Optimum Array Configurations of Maximum Output SNR for Quiescent Beamforming

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***Abstract*—In this paper, we consider optimum array configurations for multiple satellite signals in interference-free environment. The two measures of maximum output signal-to-noise ratio (SNR) and equal gains towards all sources incident on the array are considered for the array design. As it is computationally exhaustive to enumerate all configurations and implement eigenvalue decomposition to compare respective maximum eigenvalues, we resort to the relaxation of maximizing the lower bound of the output SNR. Subsequently, an iterative linear fractional programming method is proposed to maximize the spectral norm of the source covariance matrix. Simulation examples confirm that the array configuration plays a vital role in determining the array processing performance in interference-free scenarios. The selected optimum subarrays achieve maximum performance preservations with a dramatically reduced cost.**

***Index Terms*—output SNR, quiescent scenario, linear fractional programming, satellite network**

I. Introduction

Adaptive antenna arrays are capable of performing spatial filtering, which makes them an effective tool for combating interferences while providing certain gains towards desired sources. This performance requirement arises in diverse applications, such as radar, satellite network, sonar and radio astronomy to list a few [1]–[3]. The effect of array configurations on interference nulling performance in the case of single desired source was investigated in our previous work [4]–[6], where the output signal to interference plus noise ratio (SINR) is directly related to array configurations through a parameter characterizing the spatial separation between the desired source and the interference subspace. However, there is no much work in the literature examining the problem of optimum array reconfiguration and antenna placements for receiving multiple desired sources in interference-free environment or quiescent beamforming.

In this paper, we consider the general case in which the dimension of the desired signal subspace is arbitrary and not necessarily confined to a unit value. This case is encountered in multiple source emitters in the field of view with unknown or uncertain directions of arrival. It also occurs for scattered and distributed sources in astronomy [7], [8], and source spreading and fluctuation in multipath [9]. For these cases, we formulate, analyze, and solve for the two optimum array configurations which provide the maximum output signal to noise ratio (SNR) and equal gains towards all potential incoming sources are analyzed. This constitutes the main novelty of this paper. Albeit a general problem, we gear our antenna selection and placement approach towards satellite navigation application.

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With multiple source signals impinging on the receiver, the optimum weight which provides the maximum output SNR is the principal eigenvector of source covariance matrix [10]. However, the beamformer equipped with the optimum weight does not point at any particular direction, as eigenvectors are not steering vectors, and as such, are not directional. We introduce the concept of generalized inner-product to quantitatively analyze the structure of the optimum weight vector. As implementing eigenvalue decomposition to each possible configuration is prohibitively exhaustive, we resort to the relaxation of maximizing the lower bound of the maximum output SNR. We then propose an iterative linear fractional programming method for solving the antenna selection problem, which is non-convex as it maximizes a quasi-convex problem. Clearly, the optimum weight does not guarantee equal gains towards all sources, which may result in performance loss in some cases such as satellite network, where all satellite signals are identically weak and equally important [11], [12]. Thus, it becomes necessary to investigate the optimum sparse array configuration that maximizes the output SNR while providing equal sensitivities towards all sources. Thus, we also investigate the optimum beamformer which yields the maximum output SNR while providing equal gains towards all incoming sources.

The rest of this paper is organized as follows: We formulate the problem in section II, and analyze in section III the structure of the optimum weight vector utilizing the generalized inner product. Formulation of antenna selection problem is elucidated in section IV. Simulation results, presented in section V, validate the effectiveness of proposed methods. Finally, concluding remarks are provided in section VI.

II. Problem Formulation

Consider a linear array of N isotropic antennas with positions specified by multiple integer of unit inter-element spacing $x_n d, x_n \in \mathbb{N}, n = 1, \cdots, N$. Suppose that p emitter, or satellite, signals are impinging on the array from directions $\mathbf{\Theta} = \{\theta_1, \cdots, \theta_p\}$ with spatial steering vectors specified by,

$$\mathbf{u}_k = [e^{jk_0 x_1 d \cos\theta_k}, \cdots, e^{jk_0 x_N d \cos\theta_k}]^T, k = 1, \cdots, p, \quad (1)$$

respectively. The wavenumber is defined as $k_0 = 2\pi/\lambda$ with λ being the wavelength and ^{T} denotes transpose operation. The received signal at time instant t is given by,

$$\mathbf{x}(t) = \mathbf{U}\mathbf{s}(t) + \mathbf{n}(t), \tag{2}$$

where $\mathbf{U} = [\mathbf{u}_1, \cdots, \mathbf{u}_p] \in \mathbb{C}^{N\times p}$ is the source array manifold matrix. In the above equation, $\mathbf{s}(t) \in \mathbb{C}^p$ denotes the source vector and $\mathbf{n}(t) \in \mathbb{C}^N$ represents the received noise vector. The output of the N-antenna beamformer is given by,

$$y(t) = \mathbf{w}^H\mathbf{x}(t), \tag{3}$$

where $\mathbf{w} \in \mathbb{C}^N$ is the complex vector of beamformer weights and ^{H} stands for Hermitian operation. With additive Gaussian noise, i.e. $\mathbf{n}(t) \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2\mathbf{I})$ and in the absence of interfering sources, where σ_n^2 is noise power level, the optimal weight vector for maximizing the output SNR is given by [10],

$$\mathbf{w}_{\text{opt}} = \mathcal{P}\{\mathbf{R}_u\} = \mathcal{P}\{\mathbf{U}\mathbf{R}_s\mathbf{U}^H\}. \tag{4}$$

where $\mathcal{P}\{\cdot\}$ denotes the principal eigenvector of the matrix, $\mathbf{R}_u$ is defined as $\mathbf{R}_u = \mathbf{U}\mathbf{R}_s\mathbf{U}^H$ with $\mathbf{R}_s = E\{\mathbf{s}(t)\mathbf{s}^H(t)\}$ denoting the source auto-correlation matrix. There are no restrictions on the source signals, that is, the sources can be either coherent or uncorrelated.

The corresponding output SNR is,

$$\text{SNR}_{\text{opt}} = \frac{\mathbf{w}_{\text{opt}}^H\mathbf{R}_u\mathbf{w}_{\text{opt}}}{\mathbf{w}_{\text{opt}}^H\mathbf{R}_n\mathbf{w}_{\text{opt}}} = \frac{\lambda_{\max}\{\mathbf{R}_u\}}{\sigma_n^2} = \frac{\|\mathbf{R}_u\|_2}{\sigma_n^2}. \tag{5}$$

Clearly, array configuration affects the output SNR of the optimum beamformer $\mathbf{w}_{\text{opt}}$ through the term of $\|\mathbf{R}_u\|_2$ which is evident in Eq. (5). The focus of this work is to analyze the structure of the optimum beamformer in interference-free or quiescent environments and propose two methods of array reconfiguration for optimum output performance through antenna selection.

III. Structure of the Optimum Beamformer

The optimum beamformer in Eq. (4) is not directional. This means that the beamformer does not point towards a particular direction spatially. Clearly, the optimum beamformer is a linear combination of the source steering vectors, i.e. $\mathbf{w}_{\text{opt}} = \sum_{k=1}^p \beta_k\mathbf{u}_k$. Note that the coefficients $\beta_k \neq \mathbf{u}_k^H\mathbf{w}_{\text{opt}}, k = 1, \ldots, p$, as the steering vectors are not orthogonal basis. Next, we introduce the concept of generalized inner product to analyze the structure of the optimum beamformer.

Denote the subspace spanned by the source steering vectors as $\tilde{\mathbf{U}} = \text{span}\{\mathbf{U}\} = \text{span}\{\mathbf{u}_1, \ldots, \mathbf{u}_p\}$. The generalized inner product between any vector $\mathbf{u} \in \tilde{\mathbf{U}}$ and $\mathbf{u}_k, k = 1, \ldots, p$ is defined as, [13]–[15]

$$\begin{aligned} & \langle \mathbf{u}, \mathbf{u}_k | \mathbf{u}_{i_2(k)}, \ldots, \mathbf{u}_{i_p(k)}\rangle \\ = \; & \langle \mathbf{u}, \mathbf{u}_k | \mathbf{u}_1, \ldots, \mathbf{u}_{k-1}, \mathbf{u}_{k+1}, \ldots, \mathbf{u}_p\rangle \\ = \; & |\mathbf{U}_{(k)}^H\mathbf{U}|, \end{aligned} \tag{6}$$

where $\{i_2(k), \ldots, i_q(k)\} = \{1, \ldots, q\} \setminus \{k\}$ and $\mathbf{U}_{(k)}$ is the matrix with the kth column replaced by the vector $\mathbf{u}$, $|\cdot|$ means the determinant of the matrix. Accordingly, the generalized norm is defined as $\|\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_p\| := \langle \mathbf{u}_1, \mathbf{u}_1 | \mathbf{u}_2, \ldots, \mathbf{u}_p\rangle^{1/2} = |\mathbf{U}^H\mathbf{U}|^{1/2}$, which represents the square root of the volume of the p-dimensional parallelepiped spanned by $\mathbf{U}$. The decomposition of the optimum beamformer is explained by Theorem 1.

Theorem 1: For an arbitrary set of source steering vectors $\{\mathbf{u}_1, \ldots, \mathbf{u}_p\}$, the optimum beamformer $\mathbf{w}_{\text{opt}}$ in Eq. (4) can be expressed as,

$$\mathbf{w}_{\text{opt}} = \sum_{k=1}^p \beta_k\mathbf{u}_k, \tag{7}$$

where

$$\beta_k^* = \frac{\langle \mathbf{w}_{\text{opt}}, \mathbf{u}_k | \mathbf{u}_{i_2(k)}, \ldots, \mathbf{u}_{i_p(k)}\rangle}{\|\mathbf{u}_1, \ldots, \mathbf{u}_p\|^2} = \frac{|\mathbf{U}_{(k)}^H\mathbf{U}|}{|\mathbf{U}^H\mathbf{U}|}, \tag{8}$$

where $\mathbf{U}_{(k)}$ is the matrix with the kth column of $\mathbf{U}$ replaced by $\mathbf{w}_{\text{opt}}$ and $\{i_2(k), \ldots, i_p(k)\} = \{1, \ldots, p\} \setminus \{k\}$.

The proof of Theorem 1 is provided in section VII-A. The coefficient $\beta_k, k = 1, \ldots, p$ reflects the gain of the optimum beamformer towards the kth source.

IV. Antenna Selection for Optimum SNR

Denote an antenna selection vector $\mathbf{z} \in \{0, 1\}^N$ with "zero" entry denoting the corresponding antenna discarded and "one" entry for a selected antenna. As steering vectors are directional, the implementation of antenna selection is clearly expressed as $\mathbf{u}_i(\mathbf{z}) = \mathbf{u}_i \odot \mathbf{z}, i = 1, \ldots, p$ with $\odot$ denoting element-wise product and, accordingly, $\mathbf{U}(\mathbf{z}) = [\mathbf{u}_1(\mathbf{z}), \ldots, \mathbf{u}_p(\mathbf{z})]$. Ideally, the antenna array should be reconfigured through antenna selection $\mathbf{z}$ such that the spectral norm of the reduced-dimensional source auto-correlation matrix $\|\mathbf{U}(\mathbf{z})\mathbf{R}_s\mathbf{U}^H(\mathbf{z})\|_2$ is maximum. However, since maximizing the spectral norm of a matrix with respect to the selection variable $\mathbf{z}$ is a non-convex optimization problem, we resort to the following relaxation method.

Denote the maximum eigenvalues of the source auto-correlation matrix $\mathbf{U}(\mathbf{z})\mathbf{R}_s\mathbf{U}^H(\mathbf{z})$ of a full array ($\mathbf{z} = \mathbf{1}$) and a selected subarray as λ_0 and $\hat{\lambda}_0$, respectively, with their corresponding principal eigenvectors as $\mathbf{e}_0$ and $\hat{\mathbf{e}}_0$. Then, $\mathbf{e}_0 = \sum_{i=1}^p \beta\mathbf{u}_i$ and $\hat{\mathbf{e}}_0 = \sum_{i=1}^p \hat{\beta}\mathbf{u}_i(\mathbf{z})$ with $\hat{\beta}$ denoting the coefficient vector corresponding to the subarray. The source auto-correlation matrix upon implementing antenna selection can be written as,

$$\mathbf{U}(\mathbf{z})\mathbf{R}_s\mathbf{U}^H(\mathbf{z}) = \mathbf{E}(\mathbf{z})\mathbf{\Lambda}\mathbf{E}^H(\mathbf{z}), \tag{9}$$

where $\mathbf{R}_u = \mathbf{E}\mathbf{\Lambda}\mathbf{E}^H$ for the full array, with λ_0 and $\mathbf{e}_0$ denoting the maximum eigenvalue and principal eigenvector respectively. The set of vectors $\mathbf{E}(\mathbf{z})$ constitutes a basis of the reduced-dimensional source subspace with corresponding coefficients in the diagonal of the matrix $\mathbf{\Lambda}$. Furthermore, the vector $\mathbf{e}_0(\mathbf{z})$ still possesses the largest coefficient λ_0, thus we can utilize $\mathbf{e}_0(\mathbf{z})/\|\mathbf{e}_0(\mathbf{z})\|$ to approximate the principal eigenvector of the selected subarray. Note that these bases are not eigenvectors due to their non-orthonormality, i.e., $\mathbf{E}^H(\mathbf{z})\mathbf{E}(\mathbf{z}) \neq \mathbf{I}$ although $\mathbf{E}(\mathbf{z})\mathbf{E}^H(\mathbf{z}) = \mathbf{I}$. Thus, we approximate the gain vector of the subarray with β, i.e.,

$$\hat{\mathbf{e}}_0 \approx \mathbf{e}_0(\mathbf{z}) = \sum_{i=1}^p \beta\mathbf{u}_i(\mathbf{z}). \tag{10}$$

The lower bound of the optimum output SNR of the selected subarray can, therefore, be established as

$$\frac{\mathbf{e}_0^H(\mathbf{z})}{\|\mathbf{e}_0(\mathbf{z})\|}\mathbf{R}_u(\mathbf{z})\frac{\mathbf{e}_0(\mathbf{z})}{\|\mathbf{e}_0(\mathbf{z})\|} \leq \hat{\mathbf{e}}_0^H\mathbf{R}_u(\mathbf{z})\hat{\mathbf{e}}_0. \tag{11}$$

The problem of antenna selections for maximizing the optimum SNR in quiescent conditions is formulated as,

$$\begin{aligned} \max_{\mathbf{z}} \quad & \frac{\mathbf{e}_0^H(\mathbf{z})\mathbf{R}_u(\mathbf{z})\mathbf{e}_0(\mathbf{z})}{\|\mathbf{e}_0(\mathbf{z})\|^2}, \\ \text{s.t.} \quad & \mathbf{1}^T\mathbf{z} = K, \\ & \mathbf{0} \leq \mathbf{z} \leq 1. \end{aligned} \tag{12}$$

We relax the binary constraints $\mathbf{z} \in \{0,1\}^N$ in the third line of Eq. (12) to a box constraint $\mathbf{0} \leq \mathbf{z} \leq 1$, as the objective in Eq. (12) is quasi-convex and the global maximizer of a quasi-convex function locates at the extreme points of the polyhedral [16], [17]. Define the vector $\bar{\mathbf{e}}_0 = \mathbf{e}_0^* \odot \mathbf{e}_0$ and the matrix $\bar{\mathbf{U}} = [\mathbf{e}_0^* \odot \mathbf{u}_1, \ldots, \mathbf{e}_0^* \odot \mathbf{u}_p]$ with $\odot$ denoting element-wise product and * being conjugate operation. This leads to the simplification $\|\mathbf{e}_0(\mathbf{z})\|^2 = \mathbf{z}^T\bar{\mathbf{e}}_0$ and $\mathbf{e}_0^H(\mathbf{z})\mathbf{R}_u(\mathbf{z})\mathbf{e}_0(\mathbf{z}) = \mathbf{z}^T\bar{\mathbf{U}}\mathbf{R}_s\bar{\mathbf{U}}^H\mathbf{z}$, which in turn allows Eq. (12) to be rewritten as,

$$\begin{aligned} \max_{\mathbf{z}} \quad & \frac{\mathbf{z}^T\bar{\mathbf{U}}\mathbf{R}_s\bar{\mathbf{U}}^H\mathbf{z}}{\mathbf{z}^T\bar{\mathbf{e}}_0}, \\ \text{s.t.} \quad & \mathbf{1}^T\mathbf{z} = K, \\ & \mathbf{0} \leq \mathbf{z} \leq 1. \end{aligned} \tag{13}$$

Clearly the problem represented by Eq. (13) is still non-convex. We propose an iterative linear fractional programming method which linearizes the numerator globally in each iteration [18]. The problem in the $(k+1)$th iteration based on the kth solution $\mathbf{z}^k$ is formulated as

$$\begin{aligned} \max_{\mathbf{z}} \quad & \frac{-\mathbf{z}^{kT}\bar{\mathbf{U}}\mathbf{R}_s\bar{\mathbf{U}}^H\mathbf{z}^k + 2\mathbf{z}^{kT}\bar{\mathbf{U}}\mathbf{R}_s\bar{\mathbf{U}}^H\mathbf{z}}{\mathbf{z}^T\bar{\mathbf{e}}_0}, \\ \text{s.t.} \quad & \mathbf{1}^T\mathbf{z} = K, \\ & \mathbf{0} \leq \mathbf{z} \leq 1. \end{aligned} \tag{14}$$

The linear fractional programming in Eq. (14) can be further transformed into linear programming as [19], [20],

$$\begin{aligned} \max_{\mathbf{v},\alpha} \quad & -\mathbf{z}^{kT}\bar{\mathbf{U}}\mathbf{R}_s\bar{\mathbf{U}}^H\mathbf{z}^k\alpha + 2\mathbf{z}^{kT}\bar{\mathbf{U}}\mathbf{R}_s\bar{\mathbf{U}}^H\mathbf{v}, \\ \text{s.t.} \quad & \mathbf{1}^T\mathbf{v} - K\alpha = 0, \\ & \mathbf{v} \geq 0,\ \mathbf{v} - \alpha \leq 0, \\ & \alpha \geq 0,\ \bar{\mathbf{e}}_0^T\mathbf{v} = 1. \end{aligned} \tag{15}$$

Finally, we obtain the selection vector $\mathbf{z} = \mathbf{v}/\alpha$. Note that the optimum weight in Eq. (4) does not guarantee the same again towards each source as exhibited in Fig. 3 in the simulation section. If a desired specific gain toward each source is required, for example equal gains, then the coefficient vector $\hat{\beta}$ becomes known. In this case, the antenna selection problem can be formulated as

$$\begin{aligned} \max_{\mathbf{z}} \quad & \frac{\hat{\beta}^H\mathbf{U}^H\text{diag}(\mathbf{z})\mathbf{R}_u\text{diag}(\mathbf{z})\mathbf{U}\hat{\beta}}{\hat{\beta}^H\mathbf{U}^H\text{diag}(\mathbf{z})\mathbf{U}\hat{\beta}} \\ \text{s.t.} \quad & \mathbf{1}^T\mathbf{z} = K, \\ & \mathbf{0} \leq \mathbf{z} \leq 1. \end{aligned} \tag{16}$$

Utilizing the following property of Khatri-Rao product $\circ$,

$$\mathbf{A}\text{diag}(\mathbf{x})\mathbf{b} = (\mathbf{b}^T \circ \mathbf{A})\mathbf{x}, \tag{17}$$

we obtain

$$\mathbf{U}^H\text{diag}(\mathbf{z})\mathbf{U}\hat{\beta} = [(\mathbf{U}\hat{\beta})^T \circ \mathbf{U}^H]\mathbf{z}. \tag{18}$$

Define the vector $\bar{\beta} = (\mathbf{U}\hat{\beta}) \odot (\mathbf{U}\hat{\beta})^*$ and the matrix $\bar{\bar{\mathbf{U}}} = (\mathbf{U}\hat{\beta})^T \circ \mathbf{U}^H$. The problem Eq. (16) can be rewritten as

$$\begin{aligned} \max_{\mathbf{z}} \quad & \frac{\mathbf{z}^T\bar{\bar{\mathbf{U}}}^H\mathbf{R}_s\bar{\bar{\mathbf{U}}}\mathbf{z}}{\mathbf{z}^T\bar{\beta}} \\ \text{s.t.} \quad & \mathbf{1}^T\mathbf{z} = K, \\ & \mathbf{0} \leq \mathbf{z} \leq 1. \end{aligned} \tag{19}$$

The iterative linear fractional programming can then be utilized to obtain the optimal beamformer with a desired gain.

V. Simulations

In this section, simulation results are presented to validate the proposed array reconfiguration strategy and antenna selection method.

A. Example 1

Consider $K = 8$ available antennas and $N = 16$ uniformly spaced positions with inter-element spacing of $d = \lambda/2$. There are three uncorrelated source signals impinging on the array from directions $\theta_1 = 65°, \theta_2 = 75°, \theta_3 = 115°$ with SNR being 6dB, 3dB and 0dB, respectively. Assuming that all the information of the sources is known to the receiver, thus the optimum weight can be calculated as the principal eigenvector of the source auto-correlation matrix as stated in Eq. (4). Utilizing the concept of generalized inner product in section III, we can decompose the optimum weight in terms of the source steering vectors as shown in Eq. (7), where the coefficient vector β is,

$$\beta = [0.14 + j0.94,\ 0.14 + j0.12,\ 0.02]^T. \tag{20}$$

Clearly, the optimum weight puts more emphasis on the first source compared with the third one so as to obtain the maximum output SNR. We enumerate all the 12870 different configurations based on three metrics: the output SNR in the right hand side of Eq. (11), the lower bound in the left hand side and the output SNR of the equal gain beamformer in Eq. (16). The results are plotted in Fig. 1 in an ascending order of the output SNR. The following remarks are in order: (1) The optimum array can attain 4.5dB output SNR, which is 1.5dB higher than the worst array configuration. This verifies the important role of array configuration for determining the output SNR in interference-free and quiescent scenarios. (2) The lower bounds of the optimum SNR are tight for all configurations and the distance between them is no more than 0.6dB. Note that the distance between the maximum SNR among all the configurations and its lower bound is only 0.04dB. (3) The equal gain beamformer performs worse than that of the optimum weight in terms of output SNR. The maximum output SNR of the equal gain beamformer is 3.7dB, however, it is the best array that can guarantee equal gain towards each source. (4) The array configuration also affects the performance of the equal gain beamformer significantly, where the performance difference between the best and the worst array is 5.81dB.

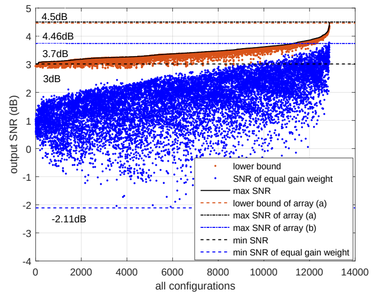

Fig. 1. The optimum output SNR, its lower bound and the output SNR of equal gain beamformer for all configurations.

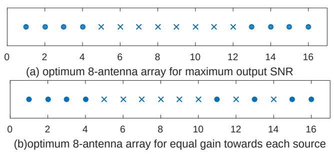

Fig. 2. The selected optimum arrays: (a) the optimum 8-antenna array for maximum output SNR; (b) the optimum 8-antenna array for equal gain towards each source.

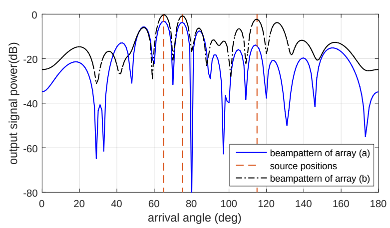

Fig. 3. The beampatterns of the two arrays (a) and (b) in Fig. 2.

We also implement the two proposed antenna selection methods of the optimum array configurations for maximum output SNR in Eq. (12) and equal gain towards each source in Eq. (16) respectively. The two selected 8-antenna optimum arrays are shown in Fig. 2 (a) and (b), respectively. The two selected arrays are exactly the optimum ones obtained by enumeration, which validates the effectiveness of the proposed iterative linear fractional programming method. The beampatterns of the two arrays are depicted in Fig. 3, which fully demonstrates the advantages of the array (b). Note that the weight vector for array (a) is calculated according to Eq. (4) and that for array (b) is according to Eq. (7) with the coefficient vector $\beta = [1, 1, 1]^T$.

B. Example 2

Next, we consider a scenario in satellite network. Assuming that prior information of the sources, such as the exact arrival directions and the power level, is unavailable. Consider three uncorrelated sources impinging on a 20-antenna uniform linear array (ULA) from the angular sectors $[60°, 65°]$, $[110°, 115°]$ and $[160°, 165°]$ respectively. We sample each angular sector and calculate an optimum 10-antenna array assuming equal source power. We assume one set of weights, in lieu of separate sets of weight for each satellite. The optimum array is shown in Fig. 4. We implement 200 trials, where a set of random angles are uniformly generated in the three angular sectors. We compare the output SNR values of the selected 10-antenna subarray and the full 20-antennas ULA, and plot the SNR difference in Fig. 5 (a). Clearly, the subarray configuration can halve the hardware cost and reduce the computational complexity by 87.5%, while the performance degradation is only 0.3dB in some scenarios with the worst case of 1.33 dB. We also compare the average correlation output of the 200 trials between the selected subarray and the full array in Figs. 5(b) and (c), which exhibits tantamount acquisition performance. This example again affirms the important role of array configuration in determining the adaptive beamformers' performance.

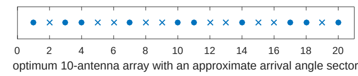

Fig. 4. The selected 10-antenna optimum subarray.

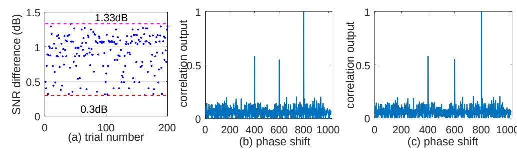

Fig. 5. (a) The output SNR difference between the full array and the subarray; (b) Average acquisition performance of the subarray in Fig. 4; (c) Average acquisition performance of the full array.

VI. Conclusions

We investigated the problem of optimum array configurations of adaptive beamformers in interference-free environments and quiescent operating conditions. A tight lower bound of the optimum SNR was derived and an iterative linear fractional programming was proposed to solve the antenna selection problem. Simulation results demonstrated that array configurations can be designated as an additional degree of freedom to improve the output SNR without increasing cost.

VII. Appendix

A. Proof of Theorem 1

Utilizing Eq. (7), the inner-product between $\mathbf{w}_{\text{opt}}$ and the steering vector $\mathbf{u}_l, l = 1, \ldots, p$ can be written as,

$$\langle \mathbf{w}_{\text{opt}}, \mathbf{u}_l \rangle = \sum_{k=1}^{p} \beta_k^* \langle \mathbf{u}_k, \mathbf{u}_l \rangle, l = 1, \ldots, p. \tag{21}$$

Considering all p generalized inner products and formulating them into a matrix form, we obtain

$$\mathbf{A}\beta^* = \mathbf{g}, \tag{22}$$

$$\begin{bmatrix} \langle \mathbf{u}_1, \mathbf{u}_1 \rangle & \ldots & \langle \mathbf{u}_p, \mathbf{u}_1 \rangle \\ \vdots & \ddots & \vdots \\ \langle \mathbf{u}_1, \mathbf{u}_p \rangle & \ldots & \langle \mathbf{u}_p, \mathbf{u}_p \rangle \end{bmatrix} \begin{bmatrix} \beta_1^* \\ \vdots \\ \beta_p^* \end{bmatrix} = \begin{bmatrix} \langle \mathbf{w}_{\text{opt}}, \mathbf{u}_1 \rangle \\ \vdots \\ \langle \mathbf{w}_{\text{opt}}, \mathbf{u}_p \rangle \end{bmatrix}.$$

According to Cramer's rule and utilizing Eq. (6) yields

$$\beta_k^* = \frac{|\mathbf{A}_{(k)}^T|}{|\mathbf{A}|} = \frac{|\mathbf{U}_{(k)}^H \mathbf{U}|}{|\mathbf{U}^H \mathbf{U}|} = \frac{\langle \mathbf{u}, \mathbf{u}_k | \mathbf{u}_{i_2(k)}, \ldots, \mathbf{u}_{i_p(k)} \rangle}{\|\mathbf{u}_1, \ldots, \mathbf{u}_p\|^2}. \tag{23}$$

The matrix $\mathbf{A}_{(k)}$ is obtained by replacing the kth column of $\mathbf{A}$ by the vector $\mathbf{g}$. □

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