OCT VOLUMETRIC DATA RESTORATION VIA PRIMAL-DUAL PLUG-AND-PLAY METHOD

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ABSTRACT

This work proposes a volumetric data restoration method, especially for data acquired through an optical coherence tomography (OCT) device. OCT is a technique for acquiring a tomographic image of a specimen object in a few μ m scale by using a near infrared laser. The authors have been trying dynamic observation of epithelium in cochlear of the inner ear. Currently, there is a problem to remove the influence of the measurement process as well as noise due to image sensor sensitivity. Therefore, in this work, on the assumption that specimen objects follow some sort of signal generation model, an OCT volumetric data restoration method is proposed. The proposed technique adopts the primal-dual plug-and-play (PDPnP) method, where the generation model is represented by a sparsity-aware regularization term explicitly or implicitly. The significance of the proposed method is verified by simulation on artificial data, followed by an experiment with actual observation data.

Index Terms— Primal-dual splitting method, BM4D, volumetric data, sparse modeling, synthesis dictionary, MS *en-face* OCT

1. INTRODUCTION

The sensory epithelium in cochlea of a living animal has nonlinearity which strongly amplifies vibration as the input sound is smaller. Such reactions are not seen in dead animals. Understanding the mechanism of the cochlea is needed from the aspect of science and medicine. In order to measure vibration of sensory epithelium in vivo, we have been developing a novel tomographic imaging device using OCT [1]. An OCT is a tomographic technique with a spatial resolution of a few μ m using a near infrared laser. It is adopted to measure layered materials such as bio-tissues and an indispensable measurement technique for fundus examination in ophthalmology [2]. However, conventional OCT techniques, such as Doppler spectral domain (SD) OCT [3-6], acquire layered structure in the depth direction (Z direction) of one point, and needs two-dimensional (2-D) scanning in X and Y directions to obtain volumetric data. Therefore, it is not suitable for temporal dynamic tomography. Our developing new device is a multi-frequency scanning (MS) full-field (en-face) OCT type that instantly acquires 2-D information in the X and Y directions and performs only Z direction scanning [7]. It is possible to acquire tomographic data in a shorter time than conventional one. On the other hand, since the light is broadened by the interference microscope, the image is weaker than that of the conventional type and it is susceptible to noise.

A target source \mathbf{u} is observed through a measurement process \mathbf{P} which consists of a coherence, i.e., local fluctuate function. Therefore, in order to estimate the tomographic data u from observed data, i.e., interference image, v, it is necessary to remove the local fluctuation as well as noise. This problem can be reduced to a signal restoration problem. In the article [8], we proposed a denoising method for OCT data with the iterative hard-thresholding algorithm. Although the significance of the denoising performance was verified, the measurement process was not taken into account. In [9], the authors propose a super-resolution technique, i.e., simultaneous interpolation and denoising, for retina SD-OCT in order to shorten the acquisition time. High resolution volumetric data is restored from incomplete OCT data by using OMP and learned dictionary. As well, article [10] proposes a speckle denoising method for retina SD-OCT. It adopts the low rank matrix decomposition for patches. Both of [9] and [10], however, have no mention on reversing the measurement process.

Removal of noise and measurement process is often formulated as a 1-norm regularized least squares problem, which can be solved by the iterative soft-thresholding algorithm (ISTA) [11–14]. ISTA guarantees to converge to a point in the exact solution set. The problem setting, however, is limited and is difficult to be modified. For example, since OCT source data **u** is a spatial distribution of reflectance of an object, the range of the value is limited to range from -1 to 1. In this case, a restoration algorithm that can incorporate hard constraints is preferable. The alternating direction method of multipliers (ADMM) works well for constrained optimization problems [15, 16] and its application to compressive video sensing is discussed in [17]. ADMM, however, requires an inverse matrix operation and has difficulty in the application to high dimensional data.

In this work, we propose to apply PDPnP method [18] to solve the OCT restoration problem. PDPnP is based on primal-dual splitting (PDS) algorithm [19, 20], which can deal with an optimization problem that ADMM can handle without inverse matrix operation. In addition, implicit regularization term is acceptable by plugging a state-of-the-art regularized Gaussian denoiser such as BM4D [21].

2. OVERVIEW OF MS EN-FACE OCT

In this section, we overview the device configuration of MS *en-face* OCT. The observation model is also discussed. Note here that we deal with only snapshot of dynamic volumetric data at a certain time, and will leave discussions on dynamic variation as a future work.

2.1. Device configuration

Fig. 1 shows the configuration of MS *en-face* OCT device. The spectrum of the broadband super-luminescent diode (SLD) light source is extracted out in a comb shape by the Fabry-Perot resonator, and

^{*} Advanced Research and Development Programs for Medical Innovation, AMED-CREST

This work was supported by AMED-CREST and JSPS KAKENHI Grant Number JP16H03164.



Fig. 1. MS en-face OCT device configuration

a multi-wavelength optical comb is generated. The position of the interference peak is scanned in the Z direction by controlling the frequency interval of the spectrum comb with a piezo actuator.

The optical comb is divided into a reference and sample beam by the beam splitter. The field of view is enlarged by the objective lenses, and the reflected lights are formed on the CMOS sensor. Interference between the reference and sample beam is acquired and rendered by image processing.

2.2. Observation model

MS *en-face* OCT successively acquires tomographic images perpendicular to the optical axis (Z direction) by changing the depth position. By aligning these images, observation volumetric data of an object such as bio-tissue is obtained. We let a discrete model of OCT observation data $\{r[n]\}_n$ be

$$r[\mathbf{n}] = b[\mathbf{n}] + \sum_{\mathbf{k} \in \Omega_{u}} u[\mathbf{k}]p[\mathbf{n} - \mathbf{k}] + w[\mathbf{n}], \ \mathbf{n} \in \Omega_{v}, \qquad (1)$$

where $\mathbf{n} = [n_x, n_y, n_z]^{\mathsf{T}} \in \Omega_v$ and $\mathbf{k} = [k_x, k_y, k_z]^{\mathsf{T}} \in \Omega_u$ are the array indexes of three-dimensional (3-D) volumetric data, each element corresponding to the position of horizontal, vertical, and depth, respectively. $\Omega_u, \Omega_v \subset \mathbb{Z}^3$ represent the range of the indexes. $\{b[\mathbf{n}]\}_{\mathbf{n}}$ represents the direct current (DC) component in the Z direction that does not contribute to interference, $\{w[\mathbf{n}]\}_{\mathbf{n}}$ denotes the noise component and $\{p[\mathbf{m}]\}_{\mathbf{m}}$ is an interference waveform representing the OCT measurement process and has a shape similar to the cosine modulated Gaussian function. $u[\mathbf{k}]$ corresponds to the reflectance in the Z direction of the object at position $\mathbf{k} \in \Omega_u$ and has a constrained value as $u[\mathbf{k}] \in [-1, 1], \mathbf{k} \in \Omega_u$.

In this paper, as an impulse response $\{p[\mathbf{m}]\}_{\mathbf{m}}$, we adopt

$$p[\mathbf{m}] = \alpha \delta[m_{\mathrm{x}}] \delta[m_{\mathrm{y}}] \exp\left(-\frac{m_{\mathrm{z}}^2}{2\sigma_{\mathrm{p}}^2}\right) \cos\left(\omega_{\mathrm{p}}m_{\mathrm{z}}\right), \ \mathbf{m} \in \mathbb{Z}^3, \ (2)$$

where $\mathbf{m} = [m_x, m_y, m_z]^T$, α , σ_p and ω_p indicate the amplitude, standard deviation and angular frequency, respectively, and $\delta[m]$ is the impulse sequence defined by 1 for m = 0 and 0 for $m \neq 0$.

In order to restore the source array $\{u[\mathbf{k}]\}_{\mathbf{k}}$, the Z direction DC component $\{b[\mathbf{n}]\}_{\mathbf{n}}$, noise $\{w[\mathbf{n}]\}_{\mathbf{n}}$ and local fluctuation by $\{p[\mathbf{m}]\}_{\mathbf{m}}$ must be removed. The Z direction DC component $\{b[\mathbf{n}]\}_{\mathbf{n}}$ can be easily removed with a high-pass filter. Therefore, as the observation data in the following discussion, we adopt

$$v[\mathbf{n}] = r[\mathbf{n}] - b[\mathbf{n}], \ \mathbf{n} \in \Omega_{\mathbf{v}}.$$
(3)

3. PROPOSED RESTORATION METHOD

In this section, we propose a method to restore the source data $\{u[\mathbf{k}]\}_{\mathbf{k}}$ from observation data $\{v[\mathbf{n}]\}_{\mathbf{n}}$ by removing the local

Algorithm 1 Primal-dual plug-and-play image restoration [18]
Input: $\mathbf{x}^{(0)}, \mathbf{y}_1^{(0)}, \mathbf{y}_2^{(0)}$
Output: $\mathbf{x}^{(n)}$
1: while A stopping criterion is not satisfied do
2: $\mathbf{x}^{(n+1)} = \mathfrak{G}_{\mathcal{R}}\left(\mathbf{x}^{(n)} - \gamma_1\left(\mathbf{\Phi}^{T}\mathbf{y}_1^{(n)} + \mathbf{\Psi}^{T}\mathbf{y}_2^{(n)}\right), \sqrt{\gamma_1}\right)$
3: $\mathbf{y}_1^{(n)} \leftarrow \mathbf{y}_1^{(n)} + \gamma_2 \mathbf{\Phi} \left(2\mathbf{x}^{(n+1)} - \mathbf{x}^{(n)} \right)$
4: $\mathbf{y}_2^{(n)} \leftarrow \mathbf{y}_2^{(n)} + \gamma_2 \boldsymbol{\Psi} \left(2\mathbf{x}^{(n+1)} - \mathbf{x}^{(n)} \right)$
5: $\mathbf{y}_1^{(n+1)} = \mathbf{y}_1^{(n)} - \gamma_2 \operatorname{prox}_{\frac{1}{\gamma_2} \mathcal{F}_{\mathbf{v}}} \left(\frac{1}{\gamma_2} \mathbf{y}_1^{(n)}\right)$
6: $\mathbf{y}_{2}^{(n+1)} = \mathbf{y}_{2}^{(n)} - \gamma_{2} P_{C} \left(\frac{1}{\gamma_{2}} \mathbf{y}_{2}^{(n)} \right)$
7: $n \leftarrow n+1$
8: end while

fluctuation due to $\{p[\mathbf{m}]\}_{\mathbf{m}}$ and noise $\{w[\mathbf{n}]\}_{\mathbf{n}}$. As a restoration algorithm, we adopt PDPnP, which can avoid using inverse matrix operation and incorporate existing regularized Gaussian denoiser [18].

3.1. Problem setting of OCT data restoration

We estimate reflectance distribution $\{u[\mathbf{k}]\}_{\mathbf{k}}$ as source array from OCT observation $\{v[\mathbf{n}]\}_{\mathbf{n}}$. Now, let $\mathbf{u} \triangleq \operatorname{vec}(\{u[\mathbf{k}]\}_{\mathbf{k}}) \in \mathbb{R}^N$, $\mathbf{v} \triangleq \operatorname{vec}(\{v[\mathbf{n}]\}_{\mathbf{n}}) \in \mathbb{R}^M$ and $\mathbf{w} \triangleq \operatorname{vec}(\{w[\mathbf{n}]\}_{\mathbf{n}}) \in \mathbb{R}^M$, i.e., the vector representations of source, observation and noise, respectively, where $N = |\Omega_{\mathbf{u}}|$ and $M = |\Omega_{\mathbf{v}}|$. With these vector notations, a linear shift invariant system with an impulse response $\{p[\mathbf{m}]\}_{\mathbf{m}}$ is represented by matrix $\mathbf{P} : \mathbb{R}^N \to \mathbb{R}^M$. From (1) and (3) with the range constraint, the observation model is represented by

$$\mathbf{v} = \mathbf{P}\mathbf{u} + \mathbf{w}, \ \mathbf{u} \in [-1, 1]^N, \tag{4}$$

Furthermore, we represent the generation process of the source data \mathbf{u} by a certain synthesis dictionary $\mathbf{D} : \mathbb{R}^L \to \mathbb{R}^N$ and a coefficient vector $\mathbf{s} \in \mathbb{R}^L$ as

$$\mathbf{u} = \mathbf{D}\mathbf{s}.\tag{5}$$

Assuming sparseness in s, the problem setting is formulated as

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}\in\mathbb{R}^L} \frac{1}{2} \|\mathbf{P}\mathbf{D}\mathbf{s} - \mathbf{v}\|_2^2 + \lambda \mathcal{R}(\mathbf{s}), \text{ s.t. } \mathbf{D}\mathbf{s} \in [-1, 1]^N, (6)$$

where $\|\cdot\|_2$ is 2-norm, $\mathcal{R}(\cdot) : \mathbb{R}^L \to [0,\infty)$ is the regularization term, and $\lambda \in [0,\infty)$ is a regularization parameter. Estimation of source is obtained as $\hat{\mathbf{u}} = \mathbf{D}\hat{\mathbf{s}}$.

3.2. Restoration algorithm

In this paper, we propose to adopt PDPnP to solve the constrained optimization problem in (6) [18]. Algorithm 1 shows the steps of PDPnP, where Steps 2 and 4 are modified from the original to be able to incorporate a generation process with **D**. $\mathfrak{G}_{\mathcal{R}}(\cdot, \sigma) : \mathbb{R}^L \to \mathbb{R}^L$ embeds processing to remove additive white Gaussian noise (AWGN) with standard deviation σ with a certain regularization $\mathcal{R}(\cdot)$. Symbols γ_1 and γ_2 are step size parameters, which should satisfy $\gamma_1 \gamma_2 (\sigma_1(\mathbf{L}))^2 \leq 1$, where $\mathbf{L} = (\mathbf{\Phi}^{\intercal} \quad \mathbf{\Psi}^{\intercal})^{\intercal}$ and $\sigma_1(\mathbf{L})$ is the maximum singular value of **L** [18].

The modified form of PDPnP aims to solve the problem

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x} \in \mathbb{R}^L} \mathcal{R}(\mathbf{x}) + \mathcal{F}_{\mathbf{v}}(\mathbf{\Phi}\mathbf{x}) \text{ s.t. } \mathbf{\Psi}\mathbf{x} \in C,$$
(7)



Fig. 2. Discrete model of coherence function defined by (2), where the standard deviation and angular frequency are set to $\sigma_{\rm p} = 8$, $\omega_{\rm p} = 0.5\pi$, and amplitude α is set so that $\sigma_1(\mathbf{P})$ is 1.

where $\mathcal{R}(\cdot) : \mathbb{R}^L \to [0, \infty)$ is a regularization term, $\mathcal{F}_{\mathbf{v}}(\cdot) : \mathbb{R}^M \to [0, \infty)$ is a data fidelity term, $\mathbf{\Phi} : \mathbb{R}^L \to \mathbb{R}^M$ is a linear measurement process, $\mathbf{\Psi} : \mathbb{R}^L \to \mathbb{R}^N$ is a linear generation process, and $C \in \mathbb{R}^N$ is a hard constraint. The conditions under which the modified PDPnP method can be applied are the same as the original [18].

3.3. Restoration model

In order to apply the PDPnP framework in (7) to the problem in (6), let $\mathbf{\Phi} = \mathbf{PD}, \mathbf{\Psi} = \mathbf{D}, \mathcal{F}_{\mathbf{v}}(\mathbf{y}) = (2\lambda)^{-1} ||\mathbf{y} - \mathbf{v}||_2^2$, and $\mathbf{x} = \mathbf{s}$. If synthesis dictionary \mathbf{D} satisfies Parseval tightness, i.e., $\mathbf{DD}^{\mathsf{T}} = \mathbf{I}$ [22, 23], then $\sigma_1(\mathbf{L}) = \sigma_1((\mathbf{P} \ \mathbf{D})) = \sigma_1((\mathbf{P} \ \mathbf{D}))$ holds, where \mathbf{I} is the identity matrix. In addition to orthonormal transforms, the undecimated Haar transform (UDHT) [24] and nonseparable oversampled lapped transform (NSOLT) [25] satisfy the Parseval tight condition. The metric projection of line 6 is given by $[P_C(\mathbf{x})]_n = [P_{[-1,1]^N}(\mathbf{x})]_n = \min\{\max\{[\mathbf{x}]_n, -1\}, 1\}$, where $[\cdot]_n$ denotes the *n*-th element of the argument vector.

Since the source data **u** is the reflectance distribution of specimen object such as bio-tissue, it is expected to be sparse by itself. It is also possible to sparsely represent the coefficient vector **s** further by using an appropriate synthesis dictionary **D** to exploit the spatial correlation of **u**. Assuming 1-norm regularizer $\mathcal{R}(\cdot) = \|\cdot\|_1$, the Gaussian denoiser $\mathfrak{G}_{\mathcal{R}}(\cdot, \sigma)$ results in

$$\left[\mathfrak{G}_{\parallel \cdot \parallel_1}(\mathbf{x},\sigma)\right]_n = \operatorname{sgn}([\mathbf{x}]_n) \max\left\{\left|[\mathbf{x}]_n\right| - \sigma^2, 0\right\}.$$
 (8)

With (8), the optimality of PDPnP is guaranteed.

As another configuration, it is also possible to plug a sophisticated Gaussian denoiser with regularization into $\mathfrak{G}_{\mathcal{R}}(\cdot, \sigma)$. Although explicit representation of the regularization term $\mathcal{R}(\cdot)$ is not always possible and the optimality may lose, high quality restoration can be expected. In Section 4, the performance is evaluated with BM4D [21] as $\mathfrak{G}_{\mathcal{R}}(\cdot, \sigma)$, as well as soft-thresholding in (8).

4. PERFORMANCE EVALUATION

In this section, let us verify the significance of the explicit regularization with dictionary \mathbf{D} and implicit one with BM4D through simulation for artificial data. Then, an experimental result for an observation array of the sensory epithelium of a guinea pig inner ear measured by the MS *en-face* OCT device is shown.

4.1. Restoration simulation

In order to verify the significance of the regularization, we compare the restoration results with different settings. An example of tomographic data \mathbf{u} is shown in Fig. 3 (a). This target source is assumed to be measured through the coherence function shown in Fig. 2. An example of observation volumetric data \mathbf{v} is shown in Fig. 3 (b).



Fig. 3. Example set of artificial volumetric arrays. (Top) YZ slice at the center of X. (Bottom) Z direction sequence at the center of XY. (a) Source **u** of $64 \times 64 \times 128$ voxels, where the reflective XY planes are randomly generated. The reflective plane generation rate is set to 0.06. The positions in the Z direction are set randomly by the uniform distribution of integers in [1, 128]. Each reflectance is randomly set by the uniform distribution of [-1, 1]. (b) Observation **v**. The function in Fig. 2 is set to the measurement process as **P**. AWGN of zero mean and 0.1 standard deviation is set as noise **w**.



Fig. 4. Simulation results of restoration. (Top) YZ slice at the center of X. (Bottom) Z direction sequence at the center of XY. The array in Fig. 3 (b) is used as observation data **v**, where $\gamma_1 = 0.01$, $\gamma_2 \simeq 47.62$, $\lambda = 0.005$ and \sharp of iterations is set to 200. "UDHT" stands for the undecimated Haar transform, and "SFTH" stands for soft-thresholding. For reference, denoising result with BM4D [21] is shown in (d), where the standard deviation of noise is set to $0.1(=\sqrt{\gamma_1})$.

Parameter settings: We use step-size parameters $\gamma_1 = \sigma_w^2 = 0.01$ and $\gamma_2 = 1/(1.05\gamma_1\tau^2) \simeq 47.62$, and set the number of iterations to 200. Under the condition that the synthesis dictionary **D** is Parseval tight and the maximum singular value $\sigma_1(\mathbf{P})$ of **P** is 1, we set $\tau = \sigma_1(\mathbf{L}) = \sqrt{\lambda_1(\mathbf{P}^{\mathsf{T}}\mathbf{P}) + 1} = \sqrt{2}$, where $\lambda_1(\mathbf{P}^{\mathsf{T}}\mathbf{P})$ is the maximum eigenvalue of $\mathbf{P}^{\mathsf{T}}\mathbf{P}$. In (2), parameter α is used for the normalization.

Synthesis dictionary and regularizer: In this simulation, we evaluate the performance on the combinations of dictionary D and



Fig. 5. PSNRs for various reflecting plane generation (RPG) rate. Ten trials are conducted for each case, where PSNR is calculated by using average MSE of whole trials. "IDNT" is the identity mapping, "UDHT" is the undecimated Haar transform, and "SFTH" denotes the soft-thresholding.

Gaussian denoiser $\mathfrak{G}_{\mathcal{R}}(\cdot, \sigma)$ as follows:

- Identity mapping and BM4D [21]
- UDHT [24] and soft-thresholding (8)
- Identity mapping and soft-thresholding (8)

UDHT is applied only to each XY slice of the 3-D volumetric data as a 2-D transform, where the number of tree levels is set to 3. λ was empirically set to 0.005.

Results and discussions: The restoration results of the simulation are summarized in Fig. 4. Fig. 4 (a) shows the highest performance among all the settings, although there is no guarantee to converge an optimal solution when using BM4D as a regularized Gaussian denoiser. The result using UDHT in Fig. 4 (b) shows higher restoration performance than that using the identity mapping in (c). It can be verified that an appropriate synthesis dictionary **D** helps improve quality. (d) shows a result of BM4D denoiser for reference.

Fig. 5 summarizes the restoration results of 10 trials under each condition by changing the reflection plane generation (RPG) rate. Peak signal to noise ratio (PSNR) is calculated by using average of total mean squared error (MSE) of all trials, where the values are biased and scaled from [-1, 1] to [0, 1]. The validity of PDPnP with BM4D is verified. Improvement in quality can be expected with soft-thresholding by selecting an appropriate synthesis dictionary **D**.

4.2. Restoration experiment

Fig. 6 shows an observation of the sensory epithelium of the guinea pig inner ear measured by an MS *en-face* OCT device, and Fig. 7 shows the result of tomographic data restored by PDPnP with the identity mapping and BM4D, where the model in Fig. 2 was set experimentally for the observation process **P**, and the same setting as the simulation in 4.1 is adopted. The data is of size $244 \times 240 \times 1024$. Although it has almost 60M voxels, sharp volumetric restoration is achieved by performing PDPnP.

5. CONCLUSIONS

In this study, we proposed an OCT volumetric data restoration method by using the PDPnP framework. We verified the significance of the proposed method by restoration simulation on artificial tomographic data. In addition, we conducted a restoration experiment on actual MS *en-face* OCT measurement data and verified



Fig. 6. Observation of sensory epithelium of a guinea pig inner ear measured by MS *en-face* OCT, where the intensity of all images is emphasized. (Top) Fifteen consecutive XY slices. (Center) YZ slice at the center of X. (Bottom) Z direction sequence at the center of XY. To remove the Z direction DC component, the output of the average filter of length 21 is subtracted from the original as preprocessing.



Fig. 7. Restored result in absolute value, where the intensity of all images is emphasized. (Top) Fifteen consecutive XY slices. (Center) YZ slice at the center of X. (Bottom) Z direction sequence at the center of XY. \sharp of iterations is set to 200.

that PDPnP is a practical method for high dimensional data. We are planning further study, such as estimation of measurement process, construction of suitable synthesis dictionary and setting of noise model, as well as improvement of MS *en-face* OCT acquisition.

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