathbf{V}, \mathbf{s}]$, and $|\cdot|$ denotes the determinant operation. The difficulty of solving the problem in Eq. (4) is twofold: the non-convexity of the objective function and boolean constraints of the selection variable $\mathbf{z}$. In order to solve the problem, the objective function is approximated by its affine upper bound iteratively and the boolean constraints are relaxed to a box constraint $0 \leq \mathbf{z} \leq 1$ [26]. The antenna selection in the (k+1)th iteration can be formulated based on the solution from the kth iteration as,

$$\begin{aligned} \min_{\mathbf{z}} \quad & \Delta\mathbf{g}^T(\mathbf{z}^{(k)})(\mathbf{z} - \mathbf{z}^{(k)}) - \log|\mathbf{V}_s^H\text{diag}(\mathbf{z})\mathbf{V}_s|, \\ \text{s.t.} \quad & \mathbf{1}^T\mathbf{z} = K, \\ & 0 \leq \mathbf{z} \leq 1, \end{aligned} \tag{5}$$

where $\Delta\mathbf{g}(\mathbf{z}^{(k)})$ is the gradient of the concave function $\log|\mathbf{V}^H\text{diag}(\mathbf{z})\mathbf{V}|$ evaluated at the point $\mathbf{z}^{(k)}$. That is,

$$\Delta\mathbf{g} = [\mathbf{v}_{r,i}^H(\mathbf{V}^H\text{diag}(\mathbf{z}^{(k)})\mathbf{V})^{-1}\mathbf{v}_{r,i}, i = 1, \ldots, N]^T,$$

with $\mathbf{v}_{r,i}$ denoting the ith column vector of the matrix $\mathbf{V}^H$. Note that the iterative relaxation in Eq. (5) is a local heuristic and its performance depends on the initial point $\mathbf{z}^{(0)}$. It is, therefore, typical to initialize the algorithm with several feasible points $\mathbf{z}^{(0)}$ and find the one with the minimum objective value over the different runs. The interior point method can be utilized to solve the optimization problem with a computational complexity of order $O(n^{3.5}L^2)$, where n and L are the number of variables and bitlength, respectively.

IV. Machine Learning based Sparse Array Design

Although convex relaxation and optimization are effective in most cases, their susceptibility to local optima and high computational complexity prohibit the practical implementation. We investigate the utilization of ML algorithms for sparse array design in this section. Below, we first summarize two principal ML techniques and then describe the formulation of antenna selection problems into the framework of ML.

A. Support Vector Machine

The support vector machine (SVM) is known as the maximum margin classifier, which calculates the optimum hyperplane $\mathbf{u}^T\mathbf{x} + b$ with the maximal margin of separation between the two classes. Given a set of training data $\mathbf{x}_i, i = 1, \ldots, L$ and the corresponding labels $y_i \in \{-1, 1\}, i = 1, \ldots, L$. The hyperplane can be calculated as follows,

$$\begin{aligned} \min_{\mathbf{u}, \mathbf{b}} \quad & \frac{1}{2}\|\mathbf{u}\|^2 + C\sum_{l=1}^{L}\epsilon_l, \\ \text{s.t.} \quad & y_i(\mathbf{u}^T\mathbf{x}_i + b) \geq 1 - \epsilon_l, l = 1, \ldots, L, \\ & \epsilon_l \geq 0, l = 1, \ldots, L, \end{aligned} \tag{6}$$

where C is a trade-off parameter. The Lagrangian dual of the problem in Eq. (6) can be expressed as the following quadratic programming form,

$$\begin{aligned} \max_{\alpha} \quad & \frac{1}{2}\alpha^T\mathbf{H}\alpha - \mathbf{1}^T\alpha, \\ \text{s.t.} \quad & \alpha^T\mathbf{y} = 0, \ 0 \leq \alpha \leq C, \end{aligned} \tag{7}$$

where $\mathbf{y} = [y_i, i = 1, \ldots, L]^T$ and $\mathbf{H} = \mathbf{G}^T\mathbf{G}$ with $\mathbf{G} = [y_1\mathbf{x}_1, \ldots, y_L\mathbf{x}_L]$. In order to increase the Vapnik-Chervonenkis (VC) dimension of the SVM classifier, which is defined as the maximum number of points that can be labelled in all possible ways, kernel mapping, $\kappa : \mathcal{X} \to \mathcal{F}$ from data space $\mathcal{X}$ to a dot product feature space $\mathcal{F}$, can be employed. The matrix $\mathbf{H}$ in Eq. (7) with kernel can be rewritten as,

$$\mathbf{H}_{ij} = y_i y_j \kappa(\mathbf{x}_i, \mathbf{x}_j). \tag{8}$$

B. Artificial Neural Network

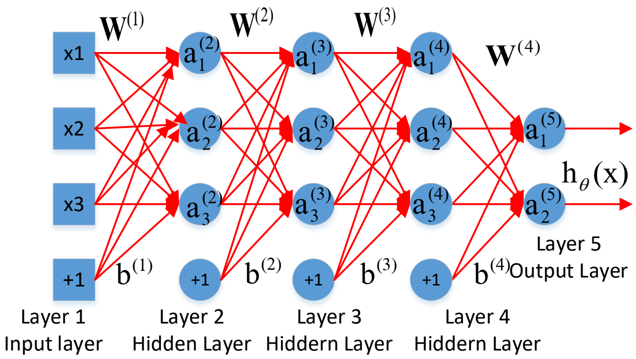

Fig. 2. The structure of a five-layer artificial neural network.

The structure of the employed artificial neural network (ANN) is shown in Fig. 2, which comprises five layers, an input layer, three hidden layers and an output layer. Let n_l indicate the number of layers and s_l denote the number of nodes in layer l. We write $a_k^{(l)}$ to denote the activation of neuron k in layer l and for l = 1, $a_i^{(1)} = x_i$, i.e. the ith feature of the input data vector $\mathbf{x}$. The circles labelled "+1" are called bias units, and correspond to the intercept term. The neural network has parameters $\theta = (\mathbf{W}^{(1)}, b^{(1)}, \mathbf{W}^{(2)}, b^{(2)}, \mathbf{W}^{(3)}, b^{(3)}, \mathbf{W}^{(4)}, b^{(4)})$, where $\mathbf{W}_{ij}^{(l)}$ denotes the weight associated with the connection between neuron i in layer l and neuron j in layer $l+1$. Thus, $\mathbf{W}^{(1)} \in \mathbb{R}^{s_l \times s_{l+1}}$. Given a fixed setting of the parameters θ, out neural network defines a hypothesis $h_\theta(\mathbf{x})$ that outputs the prediction $\mathbf{a}^{(5)}$. Specifically, the forward propagation that this neural network represents is given by,

$$\mathbf{a}^{(l+1)} = g(\mathbf{W}^{(l)}\mathbf{a}^{(l)} + b^{(l)}), l = 1, \ldots, n_l - 1 \qquad (9)$$

where the activation $g(x)$ is a sigmoid function. The neural network parameters can be trained by backward propagation using batch gradient descent algorithm.

C. Sparse Array Design using ML

The sparse array design for Capon beamforming pursues the detection of the source from a specified DOA given known directions of interferers. Antenna selection can be modelled as classification problems, where each antenna is labelled by two classes, "selected" and "discarded". The training data $\mathbf{x}$ and $\mathbf{y}$ under the two strategies can be generated either by enumeration or optimization described in section III for every possible scenario, characterized by the DOA of the source signal, the number q and DOAs of interferences. After a complete training, Capon beamformer can first sense the environment and extract feature data $\mathbf{x}$, based on which the well-trained machine then determines the status of all antennas in practical applications. The selected antennas compose the optimum sparse array corresponding to the operating environment. Note that the definition of feature space $\mathcal{X}$ is different for the SVM and the ANN, the detailed description is as follows.

The feature space $\mathcal{X}$ of the SVM is defined as $\mathbf{x}_l = [\phi_s, \phi_1, \ldots, \phi_q]$, then the dimension of the feature space is $q+1$ with the feature value within the range of $[0, 180]$. The classification variable $y_i \in \{-1, 1\}, i = 1, \ldots, N$ with value -1 denoting discarded antenna and 1 selected. We train each antenna separately and obtain the SVM parameters α^n and $b^n, n = 1, \ldots, N$. Denote $\mathbf{x}$ as the sensed electromagnetic environment, we then predict the status of each antenna according to the following formula,

$$\sum_{l=1}^{L} \alpha_l^n y_l \kappa(\mathbf{x}_l, \mathbf{x}) + b^n \begin{cases} > 0 \text{ then } y = 1 \\ < 0 \text{ then } y = -1. \end{cases} \qquad (10)$$

We employ Gaussian kernel with bandwidth τ in this paper, which is defined as

$$\kappa(\mathbf{x}_1, \mathbf{x}_2) = \exp\left(-\frac{\|\mathbf{x}_1 - \mathbf{x}_2\|_2^2}{\tau}\right). \qquad (11)$$

The Gaussian kernel has an theoretical infinite VC dimension.

For the ANN algorithm, we define the feature space $\mathcal{X}$ as $\mathbf{x}_l = \{0, 1\}^{180 \times 1}$ with value 1 indicating a signal arriving from the corresponding direction. Thus the dimension of the feature space for the ANN is 180 and the feature vector $\mathbf{x}$ is sparse with "one" entries only corresponding to the source and interference arrival angles. The classification variable is defined as $y_i \in \{0, 1\}$ with value 0 denoting discarded antenna and 1 selected. Different the separate training of SVM, we train all the N antennas at the same time for the ANN and obtain the parameters θ. For a new feature input $\mathbf{x}$, the following classification is made,

$$y = g\left\{\mathbf{W}^{(4)}\left[\mathbf{W}^{(3)}[\mathbf{W}^{(2)}(\mathbf{W}^{(1)}\mathbf{x} + b^{(1)}) + b^{(2)}] + b^{(3)}\right] + b^{(4)}\right\}.$$

Randomized initialization of the parameters $\mathbf{W}$ and b is utilized and serves the purpose of symmetry breaking.

V. Simulations

In this section, we consider $K = 8$ available antennas and $N = 16$ uniformly spaced positions with an inter-element spacing of $d = \lambda/2$..

A. Validation of Optimum Sparse Array

Assume that there are two interfering signals arriving from $\phi_1 = 58°, \phi_2 = 120°$ relative to the endfire direction with INR being 20dB. A single source is impinging on the array from $\phi_s = 64°$ with SNR being 0dB. Note that there are totally $C_{16}^8 = 12870$ different sparse array configurations. We enumerate all the different sparse arrays for Capon beamforming and calculate the output SINR, which is plotted in Fig. 3 in an ascending order. The structure of the optimum 8-antenna sparse array is presented in Fig. 4. The difference in the SINR offerings of different sparse arrays is clearly seen from Fig. 3. Thus, different array configurations play a significant impact on the Capon beamformer's performance.

B. Validation of ML Algorithms

Consider a signal arriving from the angular range $\Phi_s \in [60° \sim 120°]$ and two strong interfering signals arriving from the angular ranges $\Phi_1 \in [5° \sim 55°]$ and $\Phi_2 \in [125° \sim 175°]$, respectively. The 8-antenna optimum sparse array is constructed from a 16-antenna uniform linear array. We calculate the optimum sparse array for each possible scenario using either emuneration or optimization based techniques and prepare the feature data according to the description in section IV-C, with which we then train both the SVM and

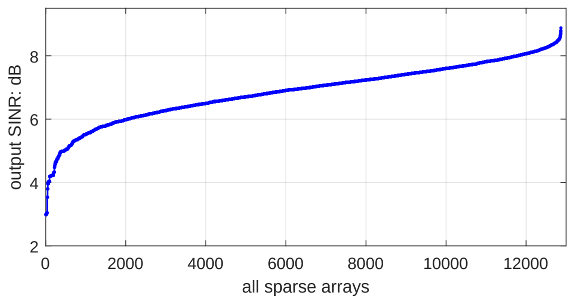

Fig. 3. Output SINR of all sparse array Capon beamformers.

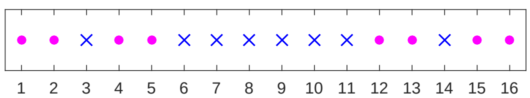

Fig. 4. The optimum 8-antenna sparse array (a). Filled circles denote selected locations and crosses denote discarded.

ANN algorithm. Note that there are $N = 16$ independent SVM classifiers with parameters $[\alpha^n, b^n], n = 1, \ldots, N$, while there is only one ANN classifier with one parameter $\theta = [\mathbf{W}^{(1)}, b^{(1)}, \ldots, \mathbf{W}^{(4)}, b^{(4)}]$. The number of neurons for the first, second and third hidden layers are $s_2 = 25$, $s_3 = 50$ and $s_4 = 25$, respectively. The input and output layers have $s_1 = 180$ and $s_5 = 16$ neurons. The feature space dimension of the SVM classifier is 3, i.e., $\mathbf{x}_i = [\phi_s, \phi_1, \phi_2]$, whereas the feature space dimension of the ANN classifier is 180, i.e., $\mathbf{x}_i \in \{0, 1\}^{180}$ with one entries corresponding to the DOAs of source and interferences. The training data of the first antenna is illustrated in Fig. 5. Clearly, it is impossible to distinguish the two sets of points in the original 3-dimensional space. Both the SVM and ANN can implement a series of transformation to project the data onto a higher dimensional feature space, where the two sets of points can be easily separated.

We compare the classification accuracy and computational time among the optimization, the ML based method and the table search in Table I. It is clear that both the SVM and ANN algorithms exhibit much higher accuracy than the optimization method. Although the optimization does not guarantee the global optimum solution, it can return a satisfactory sub-optimal solution as shown in [25], [26]. No doubt that the accuracy of table search method is always 100 percent, however, its searching time will increase dramatically with the feature space dimension. After a complete training, the SVM and ANN only requires simple matrix multiplication for classification, thus exhibiting much faster computational speed than the other two methods. The SVM is slower than the ANN as the kernel computation is computationally involved, whereas the training time that the ANN takes is much longer than the SVM. The Accuracy of the ANN can be further improved by adding more complicated hidden layer, such as convolution neurons.

Next, we consider a pragmatic scenario where the estimated DOAs of source and interferences do not exactly coincide with the training set. This can happen when there are possible biases and perturbations in the DOAs due to platform motion. Assume that the estimated DOAs of the source and interferences are deviated from the training set of integer angles. We set them as $\phi_s = 65.5°, \phi_1 = 45.4°, \phi_2 = 125.6°$. The SVM is still capable of returning the true optimum sparse array, which is plotted in Fig. 6 (b). However, the table-search based method approximate the off-grid angles to $\phi_s = 66°, \phi_1 = 45°, \phi_2 = 126°$ and return the corresponding sparse array, as plotted in Fig. 6 (c). The structure difference between the two sparse arrays (b) and (c) clearly demonstrate the robustness of the SVM based antenna selection method against DOA estimate uncertainties.

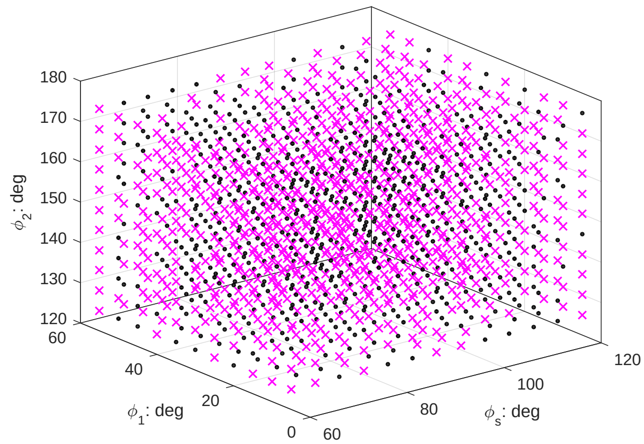

Fig. 5. The illustration of training data for the first antenna using SVM. The cross denotes the label -1, while the dot denotes the label 1.

TABLE I. THE CLASSIFICATION ACCURACY AND COMPUTATIONAL TIME OF FOUR METHODS UNDER THE UAC AND RAC.

Method	Accuracy %	Computational Time (sec)
Opt	64.3	7.17
SVM	100	0.09
ANN	97.08	2.21e-4
Table	100	5.6

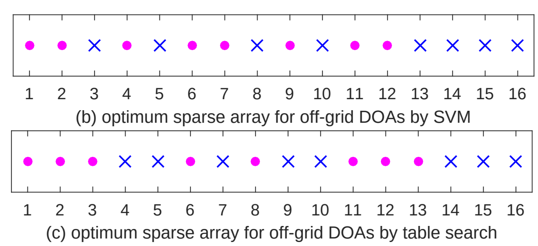

Fig. 6. The 8-antenna sparse arrays returned by the SVM and table search.

VI. Conclusions

We proposed to utilize the two principal machine learning algorithms, namely support vector machine and artificial neural network, to solve the combinatorial antenna selection problems for optimum sparse array design. Numerical examples demonstrated the utility of machine learning algorithms for optimum sparse array design. Their high accuracy, fast computational speed and robustness against DOA uncertainties manifest the ML algorithms a desirable candidate solution to adaptive sparse array reconfiguration in a rapidly changing environment. How to reduce the training time of ML algorithms for large-scaled arrays in complicated operating scenarios is important and will be the future research topic.

References

[1] L. E. Brennan and L. S. Reed, "Theory of adaptive radar," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-9, pp. 237–252, March 1973.

[2] R. L. Fante and J. J. Vaccaro, "Wideband cancellation of interference in a GPS receive array," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 36, pp. 549–564, Apr 2000.

[3] A. J. van der Veen, A. Leshem, and A. J. Boonstra, "Signal processing for radio astronomical arrays," in *Sensor Array and Multichannel Signal Processing Workshop Proceedings, 2004*, pp. 1–10, July 2004.

[4] R. Compton, "An adaptive array in a spread-spectrum communication system," *Proceedings of the IEEE*, vol. 66, no. 3, pp. 289–298, 1978.

[5] D. Johnson and S. DeGraaf, "Improving the resolution of bearing in passive sonar arrays by eigenvalue analysis," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 30, pp. 638–647, Aug 1982.

[6] Y. Selén, H. Tullberg, and J. Kronander, "Sensor selection for cooperative spectrum sensing," in *New Frontiers in Dynamic Spectrum Access Networks, 2008. DySPAN 2008. 3rd IEEE Symposium on*, pp. 1–11, IEEE, 2008.

[7] X. Shen and P. Varshney, "Sensor selection based on generalized information gain for target tracking in large sensor networks," *Signal Processing, IEEE Transactions on*, vol. 62, pp. 363–375, Jan 2014.

[8] M. Hawes and W. Liu, "Sparse array design for wideband beamforming with reduced complexity in tapped delay-lines," *Audio, Speech, and Language Processing, IEEE/ACM Transactions on*, vol. 22, pp. 1236–1247, Aug 2014.

[9] M. Wax and Y. Anu, "Performance analysis of the minimum variance beamformer," *IEEE Transactions on Signal Processing*, vol. 44, no. 4, pp. 928–937, 1996.

[10] A. Gorokhov, D. A. Gore, and A. J. Paulraj, "Receive antenna selection for MIMO spatial multiplexing: theory and algorithms," *IEEE Transactions on Signal Processing*, vol. 51, pp. 2796–2807, Nov 2003.

[11] A. Molisch and M. Win, "MIMO systems with antenna selection," *Microwave Magazine, IEEE*, vol. 5, no. 1, pp. 46–56, 2004.

[12] M. G. Amin, X. Wang, Y. D. Zhang, F. Ahmad, and E. Aboutanios, "Sparse arrays and sampling for interference mitigation and DOA estimation in GNSS," *Proceedings of the IEEE*, vol. 104, pp. 1302–1317, June 2016.

[13] A. G. Rodriguez, C. Masouros, and P. Rulikowski, "Efficient large scale antenna selection by partial switching connectivity," in *2017 42th IEEE International Conference on Acoustics, Speech and Signal Processing*, Mar 2017.

[14] P. Vaidyanathan and P. Pal, "Sparse sensing with co-prime samplers and arrays," *Signal Processing, IEEE Transactions on*, vol. 59, pp. 573–586, Feb 2011.

[15] P. Pal and P. Vaidyanathan, "Nested arrays in two dimensions, part II: Application in two dimensional array processing," *Signal Processing, IEEE Transactions on*, vol. 60, no. 9, pp. 4706–4718, 2012.

[16] X. Wang, E. Aboutanios, and M. G. Amin, "Adaptive array thinning for enhanced DOA estimation," *IEEE Signal Processing Letters*, vol. 22, pp. 799–803, July 2015.

[17] H. Gazzah and S. Marcos, "Cramer-rao bounds for antenna array design," *Signal Processing, IEEE Transactions on*, vol. 54, no. 1, pp. 336–345, 2006.

[18] H. Gazzah and K. Abed-Meraim, "Optimum ambiguity-free directional and omnidirectional planar antenna arrays for DOA estimation," *Signal Processing, IEEE Transactions on*, vol. 57, pp. 3942–3953, Oct 2009.

[19] D. Bajovic, B. Sinopoli, and J. Xavier, "Sensor selection for event detection in wireless sensor networks," *IEEE Transactions on Signal Processing*, vol. 59, pp. 4938–4953, Oct 2011.

[20] A. Bertrand, J. Szurley, P. Ruckebusch, I. Moerman, and M. Moonen, "Efficient calculation of sensor utility and sensor removal in wireless sensor networks for adaptive signal estimation and beamforming," *Signal Processing, IEEE Transactions on*, vol. 60, no. 11, pp. 5857–5869, 2012.

[21] O. Mehanna, N. Sidiropoulos, and G. Giannakis, "Joint multicast beamforming and antenna selection," *Signal Processing, IEEE Transactions on*, vol. 61, no. 10, pp. 2660–2674, 2013.

[22] N. K. Dhingra, M. R. Jovanovic, and Z. Q. Luo, "An ADMM algorithm for optimal sensor and actuator selection," in *53rd IEEE Conference on Decision and Control*, pp. 4039–4044, Dec 2014.

[23] J. Ranieri, A. Chebira, and M. Vetterli, "Near-optimal sensor placement for linear inverse problems," *IEEE Transactions on Signal Processing*, vol. 62, pp. 1135–1146, March 2014.

[24] N. B. Mehta, S. Kashyap, and A. F. Molisch, "Antenna selection in LTE: from motivation to specification," *IEEE Communications Magazine*, vol. 50, pp. 144–150, October 2012.

[25] X. Wang, E. Aboutanios, M. Trinkle, and M. G. Amin, "Reconfigurable adaptive array beamforming by antenna selection," *Signal Processing, IEEE Transactions on*, vol. 62, no. 9, pp. 2385–2396, 2014.

[26] X. Wang, E. Aboutanios, and M. G. Amin, "Slow radar target detection in heterogeneous clutter using thinned space-time adaptive processing," *IET Radar, Sonar & Navigation*, vol. 10, no. 4, pp. 726–734, 2015.

[27] X. Wang, M. Amin, X. Wang, and X. Cao, "Sparse array quiescent beamformer design combining adaptive and deterministic constraints," *IEEE Transactions on Antennas and Propagation*, vol. 65, pp. 5808–5818, Nov 2017.

[28] X. Wang, M. Amin, and X. Cao, "Analysis and design of optimum sparse array configurations for adaptive beamforming," *IEEE Transactions on Signal Processing*, vol. 66, pp. 340–351, Jan 2018.

[29] C. J. Burges, "A tutorial on support vector machines for pattern recognition," *Data mining and knowledge discovery*, vol. 2, no. 2, pp. 121–167, 1998.

[30] M. A. Hearst, S. T. Dumais, E. Osuna, J. Platt, and B. Scholkopf, "Support vector machines," *IEEE Intelligent Systems and their applications*, vol. 13, no. 4, pp. 18–28, 1998.

[31] D. W. Patterson, *Artificial neural networks: theory and applications*. Prentice Hall PTR, 1998.

[32] X. Yao, "Evolving artificial neural networks," *Proceedings of the IEEE*, vol. 87, no. 9, pp. 1423–1447, 1999.

[33] J. Capon, "High-resolution frequency-wavenumber spectrum analysis," *Proceedings of the IEEE*, vol. 57, pp. 1408–1418, Aug 1969.