SIMULTANEOUS CODED PLANE WAVE IMAGING IN ULTRASOUND: PROBLEM FORMULATION AND CONSTRAINTS

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ABSTRACT

In this paper, we propose a new emission strategy in the context of plane wave imaging. Plane wave imaging indeed implies compounding in order to preserve a good image quality. Such compounding is usually obtained using multiple, successive emissions, which in turn yields a decrease of the frame rate. As opposed to this approach, our method is based on the simultaneous emission of several coded plane waves. This allows the reconstruction of all the images corresponding to the different plane waves, by using an inverse problem approach. The proposed method is closely related to channel estimation in telecommunications, and coded excitation in synthetic aperture ultrasound imaging. In this paper, we extend these lines of work to the case of ultrasound plane wave imaging and evaluate the obtained performance from various numerical simulations.

1. INTRODUCTION

Numerous researches have been conducted to increase the data acquisition rate in medical ultrasound. A milestone in this field was the introduction of plane wave insonification [1]. In the concept of plane wave ultrasound imaging, a plane wave insonifies the full region of interest allowing the reconstruction of a complete image from only one transmission. Thus the time needed to construct a full image of the medium depends only on the sound propagation speed in soft tissues. Plane wave insonification however has a drawback, that is the lower overall quality of the image. To overcome this issue, Montaldo et al. [2] proposed the concept of Plane Wave Image Compounding (PWIC). In their work they demonstrated that a coherent summation of the images obtained from N_{pwi} steered plane wave insonifications of the same medium provides a final image with higher quality. The image quality however, can drop significantly if the imaged medium moves between the consecutive insonifications. The successive emission/reception of N_{pwi} steered plane waves affects also the acquisition rate of a high quality image, which is decreased by a factor of N_{pwi}.

This work was supported by the LABEX PRIMES (ANR-11-LABX-0063) of Universit de Lyon, within the program "Investissements d'Avenir" (ANR-11-IDEX-0007) operated by the French National Research Agency (ANR).

In this paper we propose a novel method that allows Simultaneous emission and reconstruction of N_{pwi} low quality images using Coded Emissions (SCE). The paper is organized as follows. First we present the related work in the literature, next we describe the way to formulate the approach that allows to emit simultaneously N_{pwi} plane waves, receive all the scattered echoes from the medium and detect the echoes of each plane wave. Then we present some simulations results and compare them with the classical plane wave compounding approach. In the last part of this paper a discussion on the potential ameliorations of the method is open, and a conclusion is drawn.

2. RELATED WORK

Since coded emissions are largely used in SONAR [3] and telecommunication systems [4], there are more and more applications of coded excitation in ultrasound medical imaging. Basically there are three main approaches to address coded emissions in medical ultrasound. The first approach, also used in plane wave imaging, consists in quasi-simultaneous emission of plane waves, excitation signals corresponding to each of the plane wave being multiplied by a factor of $+1$ or -1 [5]. The contributions of each plane wave are then determined by a set of linear combinations of the received echoes. The second approach, used mostly in synthetic aperture imaging [6], [7], consists in the emission of waves that carry binary codes. This method is based on the orthogonality of the codes and different matched filtering techniques are applied on the received signals to discriminate the echoes of each transmitted wave. The third approach, consists in modeling the imaged medium by its impulse response to each of the emitted codes [8], [9], [10]. Shen et al. [8] proposed a method for sector-scan ultrasound imaging, while Gran et al. [9] and Chiao et al. [11] applied this approach in synthetic aperture. In this paper we develop this method for plane wave imaging. Similar problem formulation can be found in Multiple Input Multiple Output (MIMO) wireless communications systems field as the channel estimation problem [12], [13].

3. THEORETICAL CONSIDERATIONS

Consider a simple case of a linear probe with N_e equally spaced elements that emits a single plane wave with a wavevector steered in the direction θ (see Fig. 1A).

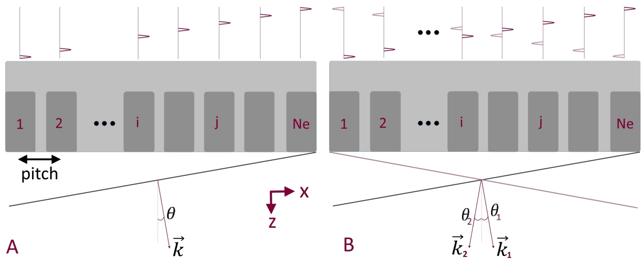

Fig. 1. A - Schematic representation of an ultrasound probe with N_e elements emitting a plane wave with a wavevector steered in the direction θ relative to the z axis, B - Schematic representation of a probe emitting simultaneously $N_{pwi} = 2$ coded plane waves. At the top of the pictures are shown the emission of each of the N_e elements

Assuming that the waveform to be sent is $a(t)$ each element j of the probe will emit a signal given by:

$$a_j(t) = a(t) * \delta(t - t_j) \tag{1}$$

with $t_j = (j-1)pitch \times tan(\theta)/c$, c being the sound propagation velocity in the medium. $\delta(t - t_j)$ is the time shifted Dirac delta function.

In telecommunication [4] different types of modulation are feasible by continuously changing the amplitude, phase, frequency of the emitted signal. For this application we decided to use Binary Shift Frequency Keying to modulate orthogonal binary sequences at the central frequency of the ultrasound probe. This choice will be discussed later in this section. Further in this paper we will refer to BPSK modulated sequences as signals $a(t) = s(t)sin(2\pi f_0 t)$, where $s(t)$ is a binary sequence and f_0 is the central frequency of the probe.

The received signal of the ith element can be written as:

$$y_i(t) = \sum_{j=1}^{N_e} a_j(t) * h_{je}(t) * g_{ji}(t) * h_{ir}(t) + v_i(t) \tag{2}$$

where $h_{je}(t)$ is the acousto-electrical impulse response of the jth element at emission, $h_{ir}(t)$ is the acousto-electrical impulse response of the ith element at reception and $v_i(t)$ is the noise on the ith element. $g_{ji}(t)$ represents the impulse response of the medium when the jth element emits and the ith receives.

Given (1), (2) can be written as:

$$y_i(t) = a(t) * g_i(t) + v_i(t) \tag{3}$$

with

$$g_i(t) = \sum_{j=1}^{N_e} h_{je}(t) * \delta(t - t_j) * g_{ji}(t) * h_{ir}(t) \tag{4}$$

In (4), the function $g_i(t)$ can be seen as the interference between all the delayed impulse responses of the medium when the jth element emits and the ith receives, convolved with the acousto-electrical responses $h_{je}(t)$ and $h_{ir}(t)$. As $g_i(t)$ is a function of the steering θ of the plane wave, we will call it the pulsed plane wave response of the medium seen by the ith element of the probe.

These N_e pulsed plane wave responses can be further processed to obtain low quality images, hence a method for their estimation is required. In medical ultrasound applications, each element emits the waves and receives the echoes which means the probe will start receiving scattered echoes from the medium after sending all the N_a samples of the signal $\mathbf{a}$. Therefore during the signal emission, $\mathbf{y}_i$ is null for all $i \leq N_a$, which corresponds to region called *blind zone*. After the end of emission the probe starts to receive and the length of the signal $\mathbf{y}_i$, N_y, depends only on the recording time T_y: $N_y = T_y/f_s$, where f_s is the sampling frequency of the received signal. For an arbitrary recording duration T_y, one cannot guarantee that all the samples of the convolution between the emitted signal $\mathbf{a}$ and the pulsed plane wave response $\mathbf{g}_i$ have been received, hence the part of the medium whose samples are missing is called *perturbation zone*. The formation of the *blind zone* and *perturbation zone*, is schematically shown in the Fig. 2.

The relation (4) can be also written as a matrix product:

$$\mathbf{y}_i = \mathbf{A}\mathbf{g}_i + \mathbf{v}_i \tag{5}$$

with:

$$\begin{aligned}
\mathbf{y}_i &= \begin{bmatrix} y_i[0] & y_i[1] & y_i[2] & y_i[3] & \cdots & y_i[N_y] \end{bmatrix}^T \\
\mathbf{g}_i &= \begin{bmatrix} g_i[0] & g_i[1] & g_i[2] & g_i[3] & \cdots & g_i[N_g] \end{bmatrix}^T \\
\mathbf{v}_i &= \begin{bmatrix} v_i[0] & v_i[1] & v_i[2] & v_i[3] & \cdots & v_i[N_y] \end{bmatrix}^T
\end{aligned}$$

N_y, N_g, N_y are respectively the lengths of the vectors $\mathbf{y}_i$, $\mathbf{g}_i$, $\mathbf{v}_i$ and N_a is the length in samples of the signal $a(t)$. Using Fig. 2 the $\mathbf{A}$ matrix is built as shown in Fig. 3.

In case of simultaneous emission of N_{pwi} plane waves (see Fig. 1B), the relation 4 transforms into:

$$y_i(t) = \sum_{k=1}^{N_{pwi}} a^k(t) * g_i^k(t) + v_i(t) \tag{6}$$

where $a^k(t)$ represents the signal carried by the kth plane wave and $g_i^k(t)$ is the pulsed plane wave response of the medium seen by the ith element of the probe when insonified with the kth plane wave. In matrix form, the relation (6) can be written as follows:

$$\mathbf{y}_i = \sum_{k=1}^{N_{pwi}} \mathbf{A}^k \mathbf{g}_i^k + \mathbf{v}_i = \mathbf{A}_c \bar{\mathbf{g}}_i + \mathbf{v}_i \tag{7}$$

where the matrix $\mathbf{A}_c$ represents the horizontal concatenation of $\mathbf{A}^k$ matrices and $\bar{\mathbf{g}}_i$ represents the vertical concatenation of the column vectors $\mathbf{g}_i^k$:

$$\begin{aligned}
\mathbf{A}_c &= \begin{bmatrix} \mathbf{A}^1 & \mathbf{A}^2 & \mathbf{A}^3 & \cdots & \mathbf{A}^{Npwi} \end{bmatrix} \\
\bar{\mathbf{g}}_i &= \begin{bmatrix} \mathbf{g}_i^1 & \mathbf{g}_i^2 & \mathbf{g}_i^3 & \cdots & \mathbf{g}_i^{Npwi} \end{bmatrix}^T
\end{aligned}$$

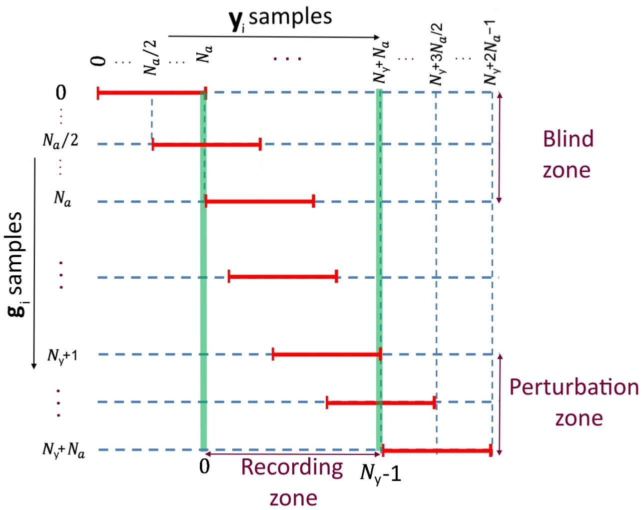

Fig. 2. Schematic representation of the convolution between the signal **a** (red) and the pulsed plane wave response $\mathbf{g}_i$. The WP-SCE method reconstructs only the part of the medium between the blind and perturbation zone while the IP-SCE reconstructs the entire medium

$$\mathbf{A} = \overbrace{\begin{bmatrix} a[N_a-1] & a[N_a-2] & \cdots & a[0] & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & a[N_a-1] & \cdots & a[1] & a[0] & \ddots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a[N_a-1] & \vdots & \ddots & a[0] & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & a[N_a-1] & \ddots & a[1] & a[0] & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 0 & \cdots & a[N_a-1] & a[N_a-2] & \cdots & a[0] \end{bmatrix}}^{N_y+N_a-1} N_y$$

Fig. 3. Matrix **A** that includes both *blind* and *perturbation zone*

Each of the matrix $\mathbf{A}^k$ is computed as shown in Fig. 3. The size of the vectors $\mathbf{y}_i$ and $\bar{\mathbf{g}}_i$ is N_y respectively $N_{pwi} \times N_g$. The matrix $\mathbf{A}_c$ has N_y rows and $N_{pwi} \times (N_y + N_a - 1)$ columns.

To reconstruct each of the N_{pwi} low quality images one need to estimate each of the $\mathbf{g}_i^k$ signals, that are the echoes received by the ith element of the probe when a plane wave, carrying an impulse signal, is sent in the kth direction. For this paper we propose to use the Least Square Estimator to solve this inverse problem:

$$\hat{\bar{\mathbf{g}}}_i = (\mathbf{A}_c^T \mathbf{A}_c)^{-1} \mathbf{A}_c^T \mathbf{y}_i \tag{8}$$

However, from Fig. 3 and equation (7) it is clear that the system matrix $\mathbf{A}_c$, has far fewer rows (N_y) than columns ($N_{pwi}(N_y + N_a - 1)$), which means that the system is underdetermined. As a consequence, its solution is not unique, but is a linear combination of the ideal solution and the null-space of $\mathbf{A}_c$. The use of BPSK modulated sequences as the signals $a^k(t)$ yields a low row and column cross-correlation in the matrix $\mathbf{A}_c$ which means that the matrix is well-conditioned. As this system has a infinite number of solutions we will refer to this method as Ill-Posed formulation of the Simultaneous Coded Emission (IP-SCE). The ill-posedness of the method is directly related to the blind and perturbation zone and, to the best of our knowledge, was never addressed for plane wave imaging in the literature.

It is however possible to set constraints on the acquisition to obtain a well-posed system. Let us assume that all the scatterers of the medium are gathered in a finite region outside the probe's blind and perturbation zone (see Fig. 2). Under this assumption, the matrices $\mathbf{A}^k$ in equation (7) simplify to:

$$\mathbf{A}^k = \begin{bmatrix} a^k[0] & 0 & 0 & \cdots & 0 \\ a^k[1] & a^k[0] & 0 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ a^k[N_a-1] & \ddots & \ddots & \ddots & a^k[0] \\ 0 & a^k[N_a-1] & \ddots & \ddots & a^k[1] \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a^k[N_a-1] \end{bmatrix} \tag{9}$$

Given that each of the $\mathbf{A}^k$ matrix has N_y rows and N_g columns the total matrix $\mathbf{A}_c$ has N_y rows and $N_{pwi} \times N_g$ columns. Considering that the condition number of the matrix $\mathbf{A}_c$ is low, props to the use of orthogonal signals $a^k(t)$, to ensure an unique solution of the linear system given by (7) the necessary condition is that the number of variables ($N_{pwi} \times N_g$) should be equal to the number of equations, i. e. N_y:

$$N_y = N_{pwi} \times N_g \tag{10}$$

Using the properties of the convolution product between the total transmitted signal and the pulsed plane wave responses it can be stated that to receive with the ith element of the probe all the signals scattered by the medium, the length of the observation $\mathbf{y}_i$ must be:

$$N_y = N_a + N_g - 1 \tag{11}$$

Using (10) and (11) we obtain the second constraint:

$$N_a = (N_{pwi} - 1)N_g + 1 \tag{12}$$

The relation (12) states that for a Well-Posed formulation of the Simultaneous Coded Emission (WP-SCE) approach the length of the emitted signal needs to be at least $(N_{pwi} - 1)$ times greater than the length of the pulsed plane wave response. As a consequence the recording time needs to be approximatively N_{pwi} times longer than the recording time needed to reconstruct a low quality image when a single plane wave is emitted. At this stage it can be stated that for a perfect reconstruction of the pulsed plane wave response $\bar{\mathbf{g}}_i^k$, a reduction of the acquisition time is impossible. A straightforward strategy to tackle this issue consists in relaxing the image quality criteria by decreasing N_y, i.e. the length of the acquisition signals.

4. SIMULATION RESULTS AND DISCUSSIONS

The simulation software used is Field II [14], [15]. Simulations on two types of media have been done: a medium that contains 3 scatterers, to illustrate the performance of the method in terms of image resolution, and a medium that contains a random distribution of scatterers with a non echogenic cyst inside, to evaluate the image contrast to noise ratio (CNR). Simulation parameters have been set to match Esaote LA523 linear array specifications, central frequency $f_0 = 5MHz$, $pitch = 0.245mm$. $N_{pwi} = 5$ plane waves steered in the following directions :$-7°$, $-3.5°$, $0°$, $3.5°$, $7°$ were used to insonify the media. Three methods have been analyzed: PWIC - successive emission of the plane waves, WP-SCE - well-posed coded approach (i.e. excluding the blind and perturbation zone), IP-SCE - ill-posed coded approach (i.e. without excluding the blind and perturbation zone). To build the B-mode images of the medium, the received signals (in PWIC) and the estimated ones (in WP-SCE abd IP-SCE) were beamformed in the time domain using the conventional Delay and Sum algorithm [2]. Envelop detection was performed and the results were normalized and log compressed for a good visual contrast.

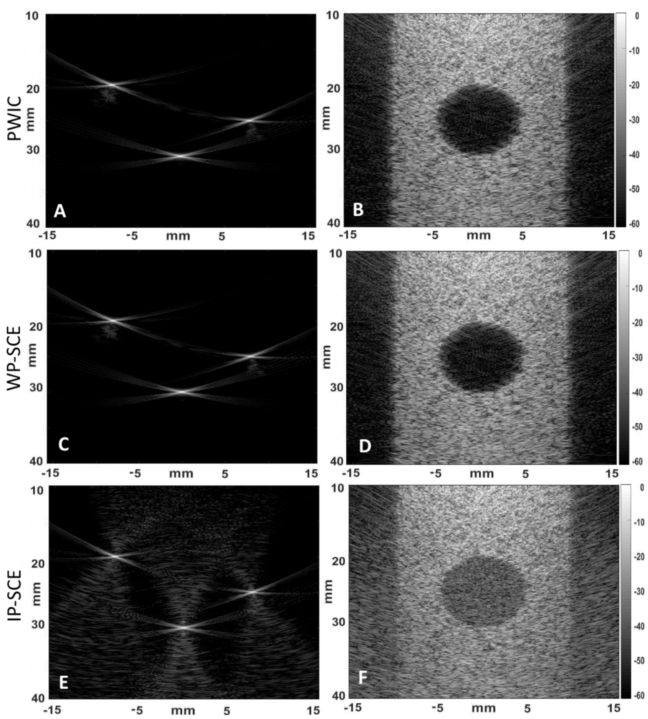

Fig. 4. A, C, E: B-mode images of a sparse medium using PWIC, WP-SCE and IP-SCE. B, D,F: B-mode images of a cyst obtained using PWIC, WP-SCE and IP-SCE. $CNR_1 = 12.3dB$, $CNR_2 = 12.3dB$, $CNR_3 = 4.9dB$

Comparing the Fig. 4A, Fig. 4C and Fig. 4E it can be observed that the 3 imaging modes yield consistent representation of the imaged scatterers, and similar axial and lateral resolutions at $-6dB$ are achieved, $0.14mm$ and $0.39mm$ respectively. However Fig. 4E clearly shows that the ill-posed coded approach (IP-SCE) yields important sidelobes in the axial direction, as opposed to the PWIC and WP-SCE. Fig. 4B and Fig. 4D indicate that same contrast can be achieved by the PWIC and the well-posed coded approach (WP-SCE), $CNR_1 = CNR_2 = 12.3dB$, which is $7.4dB$ above the contrast obtained with IP-SCE (Fig. 4F).

This first simulations allowed us to demonstrate that the well-posed coded emission of 5 plane waves can achieve the same results in terms of image quality as in the successive emission of the same plane waves if the constraints given by the relation (12) are verified.

Let us define now the time gain that WP-SCE gives regarding to the successive plane wave compounding (PWIC): $\tau = \frac{N_{y@5pwi} - N_y}{N_{y@5pwi}} 100\%$, where $N_{y@5pwi} f_s$ is the recording time needed to obtain 5 low quality images using PWIC, $N_y f_s$ is the recording time for the WP-SCE, and f_s is the sampling frequency of the received signal. A simulation where the recording time was continuously shortened has been done to trace the evolution of the contrast as a function of the time gain and the results are shown in Fig. 5. Once again, on the Fig. 5 it can be observed that WP-SCE yields the same CNR as PWIC for a time gain $\tau = 0$. Nonetheless the contrast to noise ratio drops drastically if the time gain increases, which is directly linked to the ill-posedness of the system given by the relation (7). In an ideal case, one would expect the CNR to remain constant when the value of τ increases, thus our future work consists in investigating regularization techniques that will allow a better estimation of $\overline{\mathbf{g}}_i$ in (7).

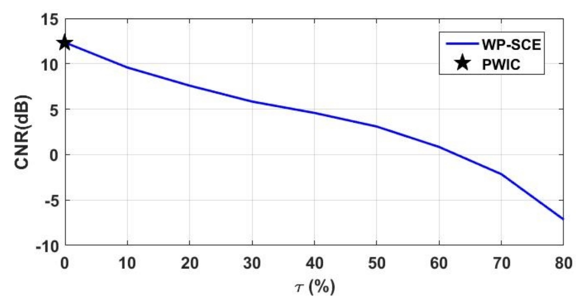

Fig. 5. Evolution of the contrast to noise ratio obtained using simultaneous coded emissions (SCE) as a function of time gain τ. $\star$ represents the CNR of the image obtained using successive plane wave image compounding (PWIC)

5. CONCLUSION

In this paper we presented the theoretical formulation of a problem that allows the emission and reconstruction of N_{pwi} plane waves using a coded approach. We demonstrated theoretically, and illustrated in simulations that in order to get an image quality close to the one obtained using the classical approach, the medium needs to be perfectly non echogenic in the blind and perturbation zone of the probe. Furthermore we showed that the quality of the image reconstruction depends also on the recording time duration, smaller recording time meaning lower image quality. Finally we show the interest of finding an appropriate regularization technique that will allow a better quality of the image reconstruction.

6. REFERENCES

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