3D EXTERIOR SOUNDFIELD REPRODUCTION USING A PLANAR LOUDSPEAKER ARRAY

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ABSTRACT

In this paper, we propose a planar array of dipole (or first-order) loudspeakers to reproduce a full three dimensional (3D) exterior soundfield over a desired spatial region. The proposed method is inspired by the spherical harmonics based solution for 3D soundfield reproduction. When decomposed in terms of spherical harmonics, we separate the corresponding soundfield coefficients into two sets (even and odd) and exploit the inherent properties of Legendre polynomials to control each set using a planar array of dipole (or first-order) speakers. We provide simulation examples to demonstrate the performance of the proposed method.

1. INTRODUCTION

Three dimensional soundfield reproduction is a well researched topic due to its plethora of applications such as home-theatre systems, in-car audio systems, communication devices, and noise cancellation. There exists a number of soundfield reproduction methods such as amplitude panning [1–4], wavefield synthesis [5–8], Ambisonics [9, 10], and higher-order Ambisonics (HOA) [11–14].

In this paper, we formulate the problem using higher-order Ambisonics in the spherical harmonics domain. Typically, HOA reproduction requires a 3D array of omnidirectional loudspeakers distributed over a spherical surface. The size of the array depends on the frequency range and the reproduction area of interest. The placement of loudspeakers on a spherical array has to follow a strict rule of orthogonality of the spherical harmonics, which limits the array flexibility. Furthermore, the spherical shape poses difficulties in implementation and integration into existing audio devices.

To overcome the limitations of spherical loudspeaker arrays, we propose a compact planar array of dipole (or fist-order) loudspeakers capable of fully controlling a 3D exterior soundfield. When developing this design, we show that a circular omnidirectional aperture can only control the even mode soundfield coefficients whereas a circular dipole aperture can control the remaining soundfield modes. Based on similar concepts, an equivalent design for 3D soundfield recording using a planar array of microphones was recently published in [15–17].

We believe the proposed loudspeaker design is largely suitable for a number of commercial products including the recently introduced Amazon Echo and Google Home, to produce directional outgoing soundfields. Due to limitations in realizing dipole loudspeaker excitation patterns at low frequencies, the proposed method is mostly suitable for mid-high frequencies.

2. SUMMARY OF SPHERICAL HARMONICS BASED 3D EXTERIOR SOUNDFIELD REPRODUCTION

2.1. Spherical harmonic decomposition of an exterior soundfield

Sound waves are space-time-frequency signals that are primarily defined by the wave equation. A comprehensive solution to the wave equation can be expressed in terms of spherical harmonic functions, which are orthogonal spatial basis functions capable of successfully decomposing any arbitrary function on the sphere. For an exterior (or outgoing) soundfield, the spherical harmonics based solution is in the form of [18]

$$p^E(\boldsymbol{x},k) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \beta_{nm}(k) h_n(kr_x) Y_{nm}(\theta_x,\phi_x) \qquad (1)$$

where $\boldsymbol{x} = (r_x, \theta_x, \phi_x)$ is an arbitrary observation point in the region of interest, $Y_{nm}(\theta_x, \phi_x)$ are the spherical harmonics, $\beta_{nm}(k)$ are the corresponding spherical harmonic coefficients, $k = \frac{2f\pi}{c}$ is the wave number with f denoting the frequency and c denoting the speed of propagation at room temperature, and $h_n(kr_x)$ is the spherical Hankel function of the first kind. The spherical harmonic functions are defined by

$$Y_{nm}(\theta,\phi) = P_{n|m|}(cos(\theta))\frac{1}{\sqrt{2\pi}}e^{im\phi} \qquad (2)$$

where $P_{n|m|}(cos(\frac{\pi}{2})) \triangleq \sqrt{\frac{(2n+1)(n-|m|)!}{4\pi(n+|m|)!}} P_{n|m|}(cos(\frac{\pi}{2}))$ is the normalized associated Legendre polynomial with $P_{n|m|}(cos(\frac{\pi}{2}))$ being the associated Legendre polynomials.

Typically, the infinite summation in (1) can be truncated at $N = \lceil kR \rceil$ where R is the distance from origin to the furthest source causing the exterior/outgoing soundfield. Note that the spherical harmonic coefficients $\beta_{nm}(k)$ are independent of the observation position. Therefore, by reproducing these coefficients, it is possible to reproduce an entire spatial soundfield.

2.2. Exterior Sound-field Reproduction using a spherical loudspeaker array

Three dimensional soundfield reproduction is a well researched topic in the past decade [19, 20] . Spherical harmonics based solutions for the problem have been proposed for both interior [11] and exterior [21] soundfield reproduction. In [12], the authors proposed a spherical array design for interior soundfield reproduction using the continuous loudspeaker array concept. It first assumes a continuous spherical loudspeaker array, which is then matched with the desired soundfield in the spherical harmonics domain. The array is realized using an equivalent discrete loudspeaker array. Our proposed solution is also based on the above concept, but on the $X-Y$ plane.

For ease of understanding, we summarize the spherical loudspeaker array design for exterior soundfield reproduction using the continuous loudspeaker concept. Consider an N^{th} order desired soundfield of the form (1), with $\beta_{nm}(k)$ representing the desired soundfield coefficients.

Let us assume a continuous aperture function $\rho(\theta,\phi,k)$ distributed over a spherical surface of radius R. Thus, the reproduced exterior soundfield as observed at any arbitrary point $\boldsymbol{x} = (r_x, \theta_x, \phi_x)$ can be expressed as [18]

$$p(\boldsymbol{x},k) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} ik \int_s \rho(\theta,\phi,k) j_n(kR) Y^*_{nm}(\theta,\phi) \cdots h_n(kr_x) Y_{nm}(\theta_x,\phi_x) ds \tag{3}$$

where $\int_s ds$ denotes the surface integral over a sphere and (R,θ,ϕ) is used to represent the loudspeaker position.

Since the aperture function $\rho(\theta,\phi,k)$ is a spherical function itself, we use the spherical harmonic expansion to write

$$\rho(\theta,\phi,k) = \sum_{p=0}^{\infty}\sum_{q=-p}^{p} \alpha_{pq}(k) Y_{pq}(\theta,\phi) \tag{4}$$

where $\alpha_{pq}(k)$ are the spherical harmonic coefficients defining the aperture function. By substituting (4) into (3), the reproduced soundfield becomes

$$p(\boldsymbol{x},k) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} ik \int_s \sum_{p=0}^{\infty}\sum_{q=-p}^{p} \alpha_{pq}(k) Y_{pq}(\theta,\phi) \ldots j_n(kR) Y^*_{nm}(\theta,\phi) h_n(kr_x) Y_{nm}(\theta_x,\phi_x) ds \tag{5}$$

Utilizing the orthogonal property of spherical harmonic functions, the integral in (5) can be evaluated to derive

$$p(\boldsymbol{x},k) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} ik\alpha_{nm}(k) j_n(kR) h_n(kr_x) Y_{nm}(\theta_x,\phi_x) \tag{6}$$

When comparing the reproduced field (6) and the desired soundfield (1), it is evident that

$$\alpha_{nm}(k) = \frac{-i\beta_{nm}(k)}{kj_n(kR)} \tag{7}$$

By Substituting (7) into (4), and by letting $p = n$ and $q = m$, the desired aperture function can be derived as

$$\rho(\theta,\phi,k) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \frac{-i\beta_{nm}(k)}{kj_n(kR)} Y_{nm}(\theta,\phi) \tag{8}$$

This aperture function can be realized via an equivalent discrete loudspeaker array by sampling the spherical surface. Based on previous work in spherical harmonic based soundfield recording [11, 22] and reproduction [11], if we have $Q \geq (N+1)^2$ equally spaced points (loudspeakers) on the spherical surface, then we can exactly reproduce $\rho(\theta,\phi,k)$ from its samples $\rho(\theta_q,\phi_q,k)$, $q = 1,\cdots,Q$. This leads to design the q^{th} loudspeaker weight as

$$w_q = \rho(\theta_q,\phi_q,k) A_q \tag{9}$$

where A_q is the compensation factor for sampling.

3. 3D EXTERIOR SOUNDFIELD REPRODUCTION USING A PLANAR ARRAY OF DIPOLE LOUDSPEAKERS

In this section, we propose a novel planar array of dipole loudspeakers to produce the full 3D soundfield. That is, we propose a compact planar aperture that is still capable of producing all the desired soundfield coefficients.

Let us observe the relationship (8) between the aperture function and the desired soundfield coefficients on the $X-Y$ plane as

$$\rho(\frac{\pi}{2},\phi,k) = \sum_{n=0}^{N}\sum_{m=-n}^{n} \frac{-i\beta_{nm}(k)}{kj_n(kR)} P_{n|m|}(cos(\frac{\pi}{2})) \frac{1}{\sqrt{2\pi}} e^{im\phi} \tag{10}$$

Note that the spherical harmonics are expanded using the definition given in (2). Next we discuss the inherent properties of the above relationship as the spherical harmonic order n and mode m varies.

3.1. Reproducing the even (when $|n+m|$) modes

We notice that when $|n+m|$ is odd the Legendre polynomial $P_{n|m|}(cos(\frac{\pi}{2}))$ in (10) is zero and and when $|n+m|$ is even it is non-zero. Therefore a circular aperture of omnidirectional loudspeakers on the $X-Y$ plane can only produce even modes of the desired soundfield. To simplify the relationship between the aperture and the even modes, lets us multiply (10) by $e^{-im'\phi}$ and integrate it over ϕ. This results in

$$\int_0^{2\pi} \rho(\frac{\pi}{2},\phi,k) e^{-im'\phi} d\phi = \sum_{n=0}^{N}\sum_{m=-n}^{n} \ldots \frac{-i\beta_{nm}(k)}{kj_n(kR)} P_{n|m|}(cos(\frac{\pi}{2})) \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} e^{im\phi} e^{-im'\phi} d\phi \tag{11}$$

By applying the orthogonality property of complex exponential (11) simplifies to,

$$\int_0^{2\pi} \rho(\frac{\pi}{2},\phi,k) e^{-im\phi} d\phi = \sum_{n=|m|}^{N} \frac{-i\beta_{nm}(k)}{kj_n(kR)} \ldots P_{n|m|}(cos(\frac{\pi}{2}))\sqrt{2\pi} \tag{12}$$

This is an interesting result as it relates the cumulative output of the aperture function and the desired even modal components with mode m. It is apparent that the right hand side of (12) may sometimes be a summation of desired coefficients (for example, for a third order desired soundfield when $m = 0$, the right hand side is $\beta_{00}(k) + \beta_{20}(k)$). At such instances, the aperture function is unable to individually control each mode. Therefore, to enable the loudspeaker array to control each desired coefficient individually, it is required to have multiple circular arrays at different radii, depending on the order of the desired soundfield. For the same example given above, it is suitable to use 2 circular apertures, where the smaller aperture only controls the desired coefficients up to the first order while the bigger aperture controls the remaining coefficients. In general, the minimum number of circles required is $L \geq \lceil N/2 \rceil$.

3.2. Case when n+m is odd

As mentioned earlier when $|n+m|$ is odd, $P_{n|m|}(cos(\frac{\pi}{2})) = 0$, thus the corresponding soundfield coefficients can not be controlled.

However, if we consider a dipole aperture function equivalent to the derivative $d\rho(\frac{\pi}{2},\phi,k)/d\theta$ on the $X-Y$ plane

$$\rho'(\frac{\pi}{2},\phi,k)=\sum_{n=0}^{N}\sum_{m=-n}^{n}\frac{-i\beta_{nm}(k)}{kj_n(kR)}P'_{n|m|}(cos(\frac{\pi}{2}))\frac{1}{\sqrt{2\pi}}e^{im\phi} \tag{13}$$

we observe that the derivative of associate Legendre polynomial $P'_{n|m|}(cos(\frac{\pi}{2}))$ is non-zero when $|n+m|$ is odd and zero when $|n+m|$ is even. Therefore by producing a dipole aperture function of the form $\rho'(\frac{\pi}{2},\phi,k)$, it is possible to reproduce the odd modes of the desired soundfield.

The normalized associate Legendre polynomial has the following relationship with its differential equation.

$$P'_{n|m|}(cos(\frac{\pi}{2}))=\sqrt{\frac{(2n+1)(n^2-m^2)}{2n-1}}P_{(n-1)|m|}(cos(\frac{\pi}{2})) \tag{14}$$

Therefore, (13) simplifies to

$$\rho'(\frac{\pi}{2},\phi,k)=\sum_{n=0}^{N}\sum_{m=-n}^{n}\frac{-i\beta_{nm}(k)}{kj_n(kR)}\sqrt{\frac{(2n+1)(n^2-m^2)}{2n-1}}\dots$$
$$P_{(n-1)|m|}(cos(\frac{\pi}{2}))\frac{1}{\sqrt{2\pi}}e^{im\phi} \tag{15}$$

In order to derive a closer relationship with each desired soundfield mode and the aperture function, we multiply (15) by $e^{-im'\phi}$ and integrate it over ϕ. This results in

$$\int_0^{2\pi}\rho'(\frac{\pi}{2},\phi,k)e^{-im\phi}d\phi=\sum_{n=|m|}^{N}\frac{-i\beta_{nm}(k)}{kj_n(kR)}\dots$$
$$\sqrt{\frac{(2n+1)(n^2-m^2)}{2n-1}}P_{(n-1)|m|}(cos(\frac{\pi}{2}))\sqrt{2\pi} \tag{16}$$

This is the direct relationship with the aperture function and the desired odd mode coefficients. Similar to the even mode case, in order to control each soundfield mode individually, multiple circular apertures of the form $rho'(\frac{\pi}{2},\phi,k)$ are required and the minimum number of circles required is $L\geq\lceil N/2\rceil$.

3.3. Summary of the aperture function requirement

In summary, the aperture function at each ring has the following relationship with the desired soundfield coefficients.
soundfield coefficients.
When $|n+m|$ is even,

$$\int_0^{2\pi}\rho(\frac{\pi}{2},\phi,k,R_\ell)e^{-im\phi}d\phi=\sum_{n=|m|}^{N}\frac{-i\beta_{nm}(k)}{kj_n(kR_\ell)}\dots$$
$$P_{n|m|}(cos(\frac{\pi}{2}))\sqrt{2\pi} \tag{17}$$

where R_ℓ ($\ell=0,\dots L$) is the radius of each concentric ring. When $|n+m|$ is odd,

$$\int_0^{2\pi}\rho'(\frac{\pi}{2},\phi,k,R_\ell)e^{-im\phi}d\phi=\sum_{n=|m|}^{N}\frac{-i\beta_{nm}(k)}{kj_n(kR_\ell)}\dots$$
$$\sqrt{\frac{(2n+1)(n^2-m^2)}{2n-1}}P_{(n-1)|m|}(cos(\frac{\pi}{2}))\sqrt{2\pi}. \tag{18}$$

4. DISCRETE LOUDSPEAKER ARRAY DESIGN

So far, we have assumed theoretical continuous aperture functions of omnidirectional and dipole loudspeaker arrays. In this section we discuss a method to (i) realize continuous apertures using discrete loudspeaker arrays and (ii) realize dipole loudspeakers (or aperture derivative) using a vertical pair of omnidirectional loudspeakers.

4.1. Spatial sampling of the continuous aperture

The discretization of the continuous loudspeaker is modeled by sampling the loudspeaker aperture functions $\rho(\frac{\pi}{2},\phi,k)$ and $\rho'(\frac{\pi}{2},\phi,k)$. Note that they are both periodic functions of ϕ with period 2π. Since our desired soundfield is mode-limited to N, according to Shannon's sampling theorem [23], we can exactly reproduce $\rho(\frac{\pi}{2},\phi,k)$ and $\rho'(\frac{\pi}{2},\phi,k)$ by its samples, if the number of samples is greater than $2N+1$. Therefore, each circular aperture can be sampled by $Q\geq 2N'+1$ samples where $N'\leq N$ is the maximum order of the desired soundfield order controlled by the circular array of interest.

4.2. Dipole loudspeakers for realizing the aperture derivative

In order to reproduce the odd modes of the desired soundfield, the derivative term $\rho'(\frac{\pi}{2},\phi_q,k)$ has to be realized at each sampling point. This derivative can also be represented by two vertical omnidirectional loudspeakers

$$\rho_q'(\frac{\pi}{2},\phi_q,k)=lim_{d\to 0}[(\rho_q(\frac{\pi}{2}+\frac{d}{2},\phi_q,k,R)\dots$$
$$-\rho_q(\frac{\pi}{2}-\frac{d}{2},\phi_q,k,R))/d] \tag{19}$$

where d is the distance between upper loudspeaker and lower loudspeaker. For an omnidirectional loudspeaker pair to produce an effective dipole pattern of the above form, their separation should be at least half a wave length, that is $d\geq\frac{c}{2f}$. Then, the time delay between them will result in a phase shift mimicking a dipole pattern. This requirement is a limitation of the proposed solution because for low frequencies, the required minimum separation is considerably high. However, for mid-high frequencies, the dipole loudspeaker pattern is achievable with a fairly small separation.

4.3. Driving signal calculation for the discrete loudspeaker array

Here, we discuss the process of deriving the loudspeaker driving signals. Similar to the method discussed in (20), the q^{th} loudspeaker weights for even mode reproduction can be designed at each concentric array using

$$w_q=\rho(\pi/2,\phi_q,k)2\pi/Q \tag{20}$$

For odd mode reproduction, the corresponding weights are

$$w_q^u=\rho'(\pi/2+d/2,\phi_q,k)2\pi/Q \tag{21}$$

for the upper set of omnidirectional loudspeakers and

$$w_q^l=\rho'(\pi/2-d/2,\phi_q,k)2\pi/Q \tag{22}$$

for the lower set of omnidirectional loudspeakers. So far, we have treated the reproduction of even and odd soundfield modes separately. However, it is important to note that only a single dipole loudspeaker array is required to control both sets of coefficients as either the upper or below set of the dipole speakers can also be used

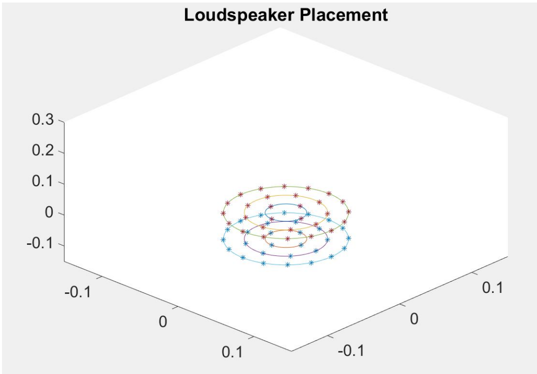

Fig. 1. Loudspeaker Arrangement

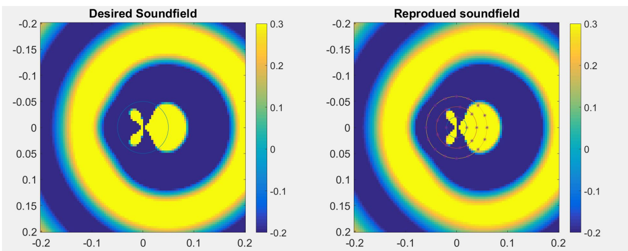

Fig. 2. Observation Plane at Z = 0

to control the even modes. That is, using the superposition property, the upper layer can produce $w_q^u + w_q$ while the lower layer produces w_q^l or the lower layer can produce $w_q^l + w_q$ while the upper layer produces w_q^u.

An alternate method to calculate the driving signals is via a least squares approach. Equations (17) and (18) can be sampled using the sampling requirement proposed earlier, and the resulting equations can be formulated in matrix format, which can then be solved using the least squares method.

5. SIMULATION

Here, we provide simulation a simulation example for the the proposed method. We assume a desired 3D exterior soundfield caused by a single point source located at $(0.05, \pi/2, 0)$. The frequency bandwidth of interest is $2000 - 4000$ Hz.

5.1. Speaker Placement

The design specifications are as follows. The separation between each dipole loudspeaker pair is $d = 0.085$ m. We use 3 concentric circular arrays with a total of 29 dipole loudspeakers. Figure 1 illustrates the resulting array.

5.2. Simulation Results

Figure 2 shows the desired sound-field and reproduced sound-field across the horizontal cross-section at elevation $z = 0$. The multiple solid circles illustrate the array geometry. It is important to only concentrate on the exterior soundfield beyond $r = 0.05$ m as that's where the exterior field lie. Similarly, Fig. 3 shows the desired and reproduced soundfields across an alternate plane at elevation $z = 0.3$. Note that in both cases, the reproduced exterior soundfield is almost identical to that of the desired field. Noise was not present in the simulations, and therefore, the performance solely depicts the mathematical accuracy of the proposed method.

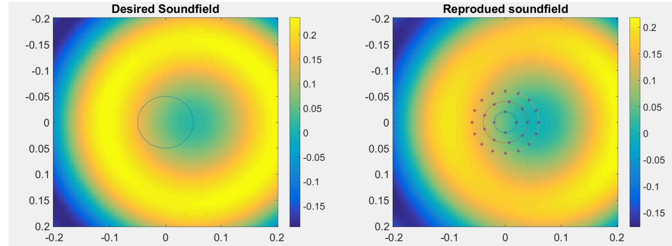

Fig. 3. Observation Plane at Z = 0.3

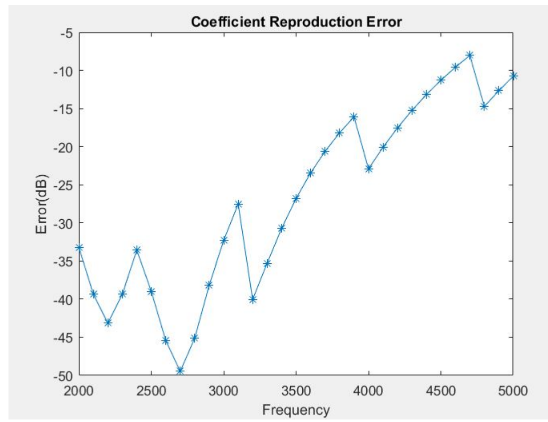

Fig. 4. Exterior Soundfield coefficient Error

6. ERROR ANALYSIS

In order to analyze the reproduction accuracy over broadband frequencies, we plot the reproduction error defined by

$$\epsilon_N(kx) = \frac{\int |S(r_x, \theta_x, \phi_x, k) - S'(r_x, \theta_x, \phi_x, k)|^2 dx}{\int |S(r_x, \theta_x, \phi_x, k)|^2 dV} \quad (23)$$

where $S(r_x, \theta_x, \phi_x, k)$ and $S'(r_x, \theta_x, \phi_x, k)$ are the desired and reproduced sound pressure at the observation point (r_x, θ_x, ϕ_x), and $\int dV$ is the volume integral over the region of interest. From figure 4, it is evident that the reproduction error is significantly low across the frequency band of interest. As expected, the error gradually increases with increasing frequency. the saw-tooth behaviour of the curve is due to the activation of new soundfield modes with increasing frequencies.

7. CONCLUSION

In this paper, we presented a planar loudspeaker design for reproducing 3D exterior soundfields. Based on simulation results, it was shown that the proposed method is capable of recreating a full 3D spatial soundfield with high accuracy, specially in mid-high frequencies. Therefore, it is a promising result for future commercial audio devices focusing on directional sound reproduction. However, the method is difficult to be realized at low frequencies due to physical limitations of dipole loudspeakers.

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