COALITIONAL GAME THEORETIC OPTIMIZATION OF ELECTRICITY COST FOR COMMUNITIES OF SMART HOUSEHOLDS

Adriana Chiş, Jarmo Lundén, and Visa Koivunen

Department of Signal Processing and Acoustics, Aalto University, Finland
Email:{adriana.chis, jarmo.lunden, visa.koivunen}@aalto.fi

ABSTRACT

In this paper we propose a novel coalitional game theory based optimization method for minimizing the cost of the electricity consumed from the power grid by a community of smart households. A smart household may own both a renewable energy source and an energy storage system (ESS), or only an ESS. We propose an optimization model in which all the members of the community jointly share their renewable resources and storage systems. We show that the proposed coalitional optimization method reduces the consumption costs both at community level and at the individual level when compared to the case in which the households would individually optimize their costs. The monetary revenues gained by the coalition are divided among the members of the coalition according to the Shapley value. Simulation examples show that the proposed coalitional optimization method may reduce the electricity costs for the community by roughly 18%.

Index Terms— Smart grids, coalitional game theory, cost reduction, smart households, renewable resources

1. INTRODUCTION

The development of sustainable energy technologies has become a major global priority. These technologies support the integration of renewable energy resources and improve the energy consumption efficiency within the power system. However, the currently existing power networks cannot sustain the integration of a large amount of renewable resources. The development of new technologies that support the efficient utilization of renewable energy is necessary.

Methods that study the cooperative approach between different types of energy users have been studied before [1, 2, 3, 4, 5]. The authors in [1] propose a noncooperative and also a cooperative demand response model to schedule the charging/discharging of energy storages and the production of dispatchable energy sources owned by a set of demand-side users. In [2] a game theory based model for load balancing at community level using community energy storage is proposed. In [3] a collaborative framework for modeling smart grid households as an exchange economy market is proposed. The users trade both the energy supplied by the utility company and the stored energy. An energy management scheme in which multiple microgrids cooperate and supply their energy surplus to a shared facility controller with the goal of gaining some income is proposed in [4]. In [5] a direct load control method based on a cooperative game model is proposed to minimize the cost of a union formed by residential households and a retailer.

In this work we propose a novel coalitional game theoretic model for energy exchange and management within a community of smart households. A smart household refers to a household that may own both a renewable energy source (RES) and an energy storage system (ESS), or a household that owns an ESS only. For the first case, an ESS is mandatory when a RES is owned, otherwise the the production of renewable energy would be inefficient. The smart households are also equipped with smart energy management meters that can predict their energy demand profiles and the profiles of renewable energy to be produced during a finite time period ahead. In case of insufficient renewable resources, the electricity demands of the members of the community are fulfilled with electricity bought from the utility company. In order to minimize their electricity costs, the households consume and store the renewable energy produced within the community and the electricity supplied from the main power grid according to the proposed game theoretic optimization method. The method takes advantage of the dynamic pricing option that utility companies offer to their customers. The main contributions of this paper are stated below:

- We propose a novel coalitional game theory based optimization method for energy exchange and management within the community. The proposed optimization implies that all members of the community may form a grand coalition and cooperate to jointly use all their available renewable energy resources, the energy supplied from the power grid and their energy storage spaces in order to reduce the energy cost for the members of the coalition. The proposed optimization consists of scheduling the energy exchange among the members of the coalition and the charging/discharging profiles of their ESSs during a finite time period.

- We formulate another optimization problem which can be used by each member of the community to individually perform cost optimization by using their own available energy resources and/or ESSs. Both of the formulated optimization problems are convex and are solved using linear programming method.

- We show that the proposed coalitional method may reduce the electricity consumption costs at the community level and for each residence participating in the game individually. The revenue of the coalitional optimization is represented by the amount of money saved by cooperating compared to the total electricity cost that the residences would pay in the case when each residence would perform individual cost optimization and not interact with the other households. The revenue obtained through the coalitional optimization is divided among the members in a fair manner, according to the amount of contribution that each member brings to the resulting profit. A method based on Shapley value is used to calculate the payoff earned by each member of the coalition.

Simulation examples show that all members of the community may significantly reduce their electricity costs if they cooperate using the proposed method. The proposed coalitional optimization reduces the overall community's electricity cost with about 18%.

2. SYSTEM MODEL

We consider a community composed of a set $\mathcal{N}$ of households, $|\mathcal{N}| = N$. By $|\mathcal{N}|$ we represent the cardinality of the set. A subset of $\mathcal{M}$ residences of the set $\mathcal{N}$, $\mathcal{M} \subseteq \mathcal{N}$, produce renewable energy and own an ESS as well. The rest of residences in the community, $\mathcal{N} \backslash \mathcal{M}$, own an ESS only. We denote by n the index of any household from the set $\mathcal{N}$ and by m the index of any household from the set $\mathcal{M}$. The energy exchange and optimization is performed over a finite time horizon T which is divided into equally long time slots indexed by t, $t = 1, \ldots, T$. The market electricity prices, $\xi = \{\xi(t)\}_{t=1}^{T}$, are given ahead by the utility company for each time slot within the period T. The set of electricity demands of each residence in the community, $\mathbf{u}_n = \{u_n(t)\}_{t=1}^{T}, n \in \mathcal{N}$, is considered known over the period T and cannot be changed. The set of per-time-slot amounts of renewable energy, $\mathbf{w}_n = \{w_n(t)\}_{t=1}^{T}$, produced by each household from the community over the period T are also considered known. Note that $\mathbf{w}_n = \{0\}_{t=1}^{T}$ for $n \in \mathcal{N} \backslash \mathcal{M}$.

We denote by C_n be the maximum storing capacity associated with each ESS_n in the community. $\mathbf{s}_n = \{s_n(t)\}_{t=1}^{T}$ represents the energy storage vector containing the total amount of energy stored in ESS_n at the end of a time slot. Let $\mathbf{r}_n = \{r_n(t)\}_{t=1}^{T}$ be the set of amounts of energy charged or discharged from each storage unit during each time slot. If $r_n(t) > 0$ at a time slot t, it means that energy is being charged to the ESS_n during that time slot, while if $r_n(t) < 0$, then energy is being discharged from the ESS_n. We denote by ρ_n the charging/discharging rate of ESS_n, parameter which indicates the maximum amount of energy that can be charged or discharged from the storage during a time slot. We denote by η_n the storage loss factor per time slot, i.e. the amount of energy that a storage loses during a time unit. This parameter is specific to each individual ESS_n and it has values between 0 and 1 (typically $\eta_n \ll 1$): 0 indicates that no energy leakage occurs, while 1 indicates that all energy from storage is lost. The coalitional energy trade implies that each member of the formed coalition may give or receive an amount of electricity to or from the other members of the coalition. Hence, we represent by $\mathbf{a}_n = \{a_n(t)\}_{t=1}^{T}$ the set of energy amounts that a member of the coalition may give away or receive from the rest of the coalition members during the period T. If $a_n(t) > 0$ during a certain time slot t, it means that household n provides this amount of energy to the rest of the members of the coalition. If $a_n(t) < 0$ it shows that in time slot t household n receives this amount of energy from the other members of the coalition. Finally, the set of energy amounts that a household n will have to buy from the power grid during period T is denoted by $\mathbf{b}_n = \{b_n(t)\}_{t=1}^{T}$.

3. COST REDUCTION OPTIMIZATION

In this section we formulate the proposed energy cost minimization problems. We first present the coalitional optimization problem which has the purpose of reducing the cost of the energy consumed from the power grid by a group of households from the community that form a coalition. In this paper, we consider that all the residences from the community decide to cooperate and form a grand coalition. Then, we formulate the optimization problem for individual members of the community. In the latter case, each member of the community would use their RES and ESS to optimize their individual consumption and minimize only their own cost, without any form of interaction with other households.

3.1. Coalitional optimization

The cost of electricity consumed by the community from the power grid has the following expression:

$$\wp_{\mathcal{N}} = \sum_{t=1}^{T} \sum_{n=1}^{N} \xi(t) b_n(t), \tag{1}$$

where $\wp_{\mathcal{N}}$ represents the community's total consumption cost over the period T, $\xi(t)$ is the cost of electricity in time slot t and $b_n(t)$ is the electricity purchased from the power grid by household n in time slot t. The coalitional optimization problem may be expressed as follows:

$$\min \wp_{\mathcal{N}}, \tag{2}$$

such that the following constraints are fulfilled:

$$u_n(t) - b_n(t) + a_n(t) - w_n(t) + r_n(t) \leq 0, \tag{2a}$$

$$b_n(t) \geq 0, \tag{2b}$$

$$0 \leq s_n(t) \leq C_n, \tag{2c}$$

$$s_n(t) = (1 - \eta_n) s_n(t-1) + r_n(t), \tag{2d}$$

$$-\rho_n \leq r_n(t) \leq \rho_n, \tag{2e}$$

$$\sum_{n=1}^{N} a_n(t) = 0, \tag{2f}$$

$$\sum_{t=1}^{T} \sum_{n=1}^{N} b_n(t) \geq \sum_{t=1}^{T} \sum_{n=1}^{N} u_n(t) - \sum_{t=1}^{T} \sum_{n=1}^{N} w_n(t), \tag{2g}$$

where (2a)-(2e) are computed for every individual household $n = 1, \ldots, N$ and for each time slot $t = 1, \ldots, T$. The optimization variables are: $\{\mathbf{b}_n, \mathbf{s}_n, \mathbf{r}_n, \mathbf{a}_n\}$. Inequation (2a) states the per-time-slot energy balance constraint: the energy bought from the grid during a time slot should compensate for the energy demand, the amount of energy charged or discharged from the storage, the energy provided or received to/from the other members of the coalition and the renewable energy resources available in that time slot. The amount of energy purchased from the power grid cannot have a negative value (2b). The total amount of energy stored in the ESS must lie between zero and the capacity, C_n, of the ESS_n (2c). Equation (2d) is expressing the dynamics of the ESS: the total storage level of ESS_n at the end of a time slot is equal to the storage level at the previous time slot, considering also the storage leakage factor, and the added or subtracted amount of energy charged or discharged from the storage in that time slot. The amount of energy charged or discharged from the storage is limited by the ESS_n's charging rate, ρ_n, in inequation (2e). Constraint (2f) states that the total amount of energy given away by some members of the community in a time slot must be equal to the total amount of energy received by the rest of the members of the community in that time slot. Finally, inequation (2g) shows that the amount of energy purchased by the community from the power grid during the whole period T should be at least equal to the total remaining energy demand of the community which cannot be satisfied by the amount of energy received from renewable sources.

3.2. Individual optimization

The individual cost of electricity consumed by a single household n from the power grid during the period T is denoted by $\wp_n$ and is expressed as:

$$\wp_n = \sum_{t=1}^{T} \xi(t) b_n(t). \tag{3}$$

The electricity cost minimization optimization for individual households may be formulated as:

$$\min \wp_n, \tag{4}$$

such that the following constraints are satisfied:

$$u_n(t) - b_n(t) - w_n(t) + r_n(t) \le 0; \tag{4a}$$

$$b_n(t) \ge 0; \tag{4b}$$

$$0 \le s_n(t) \le C_n; \tag{4c}$$

$$s_n(t) = (1 - \eta_n) s_n(t-1) + r_n(t); \tag{4d}$$

$$-\rho_n \le r_n(t) \le \rho_n. \tag{4e}$$

The optimization variables are: $\{\mathbf{b}_n, \mathbf{s}_n, \mathbf{r}_n\}$. The constraints (4a)-(4e) are computed for each time slot $t = 1 \dots T$. These constraints are closely related to the constraints (2a)-(2e) from the coalitional optimization problem. The difference between constraints consists in absence of the variable that quantifies the amounts of energy exchanged by each household per-time-slot with the rest of the members of the coalition, $a_n(t)$.

It can be observed that the objective functions and the constraints of the formulated optimization problems possess linear relationships between the variables of the problems. Hence, both the optimization problems are modeled as linear programs. The solutions of the proposed optimization problems can be easily obtained through algorithms such as the interior point algorithm [6].

4. COALITIONAL GAME MODEL FOR THE COMMUNITY

A coalitional game [7] is uniquely defined by the pair $(\mathcal{N}, \upsilon)$, where $\mathcal{N}$ represents the set of players involved in the game and $\upsilon : 2^{\mathcal{N}} \to \mathbb{R}$ is the characteristic function of the game which quantifies the worth of a coalition. In the proposed optimization problem, the players $\mathcal{N}$ of the coalitional game are the households of the smart community. These households may form coalitional groups in order to minimize their electricity costs. We denote by $\mathcal{G}$, $\mathcal{G} \subseteq \mathcal{N}$, any non-empty subset of households from the community that can form a coalitional group. If the coalition is formed by all the residences in the community, $\mathcal{G} = \mathcal{N}$, then the coalition is called a grand coalition. The worth of a coalition, $\upsilon(\mathcal{G})$, is a real value representing the total revenue received by the coalitional group for cooperating.

For the proposed cost minimization problem for the community, the worth (revenue) of a coalition is defined as the monetary amount saved by the coalitional group in the cooperative scenario in comparison with the total electricity cost that the members of the coalition would pay in the case of performing individual cost optimization. The worth of the coalition is expressed as:

$$\upsilon(\mathcal{G}) = \sum_{g \in \mathcal{G}} \wp_g - \wp_{\mathcal{G}}, \tag{5}$$

where $\wp_{\mathcal{G}}$ and $\wp_g$ are calculated according to (1) and (3), respectively, using the solutions of the two proposed optimization methods.

The worth of the coalition must be divided among its members using a fair rule, according to the amount of contribution that each of the players brought to the coalitional game and consequently to the revenue. One straightforward way of distributing the revenue among the members of the coalition in a collaborative scenario is using the Shapley value [7]. For a coalitional game defined by $(\mathcal{N}, \upsilon)$, the Shapley value, $\Phi(\upsilon)$, assigns to each player $i \in \mathcal{N}$ a payoff $\Phi_i(\upsilon)$ given by the following expression:

$$\Phi_i(\upsilon) = \sum_{\mathcal{G} \subseteq \mathcal{N} \setminus \{i\}} \frac{|\mathcal{G}|!(N - |\mathcal{G}| - 1)!}{N!} [\upsilon(\mathcal{G} \cup \{i\}) - \upsilon(\mathcal{G})], \tag{6}$$

where the sum is computed over all possible subsets $\mathcal{G}$ of $\mathcal{N}$ not containing player i. The payoff of a household taking part in a coalition represents the fraction of the total revenue of that coalition that is achieved through the participation of that household in the coalitional game.

5. SIMULATION EXAMPLES

In this section we present some simulation examples and quantitative results that demonstrate the cost savings achieved by the proposed method. For simulating and testing the performance of the proposed method we considered a smart grid community composed of N=5 households. Every household in the community owns an ESS, while a number of $|\mathcal{M}|$=3 residences own a RES as well. Here $|\mathcal{M}|$ represents the cardinality of the set $\mathcal{M}$. We considered the case in which all households from the community participate in the coalitional game. For the simulation we considered the following ESS_n capacities: $C_n = \{5kWh, 5kWh, 5kWh, 10kWh, 10kWh\}$. We also assumed the following charging/discharging rates for the storage units: $\rho_n = \{1kWh, 1kWh, 1kWh, 2kWh, 2kWh\}$. The storage loss factor is assumed to be the same for all ESSs: $\eta_n = 0.001, n = 1, \dots, N$. We perform simulations over a time horizon T=24 hours divided into hourly time slots. The pricing data used in the simulation are true pricing data taken from Finnish Nord Pool Spot database [8] for May 2013. The load modeling framework presented in [9] was used to simulate the load demand of each residence. The households were assumed to have the following number of inhabitants: $\{3, 2, 5, 2, 4\}$. The renewable energy data values were simulated using the mathematical model of wind turbines in [10] and true weather data for May 2013 in Helsinki region [11]. For solving the coalitional game and individual cost optimization problems we used the CVX package for convex optimization [12].

Fig.1 shows the total daily electricity demand of the community and the energy production of the residences owning an RES for each day of the month. We can observe that only during few days of the month the renewable electricity production within the community was sufficient to fulfill the total demand of the day. The simulations were performed considering that at beginning of day 1 the ESSs are empty. If renewable energy was left in storages at the end of a day, that amount was considered for optimizing the cost during the following day.

The bar plot in Fig.2 depicts the daily cost savings of the community for the case in which they cooperate by using the proposed coalitional game model and optimization. The obtained cost savings are compared to the case in which each residence of the community would use their own renewable energy production and storage system to individually optimize their cost without cooperating. The daily rewards, i.e. cost savings, are divided among the members of the community as shown by the blocks composing each bar. The blocks indicate the amount of monetary payoffs distributed to each household as calculated using the Shapley value: $\sum_{i \in \mathcal{N}} \Phi_i(\upsilon) = \upsilon(\mathcal{N})$. It can be observed that only during three days of the month, days 3,6 and 18, the coalitional game optimization doesn't perform better than the individual optimization. Otherwise, for the rest of the days of the month each household in the community, no matter if it owns a RES or an ESS only, receives some monetary revenue for their participation in the coalitional optimization.

Fig.3 shows the daily cumulated electricity costs for the community in case of the coalitional game theoretic optimization versus the total cumulated costs of the members of the community when they perform individual cost optimization. If the households cooperate and share their resources the total cost of the community at

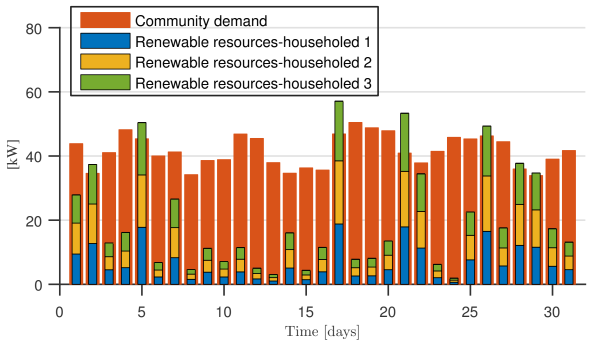

Fig. 1: Community's total electricity demand and renewable energy production of each household owning a RES during each day of a month.

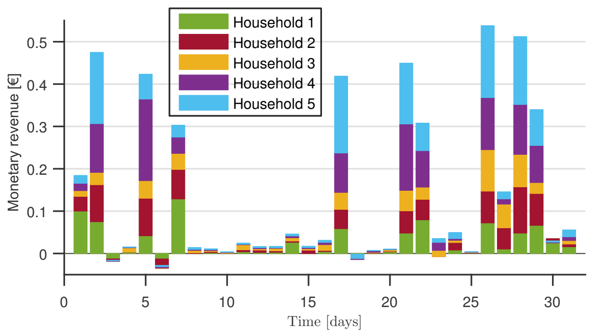

Fig. 2: The daily monetary revenues achieved by the coalition through cooperation. The blocks composing a bar each show the payoff of a household from the coalition as distributed by the Shapley value method. The coalitional optimization method provides an electricity cost reduction during majority of the days of the month.

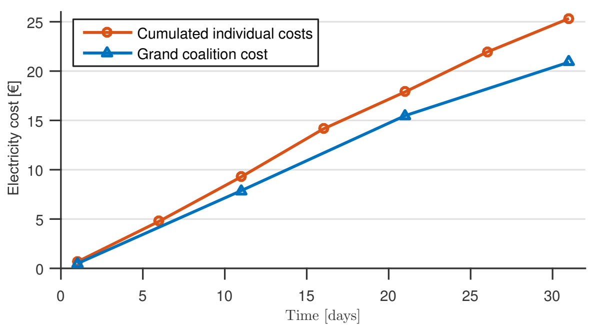

Fig. 3: Cumulated electricity costs of the two formulated problems. The proposed coalitional game cost optimization method obtains an electricity cost reduction of about 18% in comparison with the individual cost optimization method.

the end of the month would be 20.9€, while if they don't cooperate and the households would perform individual cost optimization then the total cost of the community at the end of the month would be 25.4€. The results indicate that a significant cost reduction may be achieved. The proposed coalitional optimization may provide a cost reduction of about 18% for the community.

6. CONCLUSIONS

In this paper we proposed a novel coalitional game theory based optimization method for minimizing the cost of electricity consumed by a community of smart households from the power grid. The coalitional optimization implies that the members of the community form coalitions and freely share their renewable resources and storage systems among themselves. We also formulate an optimization model through which the individual households would use their renewable energy resources and energy storage spaces to minimize only their own electricity costs. The monetary revenue of the coalition is represented by the amount of money that the coalition saves compared to the individual cost optimization case. We showed that the proposed optimization reduces the overall cost of electricity for the formed coalition and also for each member of the coalition individually. The daily monetary revenues obtained by cooperating were divided among the members of the coalition using Shapley value method. If all the households in the community participate in the coalition, then the proposed coalitional optimization may reduce the electricity consumption cost for the community with 18%.

7. REFERENCES

[1] I. Atzeni, L. G. Ordonez, G. Scutari, D. P. Palomar, and J. R. Fonollosa, "Noncooperative and cooperative optimization of distributed energy generation and storage in the demand-side of the smart grid," *IEEE Transactions on Signal Processing*, vol. 61, no. 10, pp. 2454–2472, May 2013.

[2] J. Rajasekharan and V. Koivunen, "Cooperative game-theoretic approach to load balancing in smart grids with community energy storage," in *23rd European Signal Processing Conference (EUSIPCO)*, 2015.

[3] J. Rajasekharan, J. Lunden, and V. Koivunen, "Competitive equilibrium pricing and cooperation in smart grids with energy storage," in *47th Annual Conference on Information Sciences and Systems (CISS)*, 2013.

[4] W. Tushar, C. Yuen, D. B. Smith, N. U. Hassan, and H. V. Poor, "A canonical coalitional game theoretic approach for energy management for nanogrids," in *IEEE Innovative Smart Grid Technologies - Asia (ISGT ASIA)*, 2015.

[5] Q. Cui, X. Wang, X. Wang, and Y. Zhang, "Residential appliances direct load control in real-time using cooperative game," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 226–233, Jan 2016.

[6] Yinyu Ye, *Interior point algorithms: theory and analysis*, John Wiley and Sons, 1997.

[7] Martin Osborne and Ariel Rubinstein, *A Course in Game Theory*, MIT Press, 1994.

[8] [Online]. Available: www.nordpoolspot.com.

[9] I. Richardson, M. Thomson, D. Infield, and C. Clifford, "Domestic electricity use: A high-resolution energy demand model," .

[10] Jun Wei Chuah, A. Raghunathan, and N.K. Jha, "An evaluation of energy-saving technologies for residential purposes," in *IEEE Power and Energy Society General Meeting*, July 2010, pp. 1–8.

[11] [Online]. Available: www.ilmatieteenlaitos.fi.

[12] M. Grant and S. Boyd, CVX: Matlab Software for Disciplined Convex Programming, Version 2.0 Beta, Sept 2013.