# DIRECTIVITY SYNTHESIS WITH MULTIPOLES COMPRISING A CLUSTER OF FOCUSED SOURCES USING A LINEAR LOUDSPEAKER ARRAY

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# ABSTRACT

A method to create multipoles comprising a cluster of focused sources by using a linear loudspeaker array has recently been investigated. Directivities in a listening area were confirmed with examples of primitive multipoles such as dipoles and quadrupoles. This paper describes a method to create a sound source having more complex directivity by using a superposition of multipoles comprising a collection of focused sources. An analytical method is also described with which coefficients can be obtained for each multipole by circular harmonic expansion of a sound field created by a directional sound source and Taylor expansion of the corresponding sound field. Simulation results show that a superposition of multipoles based on analytical conversion introduces desired directivities to the sound sources created in the listening area by a linear loudspeaker array.

*Index Terms*— Multipole Expansion, Focused Source, Wave Field Synthesis, Directivity Control

# 1. INTRODUCTION

Holophonic sound systems like Ambisonics [1][2] or Wave Field Synthesis (WFS) [3][4] are a key to providing high reality to audiences in theaters and gamers at home since they enable complex sound scenes to be achieved with freely movable acoustic sources. In recent live events, focused source methods [5][6] have been used to create sound sources near audience seats via loudspeakers. The methods enable closeup sound effects to be provided to audiences [7]. These sound sources are becoming increasingly important to today's audiences.

It is known that sounds with faithful reproduction of directivities help audience feel more realistic sensations [8]. Several methods have already been proposed to create directional sound sources including focused sources. Among them are multi-microphone recordings that are mapped to directional sources reproduced on a WFS setup [9], an equalization technique based on spherical harmonics [10], and an analytical method based on the Rayleigh I integral and circular harmonic expansion of the sound field created by directional sound sources [11][12][13]. Despite their accuracies in reproducing sound fields, their computational complexities are relatively large for real-time rendering. Methods for real-time rendering have also been proposed in the literature. However, they require either a sacrifice of memory consumption for pre-defined filters [10] or recalculations of the time domain driving functions [11] every time the directional sound source changes its orientation, which is often the case in scenarios where musicians or speakers frequently turn around. Modelbased methods that directly deal with directional sources have also been reported; however, they focus more on introducing directivity to the transfer functions than on creating directional sound sources [14][15]. A method of creating directional sources on the basis of multipoles comprising a cluster of focused sources was proposed as a low complexity approach [16]. Since the method deals with changes in directivities by controlling the coordinates of focused sources, it handles the changes without additional complexity. However, the way in which it reproduces complex directivities has not yet been investigated.

In this paper, we will describe a method for creating directional sound sources based on a superposition of multipoles comprising multiple focused sources. We will also describe an analytical conversion method in which circular harmonics of desired sound fields are derived into coefficients for each multipole. Simulation results will be presented to confirm the methods' effectiveness in reproducing directivities and the sound fields of directional sources.

# 2. SUPERPOSITION OF MULTIPOLES COMPRISING FOCUSED SOURCES

It has been confirmed that focused sources created infinitesimally close to each other constituted multipoles [16]. To reproduce complex directivities based on a superposition of multipoles, it is necessary to calculate the coefficients for each multipole. Several methods have been proposed to obtain coefficients for multipoles: a method based on the least square method [17], a method based on inner products of directional patterns and pre-defined orthonormal bases [18], a method of fitting by linear regression [19], and a method using measured characteristics [20]. Analytical conversion methods in the



**Fig. 1**. Dipole along the y- **Fig. 2**. Directivity of dipole axis source.



**Fig. 3**. Concept of multipole sound source comprising a collection of focused sources.

spherical harmonics, circular harmonics, and angular spectra domains have been proposed as means to provide the fastest solution to obtain coefficients for each mode [21][22]. However, no analytical method to obtain coefficients for each multipole has yet been reported. To address this issue, we derived an analytical method to calculate the coefficients for each multipole from circular harmonic coefficients of sound fields created by a directional source.

## 2.1. Multipoles

Sound radiation from a source located at the origin of a coordinate system can be characterized by a superposition of multipoles. Each multipole constitutes a collection of point sources infinitesimally close to each other with opposite phases [23]. A simple example of a multipole is the dipole shown in Fig. 1. An example of the directivity of a dipole along the y-axis is shown in Fig. 2. In the work we report in this paper, we focused on sound field reproduction in a 2D space. A sound field created by a superposition of multipoles is given by the following equation [20][23][24].

$$S(\mathbf{r},\omega) = \sum_{m,n} d_{m,n} \cdot \frac{\partial^{m+n}}{\partial x^m \partial y^n} G(\mathbf{r}), \qquad (1)$$

where  $\omega$  denotes angular frequency, d denotes a coefficient for a multipole,  $G(\mathbf{r})$  denotes the Green's function of a free field wave equation, and m, n are non-negative integers. As can be seen from (1), higher order multipoles can be obtained by higher order derivatives of the Green's function.



**Fig. 4**. Reproduced sound field and sound pressure level in dB for monopole: (a), (b), quadrupoles: (c), (d), tilted quadrupoles ( $\phi = 60^{\circ}$ ): (e), (f).

# 2.2. Focused sources in wave field synthesis

WFS aims at reproducing arbitrary sound fields using secondary sources. Typical implementations use linear distribution of secondary sources for reproducing planar sound field based on the Rayleigh I integral. If it is assumed that secondary sources are distributed along the x-axis, the driving function is given by

$$D_{2D}(\mathbf{x}_0,\omega) = \left. 2 \frac{\partial}{\partial y} S(\mathbf{x},\omega) \right|_{\mathbf{x}=\mathbf{x}_0},\tag{2}$$

where  $S(\mathbf{x}, \omega)$  denotes the desired wave field to be reproduced. Here,  $\mathbf{x} = (x, y)$  with (y > 0) and  $\mathbf{x}_0 = (x_0, 0)$ are secondary source positions. In the following discussion,  $\omega$  will be omitted. WFS creates virtual point sources between loudspeaker arrays and audiences. These sources are known as focused sources due to the relation they have with acoustic focusing. Focused source method assumes an acoustic sink defined at the position of a virtual point source [6]. By introducing the sound field created by the acoustic sink into (2), the driving function to reproduce the sound field of a focused source yields

$$D(\mathbf{x}_0, \mathbf{x}_s) = -\frac{jk}{2} c_0 \frac{y_0 - y_s}{|\mathbf{x}_0 - \mathbf{x}_s|} H_1^{(1)}(k |\mathbf{x}_0 - \mathbf{x}_s|), \quad (3)$$

where k denotes wave number,  $j = \sqrt{-1}$ ,  $H_1^{(1)}$  denotes the Hankel function of the first kind of the first order,  $\mathbf{x}_s = (x_s, y_s)$  denotes the position of the focused source for  $y_s > 0$ ,  $c_0 = \sqrt{2\pi |y_{ref} - y_0|}$  is an amplitude normalization factor, and  $y_{ref}$  is the distance of the reference line from secondary sources. As Wierstorf et al. indicated [6], the inverse Fourier transform of (3) provides efficient time domain implementation.

#### 2.3. Multipoles comprising focused sources

As shown in Fig. 3, clusters of focused sources infinitesimally close to each other function as multipoles [16]. Since a superposition of sound fields created by each focused source provides the sound field of a multipole, the driving function for a loudspeaker positioned at  $\mathbf{x}_0$  is given as

$$D(\mathbf{x}_0) = \sum_{i=0}^{N-1} g_s^{(i)} D(\mathbf{x}_0, \mathbf{x}_s^{(i)}),$$
(4)

where  $q_s^{(i)}$  is the phase of the i-th focused source  $\mathbf{x}_s^{(i)}$  in the multipole, and N is the number of focused sources in the multipole. In case of a dipole along the y-axis, the parameters in (4) are given as follows: N = 2,  $(g_s^{(0)}, g_s^{(1)}) = (-1, 1)$ ,  $\mathbf{x}_{s}^{(0,1)} = (x_m, y_m \mp \Delta)$ , where  $\mathbf{x}_m = (x_m, y_m)$  is the coordinate of the center of a multipole, and  $\Delta = d/2$ , i.e., half of the distance between mono-sources. Fig. 4 shows simulation results obtained for reproduced sound fields of a monopole and a quadrupoles created by a linear loudspeaker array centered at the origin of the coordinate along the x-axis. The speed of sound was set as 340 m/s and the sound used for the simulation was a 1-kHz monochromatic sine wave. Forty-one loudspeakers centered at the origin at 0.1 m intervals were arranged along the x-axis. As compared with the monopole case shown in Fig. 4 (a) and (b) and the quadrupole cases shown in Fig. 4 (c) and (d), the sound pressure levels around the quadrupoles exhibited null points in the direction perpendicular to the quadrupoles. By changing the coordinates of focused sources as  $\mathbf{x}_s^{(0,1)} = (x_m \mp \Delta sin\phi, y_m \mp \Delta cos\phi)$ , it was observed that the quadrupole directions lean 30° clockwise as shown in Fig. 4 (e) and (f).

#### 2.4. Conversion of circular harmonics

In Cartesian coordinates in a 2D space, a sound field at the point of  $x = (cos\alpha, sin\alpha)$  on a unit circle centered at the origin of the coordinates can be expressed on the basis of Taylor series expansion at the origin as

$$S(\mathbf{x}) = \sum_{m+n=0}^{\infty} \sum_{m=0}^{m+n} \frac{\partial^{m+n} S(0)}{\partial x^m \partial y^n} \cdot \frac{\cos^m \alpha \cdot \sin^n \alpha}{m! \cdot n!}.$$
 (5)

The same sound field can be expressed with circular harmonic expansion as

$$S(\mathbf{x}) = \sum_{\nu = -\infty}^{\infty} \check{S}_{\nu}^{(2)}(\omega) H_{\nu}^{(2)}(kr) e^{j\nu\alpha}, \qquad (6)$$

where  $\check{S}_{\nu}^{(2)}$  denotes a circular harmonic coefficient of the  $\nu$ th order and  $H_{\nu}^{(2)}$  is the Hankel function of the second kind of the  $\nu$ -th order. Inserting Eular's formula and applying binomial expansion (6) yields

$$S(\mathbf{x}) = \breve{S}^{(2)}(0)H_0^{(2)}(k) + \sum_{m+n=1}^{\infty} \sum_{m=0}^{m+n} j^n \cdot \binom{m+n}{m} \breve{S}_{m+n}^{(2)} H_{m+n}^{(2)} \cdot \cos^m \alpha \cdot \sin^n \alpha + \sum_{m+n=1}^{\infty} \sum_{m=0}^{m+n} (-j)^n \cdot \binom{m+n}{m} \breve{S}_{-m-n}^{(2)} H_{-m-n}^{(2)} \cdot \cos^m \alpha \cdot \sin^n \alpha.$$
(7)

By comparing the coefficients of  $cos^m \alpha \cdot sin^n \alpha$  in (5) and (7), we obtain the partial derivatives as

$$\frac{\partial^{m+n} S(0)}{\partial x^m \partial y^n} = j^n \cdot (m+n)! \\ \cdot \left\{ \breve{S}_{m+n}^{(2)} \cdot H_{m+n}^{(2)} + (-1)^n \cdot \breve{S}_{-m-n}^{(2)} \cdot H_{-m-n}^{(2)} \right\}.$$
(8)

These partial derivatives can be used as coefficients for a superposition of multipoles.

## 2.5. Directivity synthesis by multipole superposition

The concept of a superposition of multipoles is shown in Fig. 5. By using coefficients converted from circular harmonic coefficients as in (8), arbitrary sound fields can be reproduced on the basis of a superposition of multipoles as in (1). The driving function is given as,

$$D(\mathbf{x}_{0}) = \sum_{\substack{m,n \\ g_{s}^{m,n} \in X^{m,n} \\ g_{s}^{m,n} \in G^{m,n}}} \sum_{\substack{\partial m^{+n} S(0) \\ \partial x^{m} \partial y^{n}}} \cdot \frac{g_{s}^{m,n}}{(jdk)^{m+n}} \cdot \frac{D(\mathbf{x}_{0}, \mathbf{x}_{s}^{m,n})}{m! \cdot n!}.$$
(9)

where D is the driving function given in (3),  $\mathbf{x}_s^{m,n}$  denotes the position of each focused source, and  $g_s^{m,n}$  denotes the phase of the corresponding focused source. The positions of each focused source and phase can be obtained by the following recursive relations with an initial value of  $x^0 = x_s, y^0 = y_s, g^0 = 1$ .

$$X^{m,n} = \{(x_m, y_m) | x^m = x^{m-1} \pm \Delta, y^n = y^{n-1} \pm \Delta\},\$$
  
$$G_s^{m,n} = \{g^{m,n} = g^m \cdot g^n | g^\nu = (\pm 1) \cdot g^{\nu-1}\}.$$

### 3. SIMULATION

In simulation experiments, we first compared directivity patterns reproduced by multipole superposition with the weights



**Fig. 5**. Concept of directivity synthesis by using superposition of multipoles.

given in (8). Sound fields along a unit circle centered at the origin were calculated with (5) and (6); the normalized amplitudes are shown in Fig. 6 (a) and (b). For both cases, we used randomly generated numbers for circular harmonics  $\check{S}^{(2)}$ . The order of circular harmonics was chosen as N = 4, and this number was also applied to partial derivatives of (5). As can be seen from Fig. 6, reproduced directivities by (5) deviated from the ones by (6). This is because the contributions of higher order derivatives are discontinued. Finally, we calculated sound fields created by a linear loudspeakers and the driving function in (9). The simulation conditions were the same as those given in subsection 2.3. The sound pressure levels of the sound fields are shown in Fig. 7 (a) and (b). The circular harmonic values used for the simulation were the same as those in Fig. 6 (a) and (b). As shown in Fig. 7 (a) and (b), only the area of  $y \ge y_s (= 1m)$  was reproduced as we had expected. In addition, only the range of  $30^{\circ} < \phi < 150^{\circ}$  was consistent with the patterns shown in Fig. 6 (a) and (b). One reason for this is due to the nature of focused sources. Until radiated sounds from secondary sources pass through the focal point, they converge toward the focal point in the area of  $y < y_s$ . It can therefore be considered that radiated sounds from the focused source do not correspond to the directivities in the area of  $y < y_s$  [6]. Another reason is that a linear array with finite length makes the listening area of focused sources smaller [6]. Taking these factors into account, we conclude that the proposed method synthesized directivities in the listening area in the way we had expected.

## 4. CONCLUSION

In this paper, we described a method we propose for synthesizing directivities on the basis of a superposition of multipoles. We also described an analytical method to obtain coefficients for each multipole by conversion from circular harmonics. To derive the method, we made use of two factors. The first was that multipole expansion is an operation equivalent to Taylor expansion at the position of a sound source. Therefore partial derivatives correspond to the coefficients for each multipole. The Second was that both series obtained by circular harmonic expansion and Taylor expansion are expressed as coefficients of orthogonal bases comprising combi-



**Fig. 6**. Two examples of reproduced directivity patterns. Dotted line: circular harmonics by (6); Solid line: partial derivative by (5).



**Fig. 7**. Sound pressure level. (a): created by sound of directivity of Fig. 6 (a). (b): by directivity of Fig. 6 (b).

nations of trigonometric functions along the unit circle. Comparing these coefficients enabled us to derive analytical conversions. Simulations based on the proposed methods confirmed that the sound field of a directional source could be reproduced in the listening area of focused sources.

# 5. RELATION TO PRIOR WORK

We have already proposed a method to create directional sources based on primitive multipoles comprising a cluster of focused sources [16]; however, it can deal with only simple directivities. The method proposed in this paper provides the previously proposed method with a functionality to deal with complex directivities more efficiently than conventional methods [17][18][19]. We derived an analytical conversion from circular harmonics of a directional sound source to the coefficients for each multipole. Although analytical methods have already been proposed [21][22], they only deal with spherical harmonics, circular harmonics, and angular spectra. Therefore, they cannot be applied to directivity synthesis based on superposition of multipoles. The proposed method converts circular harmonics to coefficients for each multipole, thereby efficiently synthesizing complex directivities on the basis of a superposition of multipoles comprising a cluster of focused sources by a linear loudspeaker array.

#### 6. REFERENCES

- M. A. Gerzon, "Periphony: With-height sound reproduction," J. Audio Eng. Soc, vol. 21, no. 1, pp. 2–10, 1973.
- [2] M. A. Poletti, "Three-dimensional surround sound systems based on spherical harmonics," J. Audio Eng. Soc, vol. 53, no. 11, pp. 1004–1025, 2005.
- [3] A. J. Berkhout, D. de Vries, and P. Vogel, "Acoustic control by wave field synthesis," *Acoustical Society of America Journal*, vol. 93, pp. 2764–2778, May 1993.
- [4] J. Ahrens, R. Rabenstein, and S. Spors, "The theory of wave field synthesis revisited," in *Audio Engineering Society Convention* 124, May 2008.
- [5] S. Spors, H. Wierstorf, M. Geier, and J. Ahrens, "Physical and perceptual properties of focused virtual sources in wave field synthesis," in *Audio Engineering Society Convention 127*, Oct 2009.
- [6] H. Wierstorf, A. Raake, M. Geier, and S. Spors, "Perception of focused sources in wave field synthesis," J. Audio Eng. Soc, vol. 61, no. 1/2, pp. 5–16, 2013.
- [7] K. Tsutsumi and H. Takada, "Powerful sound effects at audience seats by wave field synthesis," *NTT Technical Review*, vol. 15, no. 12, 2017.
- [8] K. Maki, T. Kimura, and M. Katsumoto, "Reproduction of real musical instrument sounds by a spherical loudspeaker with 26 transducers: Acoustic comparison on reproducibility of sound radiation patterns of musical instruments," *Journal of the Acoustic Society of Japan*, vol. 67, no. 10, pp. 447–458, oct 2011.
- [9] B. Albrecht, D. de Vries, R. Jacques, and F. Melchior, "An approach for multichannel recording and reproduction of sound source directivity," in *Audio Engineering Society Convention 119*, Oct 2005.
- [10] E. Corteel, "Synthesis of directional sources using wave field synthesis, possibilities, and limitations," *EURASIP Journal on Advances in Signal Processing*, vol. 2007, no. 1, pp. 090509, Feb 2007.
- [11] J. Ahrens and S. Spors, "Implementation of directional sources inwave field synthesis," in 2007 IEEE Workshop on Applications of Signal Processing to Audio and Acoustics, Oct 2007, pp. 66–69.
- [12] J. Ahrens and S. Spors, "Rendering of virtual sound sources with arbitrary directivity in higher order ambisonics," in *Audio Engineering Society Convention* 123, Oct 2007.
- [13] J. Ahrens and S. Spors, "Notes on rendering of focused directional sound sources in wave field synthesis," in 34rd German Annual Conference on Acoustics (DAGA), Dresden, Germany, Mar. 2008.

- [14] J. Ahrens and S. Spors, "An analytical approach to 2.5d sound field reproduction employing linear distributions of non-omnidirectional loudspeakers," in 2010 IEEE International Conference on Acoustics, Speech and Signal Processing, March 2010, pp. 105–108.
- [15] D. de Vries, "Sound reinforcement by wavefield synthesis: Adaptation of the synthesis operator to the loudspeaker directivity characteristics," *J. Audio Eng. Soc*, vol. 44, no. 12, pp. 1120–1131, 1996.
- [16] K. Tsutsumi, H. Takada, and Y. Haneda, "Directivity control with multi-poles comprising plural focused sources in wave field synthesis," in *Proc. Autumn meeting of the Acoustic Society of Japan*, Matsuyama, Japan, 2017, pp. 469–470.
- [17] Y. Haneda, K. Furuya, and H. Itou, "Design of multipole loudspeaker array based on spherical harmonic expansion," in 2011 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), May 2011, pp. 141–144.
- [18] M. Eichler and A. Lacroix, "Broadband superdirective beamforming using multipole superposition," in 2008 16th European Signal Processing Conference, Aug 2008, pp. 1–5.
- [19] M. Noisternig and B. F. G. Katz, "Reconstructing sound source directivity in virtual acoustic environments," in *Int. Workshop on the Principles and Applications of Spatial Hearing (IWPASH)*, Miyagi, ZAO, Japan, Nov 2009.
- [20] S. Koyama, K. Furuya, Y. Hiwasaki, and Y. Haneda, "Sound field reproduction method in spatio-temporal frequency domain considering directivity of loudspeakers," in *Audio Engineering Society Convention 132*, Apr 2012.
- [21] J. Ahrens and S. Spors, "Wave field synthesis of a sound field described by spherical harmonics expansion coefficients," *The Journal of the Acoustical Society of America*, vol. 131, no. 3, pp. 2190–2199, 2012.
- [22] T. Okamoto, "2.5d higher order ambisonics for a sound field described by angular spectrum coefficients," in 2016 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), March 2016, pp. 326–330.
- [23] G. Williams, Fourier Acoustics: Sound Radiation and Nearfield Acoustical Holography, Elsevier Science, 1999.
- [24] P.A. Nelson and S.J. Elliott, Active Control of Sound, Academic Press, 1992.