BLIND ON BOARD WIDEBAND ANTENNA RF CALIBRATION FOR MULTI-ANTENNA SATELLITES

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ABSTRACT

The problem of joint Angle-of-Arrival (AoA) and calibration parameters in a wideband scenario is addressed. The system consists of multiple sources transmitting from different directions and in certain subcarriers. Sources in the same beam are regarded as transmitted from the same AoA and their signals are allocated to different subcarriers while users in different beams may transmit on the same subcarrier, i.e. the signals are multiplexed in space and frequency. Then, signals from a given beam not necessarily occupy the full bandwidth but only specific subcarriers. In addition, due to the wideband of the signal, each RF chain introduces frequency dependent gain and phase shift that need to be calibrated to perform a subsequent demultiplexing in a digital beamforming matrix. We propose a novel *blind* algorithm that (i) estimates the AoAs of all present sources in the bandwidth of interest and (ii) estimates the different gains/phases at each antenna per frequency up to an unknown impairment at a reference antenna. We provide identifiability conditions that ensure a successful parameter estimation. Finally, the potential of the proposed algorithm compared to the case of known calibration parameters is assessed by simulations.

***Index Terms*—** Satellite, blind calibration, AoA Estimation, wideband digital beamforming.

1. INTRODUCTION

Wideband Digital Beamforming (WDBF) technology has become an essential part of satellite communication systems since it combines the benefits of the digital beamforming (e.g. [1]) with the advantages of the wideband systems (e.g. [2]). The design of WDBFs is typically based on the assumption that the delay line is frequency independent (e.g. [3]). However, analog RF chains performing down-frequency conversions and Analog to Digital Converters (ADC) introduce delay contributions that are frequency dependent, thus severely degrading the performance of the WDBF network [3]. Therefore, in broadband multiantenna Satellite Systems (SS), a crucial role is played by a correct calibration to compensate for these delays. Usually, on-board calibration for bent-pipe SS equipped with WDBF is based on supervised calibration techniques using probe signals (see e.g. [3–5]). In such a context, it is appealing to develop blind calibration techniques that could outperform the supervised ones in terms of cost, spectral efficiency, power, mass of equipments, and complexity. The impairments can be modelled by a linear filter and equalized in the digital domain. In this paper, we develop a blind calibration algorithm for bent pipe Satellite Systems (SS) equipped with a *transparent* digital beamforming network which demultiplexes the signals in the space domain, but it does not perform demodulation and decoding of the transmitted data. The proposed blind algorithm jointly estimates the angles of arrival (AoA) of the different beams and estimates the linear impairments using alternating optimization. The impairments are estimated in the frequency domain and at each frequency the problem is reduced to the classical problem of joint blind AoA estimation and gain-phase error estimation in narrowband. This latter problem has been widely investigated in the eighties and nineties (e.g. [10–16]) utilizing a large variety of approaches which span from the maximum likelihood concept to the subspace method. More recently, this topic gained new attention and in [6], the authors tackled the problem by leveraging on the signal sparse model. In [7], Ollier *et al.* consider a model with block diagonal colored noise and adopt a maximum likelihood approach in the presence of a limited number of calibration sources with known angles of arrival. To assess the performance, they analyze the Cramér-Rao bound (CRB).

This work differs from the existing literature on joint AoA and calibration parameters estimation since we consider wideband signals from given beams, possibly with subcarriers, where the signals null out. Furthermore, the calibration parameters vary from one frequency to another. Therefore, in this paper, our aim is to jointly estimate the AoAs of all the sources present at all the frequencies of interest and the calibration parameters at each frequency. We propose a novel method in order to do so. We have made use of the MUSIC algorithm [17] to form a suitable optimisation problem in order to jointly estimate the AoAs and calibration parameters. First, the method estimates the AoAs using multiple one-dimensional searches, then the AoA estimates are used to estimate the calibration parameters. In addition, we provide identifiability conditions that ensure successful parameter estimation. Finally, we simulate an experiment in order to demonstrate the potential of the proposed algorithm compared to a case where calibration parameters are known.

Notations: Upper-case and lower-case boldface letters denote matrices and vectors, respectively. $(.)^T$ and $(.)^H$ represent the transpose and the transpose-conjugate operators. The matrix $\mathbf{I}$ is the identity matrix with suitable dimensions. For any matrix $\mathbf{B}$, the operator $\|\mathbf{B}\|$ denotes the *Frobenius* norm. For any finite set $\mathcal{A}$, card$(\mathcal{A})$ denotes the cardinality of $\mathcal{A}$. For any vector $\mathbf{x}$, the operator diag$[\mathbf{x}]$ returns a diagonal matrix with diagonal entries correspond-

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ing to entries of $\mathbf{x}$. Also for a set of matrices $\mathbf{A}_1 \dots \mathbf{A}_k$, the operator blkdiag$[\mathbf{A}_1 \dots \mathbf{A}_k]$ returns a block diagonal matrix formed of $\mathbf{A}_1 \dots \mathbf{A}_k$. The operator $\otimes$ stands for *Kronecker* product.

2. PROBLEM STATEMENT AND SYSTEM MODEL

We consider a digital communication Satellite System (SS) in the return link equipped with N_R receive antennas and serving N_B beams.

On board, each antenna is followed by an analog chain for down-frequency conversion, and an analog-to-digital converter (ADC). The subsequent processing, including the spatial demultiplexing by a digital beamforming network is performed by an on-board digital transparent processor (DTP). Typically, the DTP includes a module for calibration of the down-frequency conversion and analog-to-digital conversion chain and a subsequent module for digital beamforming demultiplexing. Figure 1 illustrates the on board equipment.

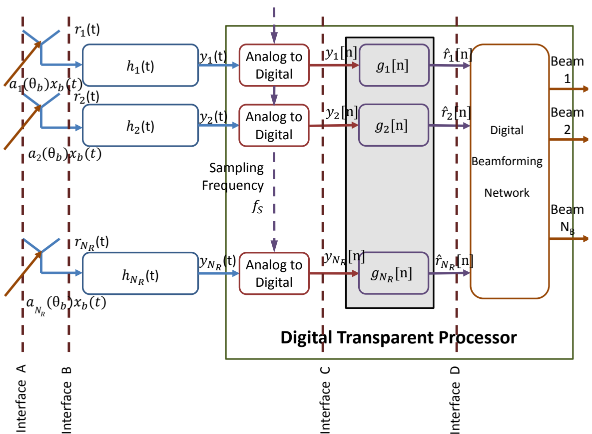


Fig. 1: On board equipment and signals

We focus on the design of the calibration module assuming no knowledge on the transmitted signals.

The signals from multiple users are multiplexed in space and frequency. The users in the footprint of beam b transmit on non-overlapping frequency bands according to a classical frequency division multiple access (FDMA) scheme. The bandwidth of the total complex multiplexed signal is $B_M/2$ and the received signals are sampled at the Nyquist rate $f_S = B_M$. Let θ_b be the angle of the planar waveforms from beam b in a reference system and $\mathbf{a}(\theta_b)$ the N_R-dimensional steering vector in the direction of θ_b. The steering vector is assumed to be constant over the band of the multiplexed signal and the angle of arrival is identical for all the users in the footprint of beam b. Throughout this paper we denote with $x_b(t)$ and $x_b[n]$ the baseband multiplexed signal from beam b at the input of the antenna array (see interface A in Fig. 1) and the corresponding sampled signal, respectively. It is worth noting that it is not necessary that each subcarrier of the SS frequency band is allocated to an active user. Thus, the Fourier transform (FT) $X_b(\omega)$ of $x_b[n]$ will be nonzero only in the frequency intervals corresponding to active users in the beam b. Given a frequency ω_k, N_{B_k} is the number of active users from different beams allocated to the subcarrier containing ω_k. We assume $N_{B_k} < N_R$. Then, the baseband multiplexed signal at the output of the antenna array (see interface B in Fig. 1) is given by

$$\mathbf{r}(t) = \sum_{b=1}^{N_B} \mathbf{a}(\theta_b) x_b(t) = \mathbf{A}(\boldsymbol{\Theta})\mathbf{x}(t) \qquad (1)$$

where

$$\boldsymbol{\Theta} = \{\theta_1 \dots \theta_{N_B}\} \qquad (2)$$

$$\mathbf{A}(\boldsymbol{\Theta}) = [\mathbf{a}(\theta_1) \dots \mathbf{a}(\theta_{N_B})] \qquad (3)$$

$$\mathbf{x}(t) = [x_1(t), x_2(t), \dots x_{N_B}(t)]^T \qquad (4)$$

Each down-frequency converter introduces a distortion in the signal that we model as a linear filter. Then, after down-frequency conversion at the m^{th} receive antenna, the signal $r_m(t)$ is filtered by an unknown filter $h_m(t)$ such that the signal at the input of the analog digital converter (ADC) is given by $y_m(t) = h_m(t) * r_m(t) + v_m(t)$, where $v_m(t)$ is an additive white Gaussian noise. After sampling at the Nyquist rate, the system model in the frequency domain is given by

$$\mathbf{Y}(\omega) = \mathbf{H}(\omega)\mathbf{A}(\boldsymbol{\Theta})\mathbf{X}(\omega) + \mathbf{V}(\omega) \qquad (5)$$

where $\mathbf{Y}(\omega)$ $(\mathbf{V}(\omega))$ is the N_R-dimensional vector whose m^{th} component is $Y_m(\omega)$ $(V_m(\omega))$, the FT of $y_m[n]$ $(v_m[n])$, the signal obtained from $y_m(t)$ $(v_m(t))$ by sampling at the Nyquist rate; $\mathbf{H}(\omega)$ is the diagonal matrix with diagonal components $H_m(\omega)$, the FT of $h_m[n]$, the signal obtained by sampling of $h_m(t)$; and $\mathbf{X}(\omega)$ is the FT of $\mathbf{x}[n]$, the signal vector obtained by sampling $\mathbf{x}(t)$.

Objective of this paper is to estimate the $N_R - 1$ frequency response that model the impairments introduced by the down-frequency converters up to an unknown distortion function $H_1(\omega)$. Since no knowledge on the signal structure, synchronization, and user position is available, we leverage only on estimations of the spectrum of the received signal vector

$$\begin{aligned}\mathbf{S}_y(\omega_k) &= \mathrm{E}\{\mathbf{Y}(\omega_k)\mathbf{Y}^H(\omega_k)\}\\ &= \mathbf{H}(\omega_k)\mathbf{A}(\boldsymbol{\Theta}^{(k)})\mathbf{S}_x(\omega_k)\mathbf{A}^H(\boldsymbol{\Theta}^{(k)})\mathbf{H}^H(\omega_k) + \sigma^2\mathbf{I}\end{aligned} \qquad (6)$$

where

$$\begin{aligned}\mathbf{S}_x(\omega_k) &= \mathrm{E}\{\mathbf{X}(\omega_k)\mathbf{X}^H(\omega_k)\}\\ &= \mathrm{diag}\left[S_{x_1}(\omega), S_{x_2}(\omega), \dots S_{x_{N_{B_k}}}(\omega)\right]\end{aligned} \qquad (7)$$

and $S_b(\omega) = \mathrm{E}\{X_b(\omega)X_b^H(\omega)\}$. This means that at each frequency ω_k, the N_{B_k} present sources are uncorrelated. Moreover, the AoAs of the sources on frequency ω_k are in the set $\boldsymbol{\Theta}$, i.e. $\boldsymbol{\Theta}^{(k)} \subset \boldsymbol{\Theta}$ with

$$\mathrm{card}(\boldsymbol{\Theta}^{(k)}) = N_{B_k} \qquad (8a)$$

and

$$\forall i \neq j \text{ such that } (\theta_i, \theta_j) \in \boldsymbol{\Theta}^{(k)} \times \boldsymbol{\Theta}^{(k)} \implies \theta_i \neq \theta_j \qquad (8b)$$

In fact, $\mathbf{S}_y(\omega)$ can be estimated from the observed signals $\mathbf{y}[n] = [y_1[n], y_2[n], \dots y_{N_R}[n]]^T$ through sample averaging. In practical implementation of the systems, the estimates of $\mathbf{S}_y(\omega)$ will be based on finite sequences of the sampled signal vector of length M.

3. PROPOSED ALGORITHM

Using spectral decomposition, each $\mathbf{S}_y(\omega_k)$ is expressed as

$$\mathbf{S}_y(\omega_k) = \mathbf{U}_s(\omega_k)\boldsymbol{\Sigma}_s(\omega_k)\mathbf{U}_s^H(\omega_k) + \sigma^2\mathbf{U}_n(\omega_k)\mathbf{U}_n^H(\omega_k) \qquad (9)$$

for all $k = 1 \dots M$. The term $\mathbf{U}_s(\omega_k) \in \mathbb{C}^{N_R \times N_{B_k}}$ contains the eigenvectors that span the *signal subspace*, which are also spanned by the columns of $\mathbf{H}(\omega_k)\mathbf{A}(\mathbf{\Theta}^{(k)})$. This is so because the first part of $\mathbf{S}_y(\omega_k)$ in equation (6) is of rank N_{B_k}, given that $N_{B_k} < N_R$ and all θ_i's are distinct. Similarly, the space spanned by the columns of $\mathbf{U}_n^H(\omega_k)$ is called the *noise subspace*. It turns out that both spaces are orthogonal, and therefore

$$\|\mathbf{U}_n^H(\omega_k)\mathbf{H}(\omega_k)\mathbf{a}(\theta_b)\|^2 = 0 \tag{10}$$

for $k = 1 \dots M$ and $b = 1 \dots N_{B_k}$. In the presence of noise and finite number of samples, we replace the quantities $\mathbf{S}_y(\omega_k)$ and $\mathbf{U}_n^H(\omega_k)$ by their estimates $\widehat{\mathbf{S}}_y(\omega_k)$ and $\widehat{\mathbf{U}}_n^H(\omega_k)$, respectively. These estimates are usually obtained through sample averaging.

$$\{\widehat{\theta}_i\}_{i=1}^{N_B} = \arg\max_{\theta} \big(\sum_{k=1}^{M} \frac{1}{\|\mathbf{U}_n^H(\omega_k)\mathbf{H}(\omega_k)\mathbf{a}(\theta)\|^2} \big) \tag{11}$$

However, applying MUSIC directly to the problem at hand is not possible, since we do not have any prior knowledge on all matrices $\mathbf{H}(\omega_k)$, except that they are diagonal. To proceed, notice that

$$\mathbf{H}(\omega_k)\mathbf{a}(\theta) = \mathbf{D}(\theta)\mathbf{h}(\omega_k) \tag{12}$$

where

$$\mathbf{D}(\theta) = \text{diag}[\mathbf{a}(\theta)] \tag{13}$$

$$\mathbf{h}(\omega_k) = [H_1(\omega_k) \dots H_{N_R}(\omega_k)]^T \tag{14}$$

Equation (12) translates to

$$\left\| \begin{bmatrix} \mathbf{U}_n^H(\omega_k)\mathbf{D}(\theta_1) \\ \vdots \\ \mathbf{U}_n^H(\omega_k)\mathbf{D}(\theta_{N_{B_k}}) \end{bmatrix} \mathbf{h}(\omega_k) \right\|^2 = 0, \quad k = 1 \dots M \tag{15}$$

More compactly, we have that $\mathbf{h}^H\mathbf{Q}(\mathbf{\Theta})\mathbf{h} = 0$, with

$$\mathbf{h} = [\mathbf{h}^T(\omega_1) \dots \mathbf{h}^T(\omega_M)]^T \tag{16}$$

$$\mathbf{Q}(\mathbf{\Theta}) = \text{blkdiag}[\mathbf{Q}(\omega_1, \mathbf{\Theta}^{(1)}) \dots \mathbf{Q}(\omega_M, \mathbf{\Theta}^{(M)})] \tag{17}$$

$$\mathbf{Q}(\omega_k, \mathbf{\Theta}^{(k)}) = \sum_{b=1}^{N_{B_k}} \mathbf{D}^H(\theta_b)\mathbf{U}_n(\omega_k)\mathbf{U}_n^H(\omega_k)\mathbf{D}(\theta_b) \tag{18}$$

$$\begin{cases} \underset{\mathbf{h},\mathbf{\Theta}}{\text{minimise}} & \mathbf{h}^H\widehat{\mathbf{Q}}(\mathbf{\Theta})\mathbf{h} \\ \text{subject to} & \mathbf{e}^H\mathbf{h} = 1 \end{cases} \tag{19}$$

where $\mathbf{e} = \mathbb{1}_M \otimes \mathbf{e}_1$, where $\mathbf{e}_1$ is the 1^{st} column of an identity matrix of size N_R. The *Lagrangian* function corresponding to the above optimisation problem is

$$\mathcal{L}(\mathbf{h}, \lambda) = \mathbf{h}^H\widehat{\mathbf{Q}}(\mathbf{\Theta})\mathbf{h} - \lambda\big(\mathbf{e}^H\mathbf{h} - 1\big) \tag{20}$$

Setting the derivative of $\mathcal{L}(\mathbf{h}, \lambda)$ with respect to $\mathbf{h}$ to 0, and with some straight-forward steps, one could verify that the vector $\widehat{\mathbf{h}}$ that minimises the optimisation problem is

$$\widehat{\mathbf{h}} = \frac{\widehat{\mathbf{Q}}^{-1}(\widehat{\mathbf{\Theta}})\mathbf{e}}{\mathbf{e}^H\widehat{\mathbf{Q}}^{-1}(\widehat{\mathbf{\Theta}})\mathbf{e}} \tag{21}$$

and $\widehat{\mathbf{\Theta}}$ is estimated (after plugging $\widehat{\mathbf{h}}$ in the cost function of equation (19), i.e. treating it as nuissance parameter) as follows

$$\widehat{\mathbf{\Theta}} = \arg\max_{\mathbf{\Theta}} \Big\{ \mathbf{e}^H\widehat{\mathbf{Q}}^{-1}(\mathbf{\Theta})\mathbf{e} \Big\} \tag{22}$$

Using the following identity:

$$\widehat{\mathbf{Q}}^{-1}(\mathbf{\Theta}) = \text{blkdiag}[\widehat{\mathbf{Q}}^{-1}(\omega_1, \mathbf{\Theta}^{(1)}) \dots \widehat{\mathbf{Q}}^{-1}(\omega_M, \mathbf{\Theta}^{(M)})] \tag{23}$$

and using the structure of $\mathbf{e} = \mathbb{1}_M \otimes \mathbf{e}_1$, we have

$$\widehat{\mathbf{\Theta}} = \arg\max_{\mathbf{\Theta}} \Big\{ \sum_{k=1}^{M} \mathbf{e}_1^H\widehat{\mathbf{Q}}^{-1}(\omega_k, \mathbf{\Theta}^{(k)})\mathbf{e}_1 \Big\} \tag{24}$$

If no knowledge about $\mathbf{\Theta}$ is given, then one has to reside to a N_B-dimensional search to optimise (24). On the other hand, if only one θ_i is known, then we could solve (24) by alternating optimization. Without loss of generality, let the known AoA be $\theta_{known} = \theta_1$. At an iteration i, the following AoAs (except for θ_{known}) are estimated from previous iterations:

$$\widehat{\mathbf{\Theta}}_{\bar{i}} = [\theta_{known}, \widehat{\theta}_2 \dots \widehat{\theta}_i] \tag{25}$$

Estimate $\widehat{\theta}_{i+1}$ as

$$\widehat{\theta}_{i+1} = \arg\max_{\theta} \Big\{ \sum_{k=1}^{M} \mathbf{e}_1^H\widehat{\mathbf{Q}}^{-1}(\omega_k, [\widehat{\mathbf{\Theta}}_{\bar{i}}, \theta])\mathbf{e}_1 \Big\} \tag{26}$$

where

$$\begin{aligned} \widehat{\mathbf{Q}}(\omega_k, [\widehat{\mathbf{\Theta}}_{\bar{i}}, \theta]) = &\sum_{\theta_j \in \widehat{\mathbf{\Theta}}_{\bar{i}}} \mathbf{D}^H(\theta_j)\widehat{\mathbf{U}}_n(\omega_k)\widehat{\mathbf{U}}_n^H(\omega_k)\mathbf{D}(\theta_j) \\ &+ \mathbf{D}^H(\theta)\widehat{\mathbf{U}}_n(\omega_k)\widehat{\mathbf{U}}_n^H(\omega_k)\mathbf{D}(\theta) \end{aligned} \tag{27}$$

Note that $\widehat{\theta}_{i+1} \in \widehat{\mathbf{\Theta}}_{\bar{i}}$ also maximize the above cost function. So we pick the highest peak of the 1-dimensional function in (26), such that $\widehat{\theta}_{i+1} \notin \widehat{\mathbf{\Theta}}_{\bar{i}}$. Therefore, $N_B - 1$ iterations, where each iteration consists of a 1-dimensional search as in (26), are needed to estimate the unknown parameters in $\mathbf{\Theta}$. After obtaining $\widehat{\mathbf{\Theta}}$, we can estimate $\widehat{\mathbf{h}}$ using equation (21); however each $\widehat{\mathbf{h}}(\omega_k)$ is estimated up to an unknown complex constant. Without loss of generality, we normalise the first entries of the estimated vectors $\widehat{\mathbf{h}}(\omega_k)$, $k = 1 \dots M$ to unity, viz.

$$\widehat{\mathbf{h}}(\omega_k) = \frac{\widehat{\mathbf{h}}(\omega_k)}{[\widehat{\mathbf{h}}(\omega_k)]_1}, \quad k = 1 \dots M \tag{28}$$

where $[\widehat{\mathbf{h}}(\omega_k)]_1$ is the first entry of $\widehat{\mathbf{h}}(\omega_k)$.

4. DISCUSSION: IDENTIFIABILITY AND POSSIBLE GENERALIZATIONS

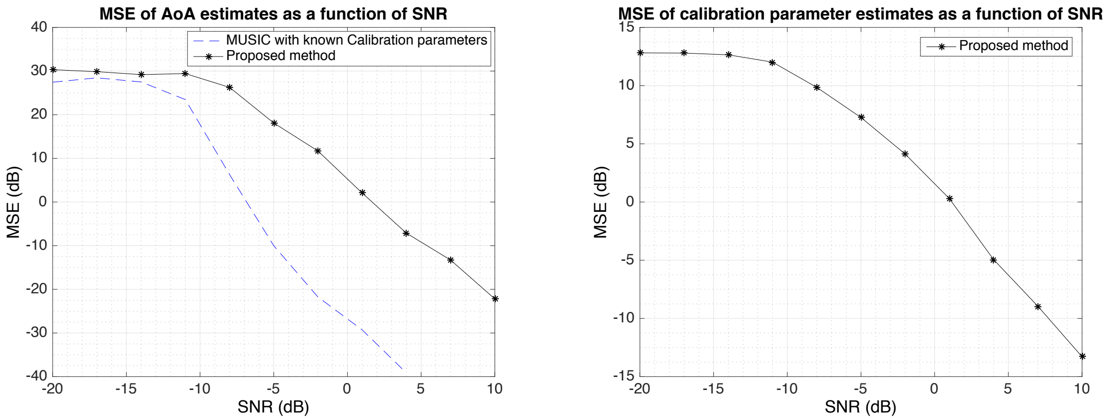
4.1. Multi-dimensional Search

Following [18], a sufficient condition for uniqueness of the solution $[\widehat{\mathbf{h}}, \widehat{\mathbf{\Theta}}]$ by first solving (24) to obtain $\widehat{\mathbf{\Theta}}$, then using equation (21) to estimate $\widehat{\mathbf{h}}$, is when then matrix $\widehat{\mathbf{Q}}^{-1}(\mathbf{\Theta})$ is invertible, or equivalently each $\widehat{\mathbf{Q}}(\omega_k, \mathbf{\Theta}^{(k)})$ (for $k = 1 \dots M$) is invertible. Recall that $\widehat{\mathbf{Q}}(\omega_k, \mathbf{\Theta}^{(k)}) = \mathbf{U}_k^H\mathbf{U}_k$, where

$$\mathbf{U}_k = \begin{bmatrix} \mathbf{U}_n^H(\omega_k)\mathbf{D}(\theta_1) \\ \vdots \\ \mathbf{U}_n^H(\omega_k)\mathbf{D}(\theta_{N_{B_k}}) \end{bmatrix}, \quad k = 1 \dots M \tag{29}$$

Therefore, $\widehat{\mathbf{Q}}(\omega_k, \mathbf{\Theta}^{(k)})$ is full rank if $\mathbf{U}_k$ is full column rank, i.e. when $(N_R - N_{B_k})N_{B_k} > N_R$, i.e. $[\widehat{\mathbf{h}}, \widehat{\mathbf{\Theta}}]$ are identifiable when the following set of inequalities are true

$$N_{B_k} + \frac{N_R}{N_{B_k}} < N_R, \quad k = 1 \dots M \tag{30}$$



(a) MSE of AoAs as a function of SNR

(b) MSE of Calibration parameters as a function of SNR

Fig. 2: MSE Experiment

4.2. Alternating optimization

The alternating optimization method proposed in equations (25) till (27) has a different identifiability condition. This is so since at iteration i, the method assumes i present sources in an attempt to estimate the $(i+1)^{th}$ AoA, namely $\widehat{\theta}_{i+1}$. Therefore, in order to ensure identifiability throughout each iteration (for $i = 2 \dots N_B$), the matricies $\widehat{\mathbf{Q}}(\omega_k, [\widehat{\mathbf{\Theta}}_{\bar{i}}, \theta])$ (for $k = 1 \dots M$) appearing in (26) should be invertible for $i = 2 \dots N_B$. Following the same reasoning as in the previous sub-section, $\widehat{\mathbf{Q}}(\omega_k, [\widehat{\mathbf{\Theta}}_{\bar{i}}, \theta])$ is invertible if and only if

$$N_{B_k} + \frac{N_R}{i} < N_R, \qquad i = 2 \dots N_B \text{ and } k = 1 \dots M \tag{31}$$

which gives $\max\left(\{N_{B_k}\}_{k=1}^{M}\right) < \frac{N_R}{2}$, for the case of 1 known AoA. This inequality shows why we have assumed knowledge for 1 AoA. This is simply due to the above inequality is not satisfied for $i = 1$. In other words, it is impossible to run the proposed alternating optimization algorithm starting with 1 present source.

In a general manner, let N_{known} be the number of known AoAs, then one could estimate $N_B - N_{known}$ AoAs in a similar manner as in the proposed alternating optimization algorithm (which is a special case, i.e. $N_{known} = 1$). The identifiability conditions of the proposed alternating optimization algorithm with N_{known} AoAs is thus

$$\max\left(\{N_{B_k}\}_{k=1}^{M}\right) + \frac{N_R}{N_{known}+1} < N_R \tag{32}$$

5. SIMULATION RESULTS

We have conducted an experiment to compare the MSE of AoAs obtained by the proposed alternating optimization (and unknown calibration parameters) in equations (25) till (27) vs. MUSIC with known calibration parameters, namely equation (11). We also plot the MSE of estimated calibration parameters. In all what follows, the experiments are conducted with 10^3 Monte-Carlo simulations. At a given SNR, let $\widehat{\theta}_k^{(j)}$ be the k^{th} estimate of θ_k at the j^{th} Monte-Carlo simulation. Similarly, let $\widehat{\mathbf{h}}(\omega_k)$ be the estimate of $\mathbf{h}^{(j)}(\omega_k)$ at the j^{th} Monte-Carlo simulation, with both vectors being normalized so that their first entries are equal to 1. Then, we define the MSE of AoA and calibration parameters as follows:

$$\text{MSE}_{\text{AoA}} = \frac{1}{500q} \sum_{j=1}^{10^3} \sum_{k=1}^{N_B} \left(\theta_k - \widehat{\theta}_k^{(j)}\right)^2 \tag{33}$$

$$\text{MSE}_{\text{Cali}} = \frac{1}{500} \sum_{j=1}^{10^3} \sum_{k=1}^{M} \frac{\left\|\mathbf{h}(\omega_k) - \widehat{\mathbf{h}}(\omega_k)\right\|}{\left\|\mathbf{h}(\omega_k)\right\|} \tag{34}$$

We fix the following simulation parameters $N_R = 10$, $M = 20$, $N_B = 3$ with $\mathbf{\Theta} = [10^\circ, 30^\circ, 40^\circ]$. We assume that at frequencies ω_k for $k = 10 \dots 15$, the 2^{nd} source ($\theta_2 = 30^\circ$) is not present. Also, at frequencies ω_k for $k = 15 \dots 20$, the the 2^{nd} and 3^{rd} source ($\theta_3 = 40^\circ$) are not present. At any other frequency, all sources are present. According to Fig. 2a, where we have plotted the MSE of AoA parameters, there is a constant "gap" of MSE between the two plots, which is of about $30dB$, when the SNR exceeds -5dB. This "gap" should be directly related to the number of calibration parameters present, which is $2MN_R = 400$ parameters (by counting the number of real parameters in $\{\mathbf{h}(\omega_k)\}_{k=1}^{M}$). Indeed, other factors contribute in this gap, such as $\mathbf{\Theta}$, N_B, $\{N_{B_k}\}_{k=1}^{M}$. In Fig. 2b, where we plot the error on calibration parameters (as in equation (34)) as a function of SNR. The performance of the error on calibration parameters is related to the estimates of AoAs (see equation (21)). Therefore, the obtained MSE curve in Fig. 2b will exhibit a similar behaviour as the one in Fig. 2a.

6. CONCLUSIONS AND FUTURE WORK

In this paper, we have addressed the problem of joint AoA and calibration estimation in a wideband scenario, where the system consists of multiple sources transmitting in different directions and occupying a certain bandwidth, with the possibility of some sources being absent at some frequencies. We propose a method that is based on MUSIC in order to formulate an optimization problem to estimate the AoAs and calibration parameters per frequency. Future work may include (i) performance analysis of the proposed method, (ii) new methods that are either more efficient or enjoy better performance in terms of MSE, and (iii) Cramer-Rao bounds of any unbiased estimator that estimates the unknown parameters of the problem in this paper.

7. REFERENCES

[1] G. Liang, W. Gong, H. Liu, and J. Yu, "Development of 61-channel digital beamforming (DBF) transmitter array for mobile satellite communication," *Progress In Electromagnetics Research*, 97, pp. 177-195, 2009.

[2] H. Li, X. Zhang, Z. He and J. Yu. "A wideband digital beamforming method based on stretch processing." *WSEAS Transactions on Signal Processing,* 5(7), pp. 251-260, 2009.

[3] L. L. Liou, D. M. Lin, M. Longbrake, P. Buxa, J. McCann, T. Dalrymple, J. B. Tsui, R. Qiu, Z. Hu, and N. Guo. "Digital wideband phased array calibration and beamforming using time reversal technique," In *Phased Array Systems and Technology (ARRAY), 2010 IEEE International Symposium on*, pp. 261-266. IEEE, 2010.

[4] A. Konovaltsev, M. Cuntz, L. A. Greda, M. V. T. Heckler and M. Meurer, "Antenna and RF front end calibration in a GNSS array receiver," *RF Front-ends for Software Defined and Cognitive Radio Solutions (IMWS), 2010 IEEE International Microwave Workshop Series on*. IEEE, 2010.

[5] G. A. Hampson and A. B. Smolders. "A fast and accurate scheme for calibration of active phased-array antennas." *IEEE Antennas and Propagation Society International Symposium*, Vol. 2. IEEE, 1999.

[6] L. Zhao, H. Liu, Y. Li, and Y. Zhou, "DOA estimation under sensor gain and phase uncertainties." *IEEE International Conference on Estimation, Detection and Information Fusion (ICEDIF)*, 2015.

[7] V. Ollier, M. N. El Korso, R. Boyer, P. Larzabal, and M. Pesavento, "Joint ML calibration and DOA estimation with separated arrays," *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)* (pp. 2996-3000), 2016.

[8] X. Cao, J. Xin, Y. Nishio, and N. Zheng, "Spatial Signature Estimation with an Uncalibrated Uniform Linear Array," *Sensors*, 15(6), 13899-13915.

[9] S. Cao, Z. Ye, D. Xu, and X. Xu. "A Hadamard product based method for DOA estimation and gain-phase error calibration," *IEEE Transactions on aerospace and electronic systems*, 49.2: 1224-1233, 2013.

[10] A. Leshem and M. Wax, "Array Calibration in the Presence of Multipath," *IEEE Transactions on Signal Processing*, 48(1):53-59, Jan 2000.

[11] B. C. Ng and C. M. S. See, "Sensor-array calibration using a Maximum Likelihood approach," *IEEE Trans. Antennas Propag.*, vol. 44, no. 6, pp. 827835, 1996.

[12] P. Stoica, M. Viberg, K. M. Wong, and Q. Wu, "Maximum-likelihood bearing estimation with partly calibrated arrays in spatially correlated noise fields," *IEEE Transactions on Signal Processing*, vol. 44, no. 4, pp. 888-899, 1996.

[13] B. Friedlander, and A. J. Weiss. "Direction finding in the presence of mutual coupling." *IEEE Transactions on Antennas and Propagation*, vol. 39, no. 3, pp. 273-284, 1991.

[14] J. Pierre and M. Kaveh. "Experimental performance of calibration and direction-finding algorithms." *IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, 1991.

[15] A. J. Weiss and B. Friedlander, "Eigenstructure methods for direction finding with sensor gain and phase uncertainties," *Circuits, Systems and Signal Processing,* 9.3 (1990): 271-300.

[16] A. Paulraj and T. Kailath. "Direction of arrival estimation by eigenstructure methods with unknown sensor gain and phase," *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)* Vol. 10, 1985.

[17] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276-280, Mar. 1986.

[18] S. Boyd, L. Vandenberghe, "Convex Optimization," *Cambridge University Press*, 2009.