MULTIPLE-MODEL AND REDUCED-ORDER KALMAN FILTERING FOR PATHOLOGICAL HAND TREMOR EXTRACTION

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ABSTRACT

Tremor extraction techniques are considered as the central component of several rehabilitative and compensatory robotic technologies, and the accuracy of such filters can directly affect the performance of the aforementioned technologies. Motivated by this fact, the paper proposes an adaptive estimation framework, referred to as Multiple Adaptive Reduced-order Kalman filtering (KFE-BMFLC), for extraction of pathological hand tremors. The proposed KFE-BMFLC framework is designed with the goal of improving the performance of an existing state-of-the-art filtering technique, i.e. Enhanced Band-limited Fourier Linear Combiner (E-BMFLC), which has shown a promising potential in extracting involuntary hand motions but uses embedded least mean square (LMS) estimation approach. The proposed technique is capable of reducing the computational overhead in comparison to that of the conventional BMFLC technique, while increasing the estimation accuracy.

1. INTRODUCTION

Parkinson's disease (PD) is a progressive movement disorder resulted from death of brain cells in mid-brain, which are responsible for producing dopamine. The four major motor symptoms associated with PD are tremor, rigidity, bradykinesia, and postural instability. As these symptoms become more pronounced, patients may develop severe difficulty in walking, talking, and ultimately performing Activities of Daily Living (ADLs) [1]. Common treatments developed for controlling PD-related hand tremors, typically, belong to one of the following two categories: (i) Prescribing dopaminergic medications, and; (ii) Deep brain stimulation [2]. The latter is a surgical option and is mostly considered if the patient develops resistance to the medication [3]. However, there could be several possible side effects and complications associated with both methods [4]. As a result, during the last decade, alternative and assistive therapeutic and technological techniques have attracted a great deal of interest [2] (with the goal of reducing/delaying the need for increasing dosage of the medications or conducting the surgical option).

In this regard, active tremor suppression [5] has been suggested by several researchers as an alternative solution. Accordingly, wearable assistive robotic exoskeleton have been proposed in the literature to compensate for pathological hand tremor, with the goal of providing the patient with a better control over upper-limb movement during performance of ADLs [6]. In addition to the above, recent literature supports the effectiveness of interactive rehabilitation for enhancing motor control in PD patients. As a result, new robotic rehabilitation techniques have been proposed in the literature that can assist patients in performing rehabilitative tasks in virtual reality environments while damping the mechanical energy of the involuntary hand tremors in a safe manner.

Performance of both rehabilitative and assistive technologies, designed for PD patients, depends critically on the real-time accuracy of the incorporated hand tremor extraction methodology. In this regard, several signal processing filtering techniques have been developed in the literature, some of which had been initially designed for extracting physiological (not pathological) hand tremor of healthy humans. An example is the Band-limited Fourier linear combiner (BMFLC) technique, which is one of the most popular methods for extracting physiological hand tremors, and is initially designed to compensate for tremor in surgeon's hand during delicate surgeries [7–10]. It is worth mentioning that, extraction of the pathological hand tremors (such as those caused by PD) is significantly more complicated in comparison to that of physiological tremors. This issue is due to the lower frequency range, higher amplitude, and higher time-based variability of the tremor signals [11]. In order to resolve the above-mentioned issues, recently, a modified format of the BMFLC has been designed in [6] to extract pathological hand tremors. Referred to as Enhanced-BMFLC (E-BMFLC), this filter: (i) Takes advantage of an enhanced harmonic model for modeling the complete motion signal, and; (ii) Implements a memory manipulation technique to enhance the performance of the BMFLC filter in dealing with non-periodic pathological tremors. The E-BMFLC technique, however, is developed based on an embedded recursive Least Mean Square (LMS) algorithm. In our recent paper [12], the LMS algorithm of conventional E-BMFLC has been replaced by Kalman filter (KF), resulting in better performance in comparison to the conventional LMS-based E-BMFLC filter.

In this paper, we take the next step to overcome potential model uncertainties and computational cost problems of the Kalman-based BMFLC solutions. In particular, we propose a new tremor extraction filtering framework, which is developed by integration of multiple models and reduced-order Kalman filtering with the E-BMFLC algorithm. Referred to as the KFE-BMFLC, the proposed framework is designed to further enhance the performance of pathological tremor extraction through a three-step strategy. In other words, while the concept of enhanced harmonic modeling and memory manipulation are embraced in the proposed framework, several reduced-order Kalman filters (RKFs), each matched to a subset of the state variables, are running in parallel. Finally, for each of these RKFs, multiple versions are incorporated to deal with the inherit uncertainty of the constructed state-space model. To validate the performance of the proposed filter, an online database of pathological hand tremors is used. The database is available in [33] and has been used in the liter-

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ature to validate performance of other tremor extraction techniques, such as the one proposed in [11]. Based on the validation tests conducted in this paper, it is shown that the proposed KFE-BMFLC technique is able to improve the performance of the filter developed in [12] by representing an acceptable computational overhead for real-time implementations.

2. PROBLEM FORMULATION

In this section, the fundamental concept of tremor extraction and the E-BMFLC is briefly outlined. We consider the following model to represent complete hand motion including the involuntary components and voluntary components

$$\eta_p(k) = \eta_{p-v}(k) + \eta_{p-i}(k), \tag{1}$$

where $\eta_p(k)$ is the original hand motion, $\eta_{p-v}(k)$ and $\eta_{p-i}(k)$ are the voluntary and involuntary (hand tremor) components, and k is the time index. To model $\eta_p(k)$, the total hand movement frequency range should be divided into a finite number $L = \frac{(f_{max}-f_{min})}{\Delta f}$ where f_{max} and f_{min} are the maximum and minimum given frequencies, and Δf is the minimum frequency difference. Consequently, the total hand motion can be modeled as follows

$$y(k)=\sum_{r=0}^{L}\left[a_r \sin(2\pi(f_{\min}+r\Delta f)k)+ b_r\cos(2\pi(f_{\min}+r\Delta f)k)\right], \tag{2}$$

where $y(k)$ is the Fourier combiner representing η_p [6]. Let us define

$$x_r(k) = \begin{cases} sin(2\pi(f_{min}+r\Delta f)k), & 0\leq r\leq L \\ cos(2\pi(f_{min}+((r-L)-1)\Delta f)k), & L+1\leq r\leq 2L+1 \end{cases}. \tag{3}$$

Considering Eq. (3), the model in Eq. (2) can be rewritten as

$$\eta_p(k) = \boldsymbol{w}^T_{\eta_p}(k)\boldsymbol{x}_{\eta_p}(k), \tag{4}$$

where T indicates transpose operator,

$$\boldsymbol{w}_{\eta_p}(k)=[a_0(k), a_1(k), \ldots, a_L(k), b_0(k), b_1(k), \ldots, b_L(k)]^T, \tag{5}$$

$$\text{and} \quad \boldsymbol{x}_{\eta_p}(k) = [x_0(k), x_1(k)\ldots, x_{2L+1}(k)]^T. \tag{6}$$

The above-given model is then used in an embedded weight estimation technique (which was originally based on LMS algorithm), to first, estimate $\boldsymbol{w}_{\eta_p}(k)$ denoted by $\hat{\boldsymbol{w}}_{\eta_p}(k)$, and then extract the involuntary and voluntary components of motions.

To extract the involuntary motion, $\hat{\boldsymbol{w}}_{\eta_p}(k)$ and $\boldsymbol{x}_{\eta_p}(k)$ should be truncated based on the frequency range of the involuntary motion. Let us consider $N_{\min} = (\omega_{\min} - f_{\min})/\Delta f$ and $N_{\max} = (\omega_{\max} - f_{\min})/\Delta f$, where $\omega_{\min}$ and $\omega_{\max}$ are the minimum and maximum frequency range of the involuntary motion. Accordingly, the involuntary hand motion is estimated as follows

$$\eta_{p-i}(k) = \boldsymbol{x}^T_{\eta_{p-i}}(k)\hat{\boldsymbol{w}}_{\eta_{p-i}}(k). \tag{7}$$

where

$$\begin{aligned} \hat{\boldsymbol{w}}_{\eta_{p-i}}(k) &= [a_{N_{\min}}(k),\ldots, a_{N_{\max}}(k), b_{N_{\min}}(k),\ldots, b_{N_{\max}}(k)]^T \\ \boldsymbol{x}_{\eta_{p-i}}(k) &= [x_{N_{\min}}(k),\ldots, x_{N_{\max}}(k), x_{L+N_{\min}}(k),\ldots, x_{L+N_{\max}}(k)]^T. \end{aligned} \tag{8}$$

Consequently, the estimation of complete hand movement can be calculated as

$$\hat{\eta}_p(k) = \boldsymbol{x}^T_{\eta_p}(k)\hat{\boldsymbol{w}}_{\eta_p}(k), \tag{9}$$

where $\hat{\eta}_p(k)$ and $\hat{\boldsymbol{w}}_{\eta_p}(k)$ are the estimates of $\eta_p(k)$ and $\boldsymbol{w}_{\eta_p}(k)$, respectively. The algorithm originally used in the design of the E-BMFLC to calculate the estimates of the underlying coefficients (a_r and b_r) is the following LMS-based technique

$$\hat{\boldsymbol{w}}^T_{\eta_p}(k) = \rho\hat{\boldsymbol{w}}^T_{\eta_p}(k-1) + 2\mu\boldsymbol{x}_{\eta_p}(k)\epsilon_{\eta_p}(k), \tag{10}$$

where $\epsilon_{\eta_p}(k) = \eta_p(k) - \hat{\eta}_p(k)$, and $\rho = \sqrt[\delta]{\alpha}$, and $\delta = \frac{1}{\Delta T}T_p$. In addition, μ is the corrective gain, ϵ_{η_p} indicates the estimation error, and ρ is the dynamic memory windowing pole (in the Z-domain). Also, T_p is the memory window width in the time domain; α is the minimum amplification gain within the window and ΔT is the sampling time. This technique was proposed to enhance the performance of the filter in presence of non-periodic tremors. Details can be found in Reference [6].

3. THE PROPOSED KFE-BMFLC

In this section, we develop the proposed KFE-BMFLC where KF-based state estimation is used instead of the conventional LMS technique. In the following sub-sections, we first start by describing the design for incorporation of the conventional KF, then extend it based on reduced-order version of the Kalman filtering (RKF), and finally describe the multiple-model formulation.

3.1. Kalman Filter

To improve the performance of the E-BMFLC filter, the algorithm used for estimating the weights can be replaced by the KF. For this, the complete hand motion (the observation model within the KF recursion) is modeled as follows

$$y(k) = \boldsymbol{x}^T_{\eta_p}(k)\boldsymbol{w}_{\eta_p}(k) + v(k), \tag{11}$$

where $v(k)$ is the observation uncertainty. The state space model is constructed as follows with the state vector defined a $\boldsymbol{w}_{\eta_p}(k)$

$$\boldsymbol{w}_{\eta_p}(k) = \boldsymbol{F}(k)\boldsymbol{w}_{\eta_p}(k-1) + \boldsymbol{\psi}(k), \tag{12}$$

where $\boldsymbol{F}(\cdot)$ denotes the state model, and $\boldsymbol{\psi}(k)$ is the state uncertainties. Within the context of tremor extraction, a random walk model is considered as the state model [29] since no preceding information can be considered for evolution of the state variables ($\boldsymbol{w}_{\eta_p}(k)$), i.e., $\boldsymbol{F}(k) \triangleq \rho\boldsymbol{I}$, where ρ is the dynamic memory windowing pole adopted from the conventional E-BMFLC, and matrix $\boldsymbol{I}$ is an identity matrix of appropriate dimension. Eqs. (11) and (12) construct the required state-space model, which is used to estimate the Fourier coefficients. Here, the state noise $\boldsymbol{\psi}(k)$ and the measurement noise $v(k)$ are considered as zero mean, uncorrelated, and white Gaussian noise processes ($v(k) \sim \mathcal{N}(0, R)$, and $\psi(k) \sim \mathcal{N}(0, \boldsymbol{Q}(k))$). Terms $\boldsymbol{Q}(k)$ and R are the state uncertainties covariance matrix and measurement uncertainties covariance, respectively. It is worth mentioning that these statistics are unknown. Another common assumption in the context of tremor extraction is to consider a constant and pre-defined state noise covariance matrix (i.e., $\boldsymbol{Q}(k) \triangleq \boldsymbol{Q}$). Based on the above modeling assumptions, the updated state estimation is

$$\hat{\boldsymbol{w}}_{\eta_p}(k|k) = \mathbb{E}\left\{\boldsymbol{w}_{\eta_p}(k) \mid \boldsymbol{Y}(k)\right\} \tag{13}$$

where $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation, and $\boldsymbol{Y}(k)$ is the total past observations (including iteration k). To estimate $\hat{\boldsymbol{w}}_{\eta_p}(k|k)$, primarily the predicted state $\hat{\boldsymbol{w}}_{\eta_p}(k|k-1) = \mathbb{E}\left\{\boldsymbol{w}_{\eta_p}(k) \mid \boldsymbol{Y}(k-1)\right\}$ and the identified covariance matrix are computed as

$$\begin{aligned}\hat{\boldsymbol{w}}_{\eta_p}(k|k-1) &= \rho \times \hat{\boldsymbol{w}}_{\eta_p}(k-1|k-1) \quad (14)\\ \boldsymbol{P}(k|k-1) &= \rho^2 \times \boldsymbol{P}(k-1|k-1) + \boldsymbol{Q}, \quad (15)\end{aligned}$$

where ρ is the memory manipulation factor. By considering $y(k)$, the updated state estimation $\hat{\boldsymbol{w}}_{\eta_p}(k|k)$ and its corresponding error covariance matrix $\boldsymbol{P}(k|k)$ are obtained as

$$\begin{aligned}\boldsymbol{K}(k) &= \boldsymbol{P}(k|k-1)\boldsymbol{x}^T_{\eta_p}(k)\left[\boldsymbol{x}_{\eta_p}(k)\boldsymbol{P}(k|k-1)\boldsymbol{x}^T_{\eta_p}(k) + R\right]^{-1}\\ \hat{\boldsymbol{w}}_{\eta_p}(k|k) &= \hat{\boldsymbol{w}}_{\eta_p}(k|k-1) + \boldsymbol{K}(k)\big(y(k) - \boldsymbol{x}^T_{\eta_p}(k)\hat{\boldsymbol{w}}_{\eta_p}(k|k-1)\big)\\ \boldsymbol{P}(k|k) &= \left[\boldsymbol{I} - \boldsymbol{K}(k)\boldsymbol{x}_{\eta_p}(k)\right]\boldsymbol{P}(k|k-1). \quad (16)\end{aligned}$$

Here, $\boldsymbol{K}(k)$ is the Kalman gain that is updated at each time sample. Let us represent $\hat{\boldsymbol{w}}_{\eta_p}(k|k)$ as

$$\hat{\boldsymbol{w}}_{\eta_p}(k|k) = \quad (17)$$
$$[a_0(k|k), a_1(k|k), \ldots, a_L(k|k), b_0(k|k), b_1(k|k), \ldots, b_L(k|k)]^T.$$

Considering Eq. (8) and Eq. (18), we have

$$\hat{\boldsymbol{w}}_{\eta_{p-i}}(k|k) = \quad (18)$$
$$[a_{N_{\min}}(k|k), \ldots, a_{N_{\max}}(k|k), b_{N_{\min}}(k|k), \ldots, b_{N_{\max}}(k|k)]^T.$$

As a result the involuntary tremor can be estimated as

$$\eta_{p-i}(k|k) = \boldsymbol{x}^T_{\eta_{p-i}}(k)\hat{\boldsymbol{w}}_{\eta_{p-i}}(k|k). \quad (19)$$

3.2. Reduced-Order Kalman Filtering

Instead of using the high-dimension KF (explained in Sub-section 3.1), the proposed reduced-order tremor extraction approach runs $N_{\text{RO}} > 1$ number of reduced-order state-space models. This can be achieved by spatially decomposing the state mode given by Eq. (12) as follows

$$\begin{bmatrix}\boldsymbol{w}^{(1)}_{\eta_p}(k)\\ \vdots\\ \boldsymbol{w}^{(l)}_{\eta_p}(k)\\ \vdots\\ \boldsymbol{w}^{(N_{\text{RO}})}_{\eta_p}(k)\end{bmatrix} = \begin{bmatrix}\rho^{(1)}\boldsymbol{w}^{(1)}_{\eta_p}(k-1)\\ \vdots\\ \rho^{(l)}\boldsymbol{w}^{(l)}_{\eta_p}(k-1)\\ \vdots\\ \rho^{(N_{\text{RO}})}\boldsymbol{w}^{(N_{\text{RO}})}_{\eta_p}(k-1)\end{bmatrix} + \begin{bmatrix}\boldsymbol{\psi}^{(1)}(k)\\ \vdots\\ \boldsymbol{\psi}^{(l)}(k)\\ \vdots\\ \boldsymbol{\psi}^{(N_{\text{RO}})}(k)\end{bmatrix}. \quad (20)$$

Note that, Eq. (20) illustrates a state-space decomposition we refer to it as reduced-order filtering. In the next step, N_{RO} KFs are applied in parallel where the l^{th} filter, for, $(1 \leq l \leq N_{\text{RO}})$, provides localized state estimates, i.e., $\hat{\boldsymbol{w}}^{(l)}_{\eta_p}(k|k) = \mathbb{E}\{\boldsymbol{w}^{(l)}_{\eta_p}(k) \mid y^{(l)}(k)\}$. The measurement model combines the localized KFs since only one measurement $y^{(l)}(k)$ is obtainable at each time step. The l^{th} localized KF performs the prediction as

$$\begin{aligned}\hat{\boldsymbol{w}}^{(l)}_{\eta_p}(k|k-1) &= \rho^{(l)} \times \hat{\boldsymbol{w}}^{(l)}_{\eta_p}(k-1|k-1) \quad (21)\\ \boldsymbol{P}^{(l)}(k|k-1) &= [\rho^{(l)}]^2 \times \boldsymbol{P}^{(l)}(k-1|k-1) + \boldsymbol{Q}^{(l)}, \quad (22)\end{aligned}$$

where $\rho^{(l)}$ is the localized memory manipulation factor. It should be noted that, the localized memory manipulation factors can be vary from one sub-filter to another based on the frequency content which is estimated by the l^{th} KF. By considering $y^{(l)}(k)$, the updated state estimate $\hat{\boldsymbol{w}}^{(l)}_{\eta_p}(k|k)$ and the corresponding error covariance matrix $\boldsymbol{P}^{(l)}(k|k)$ are computed as

$$\begin{aligned}&\boldsymbol{K}^{(l)}(k) =\\ &\quad \boldsymbol{P}^{(l)}(k|k-1)\boldsymbol{x}_{l\eta_p}(k)\left[\boldsymbol{x}^T_{l\eta_p}(k)\boldsymbol{P}^{(l)}(k|k-1)\boldsymbol{x}_{l\eta_p}(k) + R^{(l)}\right]^{-1}\\ &\hat{\boldsymbol{w}}^{(l)}_{\eta_p}(k|k) =\\ &\quad \hat{\boldsymbol{w}}^{(l)}_{\eta_p}(k|k-1) + \boldsymbol{K}^{(l)}(k)\big(y^{(l)}(k) - \boldsymbol{x}^T_{l\eta_p}(k)\hat{\boldsymbol{w}}^{(l)}_{\eta_p}(k|k-1)\big)\\ &\boldsymbol{P}^{(l)}(k|k) = \left[\boldsymbol{I} - \boldsymbol{K}^{(l)}(k)\boldsymbol{x}^T_{l\eta_p}(k)\right]\boldsymbol{P}^{(l)}(k|k-1). \quad (23)\end{aligned}$$

Here, $\boldsymbol{K}^{(l)}(k)$ is the Kalman gain that is updated at each time step. In order to estimate $\hat{\boldsymbol{w}}_{\eta_p}(k|k)$, we need to combine $\hat{\boldsymbol{w}}^{(l)}_{\eta_p}(k|k)$s, as

$$\hat{\boldsymbol{w}}_{\eta_p}(k|k) = \quad (24)$$
$$[a_0(k|k), a_1(k|k), \ldots, a_L(k|k), b_0(k|k), b_1(k|k), \ldots, b_L(k|k)]^T.$$

Considering (8) and (24) we have

$$\hat{\boldsymbol{w}}_{\eta_{p-i}}(k|k) = \quad (25)$$
$$[a_{N_{\min}}(k|k), \ldots, a_{N_{\max}}(k|k), b_{N_{\min}}(k|k), \ldots, b_{N_{\max}}(k|k)]^T.$$

As a result, considering Eqs. (7), (9), and (25), the estimated involuntary motion can be calculated as follows

$$\eta_{p-i}(k|k) = \boldsymbol{x}^T_{\eta_{p-i}}(k)\hat{\boldsymbol{w}}(k|k)_{\eta_{p-i}}. \quad (26)$$

The computational overhead of the proposed RKF approach for tremor extraction is approximately of $O(3(N_W/N_{\text{RO}})^2)$ considering the localized KFs running in parallel, where N_W is dimension of the state vector and N_{RO} is considered as the number of reduced-orders. The above number is based $N_{\text{RO}} = 3$, which is considered by intuitively dividing the overall state into low frequency, middle frequency and high frequency components. Accordingly, there is a computational cost saving in comparison with the Kalman-based E-BMFLC with the computational complexity of $O(3N_W^2)$ [32].

3.3. Multiple Adaptive Reduced-Order Kalman Filtering

The main drawback of the RKFs, introduced in Sub-section 3.2, is the inherit uncertainty of the underlying state-space model developed in Eqs. (11) and (12). In other words, Terms $\rho^{(l)}$, $\boldsymbol{Q}^{(l)}$, and $R^{(l)}$ are assumed known and fixed through out the filtering iterations. To address this issue, one solution is to keep the localized manipulation factor $\rho^{(l)}$, for $(1 \leq l \leq N_{\text{RO}})$, fixed a-priori and adaptively learn/compute the noise statistics using adaptive extensions of the KF [35]. Alternatively, we propose to use multiple adaptive models for each of the original RKFs to not only consider the effects of localized noise statistics but also consider deferent localized manipulation factors.

It should be mentioned that the proposed framework is composed of a bank of $N_f > 1$ filters for each of the N_{RO} RKFs developed in the previous sub-section. Each RKFs is initialized based on a different set of initial parameters depending on the frequency band that the RKF is estimating. All individual RKFs run in parallel and independently, each producing their own estimate $\hat{\boldsymbol{w}}^{(l)}_{i\eta_p}(k|k)$ and covariance matrix $\boldsymbol{P}^{(l)}_i(k|k)$, for $(1 \leq i \leq N_f)$, and $(1 \leq l \leq N_{\text{RO}})$. The localized state estimates associated with each RKF are fused through a collapsing step [22, 32] which forms the optimal single Gaussian distribution in the mean-square error (MSE) sense with mean $\hat{\boldsymbol{w}}^{(l)}_{\eta_p}(k|k)$ and covariance matrix $\boldsymbol{P}^{(l)}(k|k)$. By utilizing the

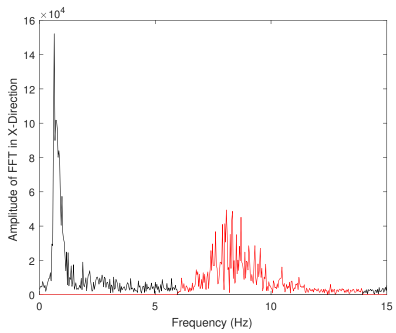

Fig. 1. The tremor truncation in the frequency domain. The red part of the signal represents the involuntary hand motion regarding a PD patient.

parameter $\gamma_i^{(l)}(k|k-1)$, the weights $\omega_i^{(l)}(k|k)$ can be updated linearly and in an autoregressive way. This value illustrates the confidence amount corresponding to the i^{th} model constructed for the l^{th} RKF at each time step. The innovation residual $\boldsymbol{z}_i^{(l)}(k|k-1)$ and innovation covariance $S_i^{(l)}(k|k-1)$ provide the prediction error of each filter and can be defined as

$$\boldsymbol{z}_i^{(l)}(k|k-1) = \boldsymbol{w}_{i\eta_p}^{(l)}(k) - \hat{\boldsymbol{w}}_{i\eta_p}^{(l)}(k|k-1) \tag{27}$$

$$S_i^{(l)}(k|k-1) = [\boldsymbol{x}_{i\eta_p}^{(l)}(k)]^T \boldsymbol{P}_i^{(l)}(k|k-1)\boldsymbol{x}_{i\eta_p}^{(l)}(k) + R_i^{(l)} \tag{28}$$

which are deployed for the calculation of $\gamma_i(k|k-1)$. Accordingly, the weights $\omega_i(k|k)$ can be computed as follow

$$\gamma_i^{(l)}(k|k-1) = \det(S_i^{(l)}(k|k-1))^{-\frac{1}{2}} \tag{29}$$

$$\times \ \exp\left[-\frac{1}{2}[\boldsymbol{z}_i^{(l)}(k|k-1)]^T[S_i^{(l)}(k|k-1)]^{-1}\boldsymbol{z}_i^{(l)}(k|k-1)\right]$$

$$\Theta_i^{(l)} = \gamma_i^{(l)}(k|k-1)\omega_i^{(l)}(k-1|k-1) \tag{30}$$

Furthermore, the normalized weights can be represented as follows

$$\omega_i^{(l)}(k|k) = \frac{\Theta_i^{(l)}}{\sum_{j=1}^{n}\Theta_j^{(l)}}. \tag{31}$$

Finally by fusing all the n estimates we will have the final estimates

$$\hat{\boldsymbol{w}}_{\eta_p}^{(l)}(k|k) = \sum_{i=1}^{n}\omega_i^{(l)}(k|k)\hat{\boldsymbol{w}}_{i\eta_p}^{(l)}(k|k) \tag{32}$$

$$\boldsymbol{P}^{(l)}(k|k) = \sum_{i=1}^{n}\omega_i^{(l)}(k|k)\times \tag{33}$$

$$\left[\boldsymbol{P}_i^{(l)}(k|k)+\{\hat{\boldsymbol{w}}_{\eta_p}^{(l)}(k|k)-\hat{\boldsymbol{w}}_{i\eta_p}^{(l)}(k|k)\}\{\hat{\boldsymbol{w}}_{\eta_p}^{(l)}(k|k)-\hat{\boldsymbol{w}}_{i\eta_p}^{(l)}(k|k)\}^T\right]$$

This completes the definition of multiple adaptive reduced-order Kalman filter.

4. EXPERIMENTAL RESULTS

In this section, the evaluation of the proposed KFE-BMFLC filter is provided. The dataset used in this evaluation has been published online by Motus Bioengineering Inc. [33]. This dataset involves real measurements of hand tremor in PD patients. Regarding the hand movement estimation, the *actual* tremor signal is extracted from the complete hand movement using an *offline post-processing* method introduced by Atashzar *et al.* in [6]. The result is demonstrated in Fig. 1, where the red part indicates the involuntary hand motion in

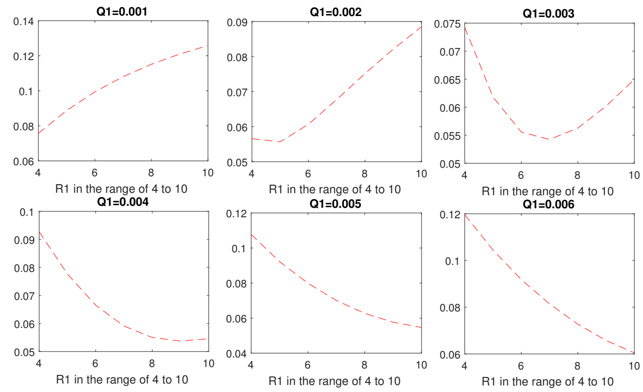

Fig. 2. NRMSE sensitivity analysis regarding multiple adaptive reduced-order Kalman filtering based on the variation of $Q_1^{(l)}$ and $R_1^{(l)}$.

Table 1. Accuracy evaluation of KFE-BMFLC and E-BMFLC for the extracted tremor over four different tremor data. It is observed that the KFE-BMFLC improves estimation accuracy in comparison with E-BMFLC.

Estimation Methods	NRMSE 1	NRMSE 2	NRMSE 3	NRMSE 4
KFE-BMFLC	0.0426	0.0538	0.0568	0.0517
E-BMFLC	0.0551	0.0603	0.0654	0.0582

the frequency domain. The extracted actual tremor is considered as the reference for evaluation of the online technique proposed in this paper. In this experiment, the KF is reduced into three subsystems, i.e., $N_{\text{RO}} = 3$. Accordingly, one localized KF is allocated to low frequencies, one is allocated to the frequencies of the considered tremor (i.e., $6-14Hz$) and one is allocated to high frequencies. On the other hand, there are three RKFs which are running in parallel. Moreover the frequency difference has been considered as $\Delta f = 0.5Hz$.

The Normalized Root Mean-Square Error (NRMSE) is used in this paper to measure the estimation inaccuracy. The NRMSE is defined as NRMSE $=$ RMSE$/(s_{\max} - s_{\min})$, RMSE $= \sqrt{(\sum_{k=1}^{n}(\hat{s}(k) - s(k))^2)/n}$. Here, $s(k)$ is the input tremor signal, $\hat{s}(k)$ is the estimated tremor, and n is the number of samples. Terms $s_{\max}$ and $s_{\min}$ represent the minimum and maximum value of the observed signal. The comparisons is performed between the proposed KFE-BMFLC and the E-BMFLC. The summary of the results for 4 subjects are provided in Table 1. In addition, in Fig. 2, the sensitivity analysis regarding the multiple adaptive reduced-order Kalman filtering is represented based on the variation of $Q_1^{(l)}$ and $R_1^{(l)}$. It is observed that the proposed KFE-BMFLC has enhanced the performance of the E-BMFLC.

5. CONCLUSION

In this paper, a new estimation framework is proposed to extract pathological hand tremor by integration of the multiple-model and reduced-order Kalman filtering with recently developed E-BMFLC technique. Referred to as the KFE-BMFLC, the goal is to increase the accuracy in estimation of involuntary hand motions while reducing the overall computational complexity within the context of tremor extraction, which consists of an extensively large number of states. The proposed KFE-BMFLC framework is capable of reducing the computational overhead of the KF-based implementation of E-BMFLC by decomposing the overall large scale estimation problem into several lower dimensional sub-systems. Besides, the proposed KFE-BMFLC framework deals with the inherit structural uncertainty of the constructed reduced-order state-space model by utilization of multiple-models adaptive estimation techniques.

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