HYBRID BEAMFORMING DESIGN WITH FINITE-RESOLUTION PHASE-SHIFTERS FOR FREQUENCY SELECTIVE MASSIVE MIMO CHANNELS

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ABSTRACT

Massive multiple-input multiple-output (MIMO) theoretical performance results have attracted the attention of the community due to the possibility of increasing the spectral efficiency in wireless communications. The performance potential is mainly conditioned to the use of digital beamforming techniques which demand one radio-frequency (RF) chain per antenna element. For large arrays, this implementation may result in high complexity, power consumption, and cost. To reduce the number of RF chains, we use a hybrid beamforming (HB) architecture of an analog beamformer implemented by using phase-shifters and a low-dimensional digital beamformer. The performance of the HB depends on the resolution of the phase-shifters. However, very few works in the literature take into account finite phase-shifters. In this paper, we address the problem of designing HB in frequency selective channels using finite-resolution phase-shifters. The strategy is to exploit the second-order statistics of the channel and a least-square formulation to obtain the discrete phase of each phase-shifter. The digital part is derived based on analog solution to maximize the single-user MIMO system sum-rate. This solution requires a number of RF chains compared to the rank of the spatial covariance matrix which is far lower than ones demanded to implement the full digital beamforming. The simulation results show that the proposed technique can achieve a sum-rate performance very close to that of the digital beamforming assuming low-rank channels.

Index Terms— Massive MIMO, hybrid beamforming, phase-shifters, alternating least-squares

1. INTRODUCTION

Massive multiple-input multiple-output (MIMO) is the next wireless communication systems due to its capability of increasing the system spectral efficiency. The large number of antennas at the base station (BS) makes possible to spatially multiplex many independent signals into the same frequency-time communication resource [1]. However, its theoretical results depend upon a digital beamforming (DB), which enables both phase and amplitude control of the signals, but demands a dedicated radio frequency (RF) chain per antenna element. This solution can prohibit the implementation of massive MIMO because of the high cost and the RF chain power consumption [2, 3].

The analog-digital beamforming architecture, also known as hybrid beamforming (HB), has gained attention due to the use of less RF chains than the number of antennas, which is particularly of practical importance in massive MIMO systems. This beamforming is a concatenation of two beamformers, a low-dimension digital one that scales with the number of RF chains, while the analog one is a phase-shifter network that maps the signal of each RF chain to a set of antenna elements [4]. Using this architecture, the authors in [2, 5] provide a solution for data transmission that maximizes the single-user MIMO sum-rate. Another work designed the HB for pilot transmission and consequently improves the channel estimation quality as shown in [6]. In this work, the authors propose that the BS uses the analog and digital beamformers to spatially multiplex the pilots. The benefit of it is that the system can use shorter pilot sequences and consequently deal with the downlink overhead problem in massive MIMO. The common assumption in the literature is the use of infinite resolution phase-shifter; very few works have taken into account the phase-shifter resolution, such as [3].

The goal of this paper is to design a HB solution that maximizes the single-user MIMO sum-rate using low-resolution phase-shifters that operate over frequency-selective channels. The proposed solution exploits only partial information of the channel by using the spatial covariance matrix to determine a reduced subspace from the eigen decomposition. We then propose a heuristic to implement the analog and digital parts at the BS. The method uses the alternate least-square with projection (ALSP) algorithm to construct a wideband HB subject to a finite-resolution phase-shifter constraint. The ALSP algorithm appeared earlier in [7] as a solution for the blind separation of co-channel signals. Herein, we identify this algorithm as a solution to design the wideband beamforming vectors under finite-alphabet phase shifters, which is of practical relevance in hardware-constrained massive MIMO systems. Once the wideband beamforming is designed, we calculate a set of narrowband beamformers that are based on the baseband frequency channel response. We show that, with the proposed HB, the number of RF chains needs to scale with the rank of spatial covariance matrix, which is often smaller than the number of antennas.

2. SYSTEM MODEL AND PROBLEM DEFINITION

A large number of antennas at the BS prohibits the use of DB solutions especially for high frequencies. This type of implementation has dedicated RF chains per antenna element, and each chain uses a digital to analog converter (DAC) for downlink transmission. This component is responsible for using considerable amount of power consumption as shown in [8]. This means the DB may reach cost-inefficient levels in massive MIMO system due to DAC. To address this problem, we can reduce the number of RF chains (consequently the number of DACs) by assuming a HB solution at the BS. Its structure is a concatenation of a full-dimensional analog beamformer $\mathbf{A}$, which has the same dimension as the number of transmit anten-

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nas, and a low-dimensional digital one $\mathbf{D}$. The analog part is implemented using a phase-shifter network that connects a set of antennas to each one of the J RF chains. The digital part corresponds to the baseband processing connected to the J RF chains.

The scenario considered is a single-user MIMO system in which the BS is employed with N antennas and transmits $L < N$ parallel streams to a user equipment (UE) using $M \geq L$ antennas. The channel between BS and UE is frequency selective and follows the block-fading model, where $\mathbf{H}(k) \in \mathbb{R}^{M \times N}$ $\forall k$ $k = \{1, 2, \ldots, K\}$ denotes the matrix associated with the kth block. Herein, we define the term "block" as a set of frequencies whose channel response is approximately constant, and the number of subcarriers F within the set defines the coherence bandwidth. For this paper, we assume that the spatial channel covariance matrix is known perfectly at the BS and UE, and the second-order statistics remain constant over many blocks. Given this, the BS uses such an information to determine a $N \times J$ matrix $\mathbf{A}$ whose entries have constant modulus, i.e. $|\mathbf{A}(n, j)| = 1$, while the $J \times L$ matrix $\mathbf{D}(k)$ is designed from the equivalent channel $\mathbf{H}_{\rm e}(k) \triangleq \mathbf{H}(k)\mathbf{A}$. Note that the digital part is frequency selective whereas the analog one is wideband filter.

Let $\mathbf{S}(k) \in \mathbb{C}^{L \times F}$ denote the signal transmitted during the kth fading-block. The matrix $\mathbf{S}(k)$ is such that $\mathrm{E}\left[\mathbf{S}(k)\mathbf{S}(k)^H\right] = \mathbf{I}_L$. The received signal in the kth fading-block can be expressed as

$$\mathbf{Y}(k) = \mathbf{H}(k)\mathbf{A}\mathbf{D}(k)\mathbf{S}(k) + \mathbf{Z}(k), \tag{1}$$

where $\mathbf{Z}(k) \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2\mathbf{I}_L\right)$ is the additive Gaussian noise. To extract $\mathbf{S}(k)$, the UE employs a $L \times M$ combiner $\mathbf{W}(k)$. The number of antennas at the receiver is expected to be much lower than that at the BS, which makes the use of a DB possible with affordable complexity. Thus, the final baseband signal is represented as

$$\mathbf{R}(k) = \mathbf{W}(k)\mathbf{Y}(k). \tag{2}$$

Regarding practical implementation of HB, the design of a phase-shifter network plays a fundamental role on the overall precoder performance because it controls the phase rotation of every continuous signal. The continuous phase control requires accurate components which can be expensive [9]. On the other hand, finite-resolution phase-shifters are attractive solutions in terms of cost because they require simpler hardware implementation than those with infinite resolution. The analog beamformer with discrete phases is a matrix $\mathbf{A} \in \mathcal{A}$, where $\mathcal{A} = \{1, a, a^2, \ldots, a^{I-1}\}$, $a = \mathrm{e}^{\jmath\frac{2\pi}{I}}$, and I is the number of discrete phases. Our goal is to design an HB solution to maximize the downlink single-user MIMO sum-rate.

Let us assume a Gaussian signaling in $\mathbf{S}(k)$. The average rate per block is

$$R(k) = \log_2\left|\mathbf{I}_L + \frac{1}{\sigma^2}\mathbf{W}(k)^{\dagger}\mathbf{Q}(k)\right|, \tag{3}$$

where $\mathbf{Q}(k) = \mathbf{W}(k)\mathbf{H}(k)\mathbf{A}\mathbf{D}(k)\mathbf{D}(k)^H\mathbf{A}^H\mathbf{H}(k)^H\mathbf{W}(k)^H$ and the operator $(.)^{\dagger}$ denotes the pseudoinverse [3, 10]. The optimal HB and combiner are obtained as a solution of the following optimization problem

$$\begin{aligned} \max_{\mathbf{A}, \{\mathbf{D}(k), \mathbf{W}(k)\}_{k=1}^K} \quad & 1/K \sum_{k=1}^{K} R(k) \\ \text{s.t.} \quad & \mathrm{trace}\{\mathbf{A}\mathbf{D}(k)\mathbf{D}(k)^H\mathbf{A}^H\} \leq P_k/F \ \forall k, \end{aligned} \tag{4}$$

where P_k is the power budget of the kth block[1]. Therefore, the problem consist of designing $\mathbf{D}(k)$ and $\mathbf{W}(k)$ per block while $\mathbf{A}$ has to be the same for all k.

3. HYBRID BEAMFORMING AND DIGITAL COMBINER

The precoder and combiner to solve (4) can be obtained by optimizing jointly $\mathbf{A}$, $\mathbf{D}(k)$, and $\mathbf{W}(k)$ $\forall k$ $k = \{1, 2, \ldots, K\}$; however, the non-convexity of the problem leads us to search alternative strategies to solve it. Our approach consists of decoupling the HB precoder and the digital combinyer by solving firstly the mutual information maximization problem of $\mathbf{Y}(k)$ and $\mathbf{S}(k)$ to obtain the HB and subsequently the design of $\mathbf{W}(k)$ is obtained from (4) for a fixed HB.

Firstly, let us consider the mutual information function for the block k as

$$R_1(k) = \log_2\left|\mathbf{I}_L + \mathbf{H}(k)\mathbf{A}\mathbf{D}(k)\mathbf{D}(k)^H\mathbf{A}^H\mathbf{H}(k)^H\right|. \tag{5}$$

The precoder design can be expressed as

$$\begin{aligned} \max_{\mathbf{A}, \{\mathbf{D}(k)\}_{k=1}^K} \quad & 1/K \sum_{k=1}^{K} R_1(k) \\ \text{s.t.} \quad & \mathrm{trace}\{\mathbf{A}\mathbf{D}(k)\mathbf{D}(k)^H\mathbf{A}^H\} \leq P_k/F \ \forall k, \\ & \mathbf{A} \in \mathcal{A}. \end{aligned} \tag{6}$$

In (6), the analog beamforming solution involves an exhaustive search over the set $\mathcal{A}$. To design $\mathbf{A}$, we first note that every digital beamformer can be expressed as $\mathbf{D}(k) = \mathbf{B}\mathbf{C}(k)$, where $\mathbf{B}$ is a $J \times J$ matrix that combines output of the phase-shifters, and $\mathbf{C}(k)$ is a $J \times L$ matrix that precodes the signal $\mathbf{S}(k)$. The motivation behind this factorization is to adjust the signal amplitude by performing a linear combination over the phase-shifters. Assuming that $\mathbf{C}(k)\mathbf{C}(k)^H = \gamma^2\mathbf{I}_L$, the remaining problem consists of optimizing jointly two wideband beamformers $\mathbf{A}$ and $\mathbf{B}$. The advantage of working only with $\mathbf{A}$ and $\mathbf{B}$ is that we can remove the dependence on the block index k and optimize them based on the channnel covariance matrix $\mathbf{R} = \mathrm{E}\left[\mathbf{H}(k)^H\mathbf{H}(k)\right]$.

The second part of our problem involves the design of the digital beamformers $\mathbf{C}(k)$ and $\mathbf{W}(k)$. This can be done by first fixing the wideband matrices $\mathbf{A}$ and $\mathbf{B}$, and then calculating the digital beamformers using the eigenbeamforming solution. More specifically, the matrices $\mathbf{C}(k)$ and $\mathbf{W}(k)$ can be obtained from the left and right singular vectors of the effective channel matrix defined as

$$\mathbf{H}_{\rm e}(k) \triangleq \mathbf{H}(k)\mathbf{A}\mathbf{B}. \tag{7}$$

We now focus on the design of the wideband beamformer $\mathbf{AB}$. Two solutions are discussed for this purpose. The first one is a baseline method originally applied to narrowband channels in [2]. The second solution is the main contribution of this paper, where we propose to find $\mathbf{AB}$ using a least squares formulation that is solved by the ALSP algorithm.

3.1. Statistical Wideband HB

We design the RF beamformer assuming $\mathbf{C}(k)\mathbf{C}(k)^H = \alpha^2\mathbf{I}_L$, where $\alpha = \sqrt{P_k/F\|\mathbf{AB}\|_{\mathcal{F}}^2}$ to ensure the power constraint in (6),

[1] Equal power allocation in frequency is non-optimum in principle, but it is assumed for simplicity.

and the operator $\|.\|_{\mathcal{F}}^2$ denotes the Frobenius norm. Therefore, the problem can be rewritten as

$$\max_{\mathbf{A},\mathbf{B}} \ 1/K \sum_{k=1}^{K} \log_2 \left| \mathbf{I}_L + \frac{\gamma^2}{\sigma^2} \mathbf{B}^H \mathbf{A}^H \mathbf{H}(k)^H \mathbf{H}(k) \mathbf{A} \mathbf{B} \right| \quad \text{s.t. } \mathbf{A} \in \mathcal{A}. \tag{8}$$

The phase-shifter network remains constant over the frequency domain while the channel frequency response does not. Such a contradiction motivates us to express the problem in function of the channel spatial covariance matrix instead, as the second order statistics are constant over the transmission bandwidth. Assuming a large number of blocks, which means a large bandwidth, we can use the expectation operator $\mathrm{E}[.]$ to express (8), and Jensen's inequality to establish the relationship

$$\mathrm{E}\left[\log_2 \left| \mathbf{I}_L + \mathbf{G}(k)^H \mathbf{G}(k) \right|\right] \leq \log_2 \left| \mathbf{I}_L + \mathrm{E}\left[\mathbf{G}(k)^H \mathbf{G}(k)\right] \right|, \tag{9}$$

where $\mathbf{G}(k) = \sqrt{\frac{\gamma^2}{\sigma^2}} \mathbf{H}(k)\mathbf{A}\mathbf{B}$. Using the right-hand side of (9) into (8), the dependence of block k is removed and the problem is

$$\max_{\mathbf{A},\mathbf{B}} \ \log_2 \left| \mathbf{I}_L + \frac{\gamma^2}{\sigma^2} \mathbf{B}^H \mathbf{A}^H \mathbf{R} \mathbf{A} \mathbf{B} \right| \quad \text{s.t. } \mathbf{A} \in \mathcal{A}. \tag{10}$$

The objective function is still not concave, yet (10) is convenient because the formulation has no frequency dependence. Moreover, the use of the channel second-order statistics gives the advantage of updating $\mathbf{A}$ and $\mathbf{B}$ with low periodicity, as $\mathbf{R}$ changes slower than $\mathbf{H}(k)$.

Based on (10), the design of $\mathbf{A}$ and $\mathbf{B}$ is done from the eigenvectors of $\mathbf{R}$. We exploit two solutions in this paper: 1) the RF chains are combined into pairs using the rows of $\mathbf{B}$ and 2) $\mathbf{A}$ and $\mathbf{B}$ are jointly determined by minimizing $\|\mathbf{\Gamma}_E - \mathbf{A}\mathbf{B}\|$, where $\mathbf{\Gamma}_E$ are the first E eigenvectors of $\mathbf{R}$.

The number E represents the amount of channel knowledge used by the BS to design the HB solution. The critical case is $E = L$, where the BS uses the same number of eigenmodes as the number of layers. For this design, there is no extra degree of freedom to exploit. If $E > L$, there are more eigenmodes than the number of layers and more information about the channel. With this design, the signal-noise ratio (SNR) tends to increase because the BS uses more relevant modes to represent the instantaneous channel.

3.1.1. Paired Phase-shifter

The first solution uses the fact that any vector is obtained from a combination of two other vectors with constant modulus entries. Using this result, it is possible to combine two columns of $\mathbf{A}$ using two rows of $\mathbf{B}$ to implement one eigenvector. The equation to be solved is given by [2]:

$$\mathbf{A}(n,j)\mathbf{B}(j,e) + \mathbf{A}(n,j+1)\mathbf{B}(j+1,e) = \mathbf{\Gamma}(n,e), \tag{11}$$

where e is the eigenmode index. Using the procedure described in [2], the solution for $\mathbf{B}$ is as follows

$$\mathbf{B}(j,e) = \frac{\gamma_{\max}(e) + \gamma_{\min}(e)}{2}, \tag{12}$$

$$\mathbf{B}(j+1,e) = \frac{\gamma_{\max}(e) - \gamma_{\min}(e)}{2}, \tag{13}$$

Algorithm 1 ALSP

Require: $\mathbf{A} \in \mathcal{A}$
Ensure: $\min_{\mathbf{A},\mathbf{B}} \|\mathbf{\Gamma} - \mathbf{A}\mathbf{B}\|_{\mathcal{F}}^2$
Start the algorithm with a random choice for $\hat{\mathbf{A}}$
while $\|\hat{\mathbf{A}}_i - \hat{\mathbf{A}}_{i-1}\|_F^2 \geq \epsilon$ **do**
$\hat{\mathbf{B}}_{i+1} = \mathbf{A}_i^{\dagger}\mathbf{\Gamma}_E$
$\hat{\mathbf{A}}_{i+1} = \mathbf{\Gamma}_E \mathbf{B}^{\dagger}$
$\hat{\mathbf{A}}_{i+1} = \mathrm{proj}_{\mathcal{A}}\left[\hat{\mathbf{A}}_{i+1}\right]$
end while

where $\gamma_{\max}(e) = \max_n\{|\mathbf{\Gamma}(n,e)|\}$ and $\gamma_{\min}(e) = \min_n\{|\mathbf{\Gamma}(n,e)|\}$. The solution for the phase shifters is then given by [2]

$$\phi_{n,j} = \angle\mathbf{\Gamma}(n,e) + \cos^{-1}\left(\frac{|\mathbf{\Gamma}(n,e)| + \gamma_{\max}(e)\gamma_{\min}(e)}{|\mathbf{\Gamma}(n,e)|(\gamma_{\max}(e)+\gamma_{\min}(e))}\right), \tag{14}$$

$$\phi_{n,j+1} = \angle\mathbf{\Gamma}(n,e) + \cos^{-1}\left(\frac{|\mathbf{\Gamma}(n,e)| - \gamma_{\max}(e)\gamma_{\min}(e)}{|\mathbf{\Gamma}(n,e)|(\gamma_{\max}(e)-\gamma_{\min}(e))}\right), \tag{15}$$

where $\phi_{n,j} = \angle\mathbf{A}(n,j)$ and $\phi_{n,j+1} = \angle\mathbf{A}(n,j+1)$. In this technique, each eigenvector is implemented by combining two RF chains. Therefore, the total number of RF chains to transmit the E eigenmodes is scaled by two, i.e. $J = 2E$. This means the BS can eliminate the hardware limitations of the analog beamforming at the cost of twice more RF chains. Note, however, that this solution assumes phase-shifters with infinite resolution. We then need to round the values $\phi_{n,j}$ and $\phi_{n,j+1}$ to the closest phase in a codebook of I possible values.

3.1.2. LS criterion

If we could relax the constant modulus restriction in (10), the optimum solution is $\mathbf{A}\mathbf{B} = \mathbf{\Gamma}_E$, where $\mathbf{\Gamma}_E$ corresponds to the E dominant eigenvectors. Since we cannot do this relaxing in practice, to obtain a solution for $\mathbf{A}$ and $\mathbf{B}$, we propose to solve the following problem:

$$\min_{\mathbf{A},\mathbf{B}} \ \|\mathbf{\Gamma}_E - \mathbf{A}\mathbf{B}\|_{\mathcal{F}}^2 \ \text{ s.t. } \mathbf{A} \in \mathcal{A}. \tag{16}$$

An attractive solution to this problem is given by the ALSP algorithm proposed earlier in [7] in the context of co-channel signals blind separation. In that work, the convergence is proven by assuming one of the matrices belonging to a finite set. This assumption matches up with the one in the problem (16), where the analog beamformer follows a finite-alphabet.

The algorithm starts by assuming an initial guess $\hat{\mathbf{A}}$ to obtain $\hat{\mathbf{B}}$ in the LS sense, yielding $\hat{\mathbf{B}} = \hat{\mathbf{A}}^{\dagger}\mathbf{\Gamma}_E$. Given $\hat{\mathbf{B}}$, we then conditionally update $\hat{\mathbf{A}}$ in the LS sense, which gives $\tilde{\mathbf{A}} = \mathbf{\Gamma}_E\hat{\mathbf{B}}^{\dagger}$. Since the solution found for $\hat{\mathbf{A}}$ in the equation above is unstructured and does not meet the finite resolution requirement for the phase shifters, we project the elements of $\hat{\mathbf{A}}$ onto the admissible finite alphabet set $\mathcal{A}$, i.e. $\hat{\mathbf{A}} = \mathrm{proj}_{\mathcal{A}}\left[\tilde{\mathbf{A}}\right]$. The overall algorithm is summarized in the Table 1.

Note that, in the proposed solution, the number of RF chains scales with the rank of covariance matrix, i.e. $J = E$. Thus, this technique requires half of the number of RF chains needed in the paired phase shifter solution of Section 3.1.1. Assuming the critical case $E = L$ consequently the number of RF chains is $J = L$, otherwise $J > L$.

3.2. Narrowband Digital Beamformers and Combiners Design

We now consider the design of $\mathbf{C}(k)$ and $\mathbf{W}(k)$ at the BS and UE, respectively, provided that the wideband part $\mathbf{A}\mathbf{B}$ has been found

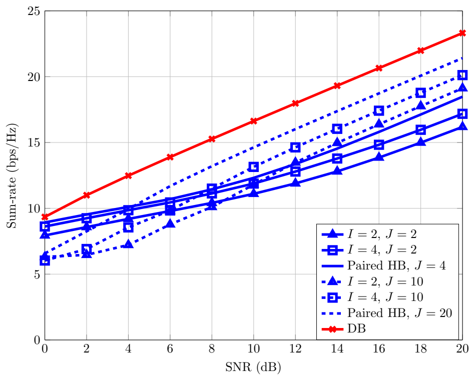

Fig. 1: Sum-rate for different numbers of RF chains and different phase-shifter resolutions.

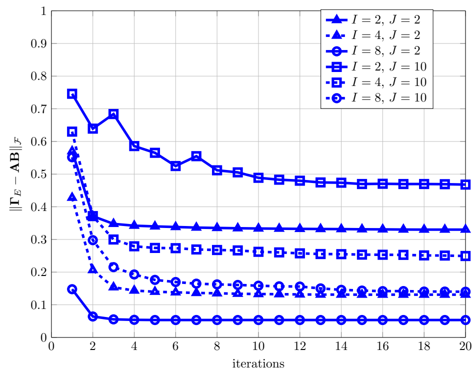

Fig. 2: The plot shows the error $\|\mathbf{\Gamma}_E - \mathbf{AB}\|_{\mathcal{F}}$ per iteration of the ALSP algorithm. The number of RF chains is fixed to $J = E$.

using one of the two methods derived in the previous section. For a fixed $\hat{\mathbf{A}}\hat{\mathbf{B}}$, the set of digital beamformers can be found in closed-form by maximizing the overall spectral efficiency. To this end, we work on the effective channel matrix $\mathbf{H}_e(k) \triangleq \mathbf{H}(k)\hat{\mathbf{A}}\hat{\mathbf{B}}$ and propose to find $\mathbf{C}(k)$ and $\mathbf{W}(k)$ by solving the following problem:

$$\max_{\{\mathbf{C}(k),\mathbf{W}(k)\}_{k=1}^{K}} \quad 1/K \sum_{k=1}^{K} \log_2 \left| \mathbf{I}_L + \frac{\gamma^2}{\sigma^2}\mathbf{Q}(k), \right|,$$
$$\text{s.t. } \operatorname{trace}\{\mathbf{C}(k)\mathbf{C}(k)^H\} = 1,$$
$$\mathbf{Q}(k) = \mathbf{W}(k)\mathbf{H}_e(k)\mathbf{C}(k)\mathbf{C}(k)^H\mathbf{H}_e(k)^H\mathbf{W}(k)^H.$$

The problem solution is obtained from the singular value decomposition $\mathbf{H}_e(k) = \mathbf{U}(k)^H\mathbf{\Sigma}(k)\mathbf{V}(k)^H$. The UE uses the L left singular vectors, $\mathbf{W}(k) = \mathbf{U}(k)_L^H$, while the BS uses the L right singular vectors, $\mathbf{C}(k) = \mathbf{V}(k)_L$.

4. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we present numerical results to evaluate the performance of the proposed HB solution based on the ALSP algorithm in terms of the sum-rate (bps/Hz). The plots consider different numbers of RF chains and different phase-shifter resolutions. The benchmarks considered are the upper-bound obtained from the DB sum-rate by assuming knowledge of $\mathbf{H}(k)$ $\forall k$ $k = \{1, 2, \dots K\}$ and the paired HB implementation using infinite phase-shifter resolution. The simulated 128×2 MIMO channel has a 10MHz bandwidth, and is generated using an extension of the COST2100 channel model for massive MIMO systems, assuming a coherence block of 300 KHz [11]. The number of layers for all simulations is $L = 2$.

In Fig. 1, we first consider the case with $J = 2$ RF chains and phase-shifters with resolution given by $I = 2$ and $I = 4$ discrete phases. The bechmark is the paired HB solution with $J = 4$. Note that the ALSP solution provides the possibility of reducing the number of RF chains and the phase-shifter resolution without a great sacrifice in the sum-rate. For instance, the scheme with $I = 4$ and $J = 2$ only looses 1 bps/Hz compared to the benchmark one.

To further improve the spectral efficiency, the system must increase E which means the transmitter must use more eigemodes, i.e. the BS should employ more RF chains. The rank of covariance matrix rank $(\mathbf{R}) = 10$ determines the number of RF chains to reach the best performance. The relationship between the rank and the number of RF chains depends on the technique used to implement the eigenvectors. The paired HB solution demands $J = 20$ RF chains whereas the proposed technique demands $J = 10$ only. Thus, using the ALSP solution to the HB problem, the number of RF chains scales with the rank of the covariance matrix which in this example is an order of magnitude less than the number of antennas.

The results in Fig. 1 also show that the sum-rate with $J = 2$ is better than that with $J = 4$ in low SNR, which is explained by the lack of power allocation solution performed across the beams. More specifically, there are eigenmodes whose ratio between the eigenvalue and the noise variance is high consequently more power could be allocated to them. This would improve the overall performance.

A disadvantage of the statistical approach is the natural loss identified between the full DB and the paired HB. While the first uses the perfect instantaneous channel state information, the second represents the channel using a reduced space. This leads to a reduction of the channel degrees of freedom and, consequently, an approximation error, which explains the gap shown in Fig. 1. However, we believe it is still interesting to exploit the second-order statistics because the BS and UE can properly represent the channel in a compressed space. With this strategy, it is also possible to reduce the pilot overhead, which is a bottleneck in donwlink channel estimation for massive MIMO systems.

Figure 2 depicts the approximation error $\|\mathbf{\Gamma}_E - \mathbf{AB}\|_{\mathcal{F}}^2$ per iteration of the ALSP algorithm. Note that fixing the number J of RF chains and increasing the phase-shifter resolution I, the error tends to decrease, as expected. However, the error tends to increase by using more eigenmodes (i.e. more RF chains). To reduce the losses, the BS has to use phase-shifters with higher resolution.

5. CONCLUSION

This paper presents a HB that uses a limited number of RF chains and phase-shifters with finite resolution for massive MIMO systems. The proposed solution exploits the second order statistics of the channel to derive the wideband part of the HB, and instantaneous channel knowledge of significantly reduced dimension, to derive the narrowband part of the HB. Comparing our solution with the scheme that uses only a pair of phase-shifters, the number of RF chains in the latter scales with twice the rank of spatial covariance matrix, while the proposed solution only scales with the rank of covariance matrix, thus being more cost-effective. Finally, it is worth mentioning that the proposed framework can also be applied for pilot transmission in the downlink of a hardware-constrained massive MIMO system that operates with finte resolution phase-shifters. The use of the proposed HB design in the problem of downlink channel estimation is under investigation and will be subject of a future work.

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