DISTRIBUTED TARGET DETECTION WITH PARTIAL OBSERVATION VIA MATRIX COMPLETION

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ABSTRACT

This paper considers the detection of a distributed target, which is important in high resolution radars (HRRs). We focus on the practical scenario where only partial observation is available. The key contribution of this work is the proposition of a generalized likelihood ratio test (GLRT) detector using matrix completion (MC). Firstly, a decision rule is obtained for the hypothesis test model with missing data. Then the estimation of the unknown parameters involved in the detector is derived via the maximum-likelihood estimator (MLE). To overcome the problem that the full covariance matrix of the disturbance can not be estimated analytically, we adopt the MC technique, in which the estimate is obtained by solving an optimization problem concerning both the MLE expression and the low rank interference. An alternating iterative algorithm is followed to achieve the final estimate. Numerical results are presented to validate the effectiveness of the proposed method when the missing data problem occurs.

Index Terms— distributed target, partial observation, generalized likelihood ratio test, matrix completion, covariance matrix estimation

1. INTRODUCTION

The distributed target has received extensive attention in the past decades. Distributed target (also known as extended target or spread target) means a target that takes multiple resolution cells, which is caused by the high resolution of the radar and the relatively large size of the target. The conventional point-target model which concerns only a single resolution cell fails in this case [1].

The detection of distributed target has been considered in many literature. The integrated detector is proposed in [2] and is shown to perform much better than the single range cell detection. Other works are based on the generalized ratio test (GLRT) criterion with various disturbance models. The Gaussian noise described with complex, zero-mean, circular Gaussian distribution and the non-Gaussian clutter modeled as spherically invariant random vector (SIRV), are considered in [1] and [3], respectively. Different from these two methods, in [4], the constrained maximization likelihood estimate (MLE) of unknown parameters without secondary data, i.e., data that are free of cells under test, is derived.

Note that the current approaches are mostly based on the assumption that a set of complete observation data in continuous range cells under test is available. However, this assumption may not be satisfied in many practical applications. Typical scenarios that would cause incomplete observation data include element failures in array antenna [5], channel occupancy by other device in spectrum sharing [6], or the compressed sampling for lowing down of data rate in many electronic systems [7]. The incomplete observation results in data missing in one or more domains of space, frequency and time, which is referred to as partial observation in the following.

The issue of partial observation for point-target model has been discussed in the existing works. For instance, in [8], the well-known compressed sampling (CS) model [9, 10] is adopted to recover the sparse signal from partial observation. Meanwhile, the matrix completion (MC), which is an effective method for the reconstruction of the full matrix from partial observation [11, 12], is also applied to the point-target model in [13]. Though the point-target with partial observation is sufficiently discussed, the case of distributed target is more complicated and has not been extensively studied yet.

In this paper, we consider the detection of distributed target with partial observation, and propose an approach based on GLRT and MC to deal with this issue. In this work, the target is assumed to be surrounded by disturbance (interference plus noise) with unknown covariance matrix. We establish the hypothesis test model by dividing the partial observation into multiple groups according to different types of data missing. Then the GLRT detector is obtained with the Neyman-Pearson criterion. The MLE is imposed on the estimation of the unknown parameters involved in the GLRT detector, i.e., the complex amplitude of target and the covariance matrix of disturbance. The main challenge in the MLE, i.e., the estimation of the covariance matrix can not be obtained analytically, is considered by introducing the MC technique. With the full covariance matrix of interference obtained via MC, the covariance matrix associated with the partial observation is acquired and the final GLRT detector is accomplished. Furthermore, a weighted Frobenius norm is adopted to enhance

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the estimation of the covariance matrix. Numerical results verify the effectiveness of the proposed method.

The rest of this paper is organized as follows. Section 2 specifies the detection problem and the GLRT decision rule. Section 3 presents the estimating process of unknown parameters. Simulation results are provided in Section 4. Conclusions are drawn in the final section.

2. PROBLEM FORMULATION AND THE GLRT DETECTOR DERIVATION

Consider a radar composing of N channels and detecting the presence of a distributed target across at most K range cells. The channel may denote the element of an array antenna, the pulse of a coherent radar, or a combination of both, depending on the specific detection scenario. Specifically, considering a matrix expression, the detection problem can be formulated as the following binary hypothesis test

$$\begin{cases} H_0 : \boldsymbol{R} = \boldsymbol{W} \\ H_1 : \boldsymbol{R} = \boldsymbol{p}_t \boldsymbol{\alpha}^T + \boldsymbol{W}, \end{cases} \tag{1}$$

where $\boldsymbol{R} = [\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_K]$, $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_K]^T$ and $\boldsymbol{W} = [\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_K]$. $\boldsymbol{r}_k$ ($k = 1, 2, \ldots, K$) is the data vector collected from the k-th range cell. α_k denotes the uncorrelated unknown complex amplitude of the desired target's scatterer in the k-th range cell accounting for both target reflectivity and channel effects. $\boldsymbol{p}_t$ is the deterministic steering vector of the target. $\boldsymbol{w}_k$ denotes the disturbance consisting of interference $\boldsymbol{c}_k$ and white noise $\boldsymbol{n}_k$. Suppose $\boldsymbol{w}_k$s are independent and identically distributed (i.i.d.) zero-mean complex circular Gaussian vectors sharing the same unknown covariance matrix, namely $E\left\{\boldsymbol{w}_k \boldsymbol{w}_k^H\right\} = \boldsymbol{X} \succ \boldsymbol{0}$ ($k = 1, 2, \ldots, K$).

In this paper, we focus on the scenario where only partial observation data is available. The positions of the missing entries are randomly distributed and known after sampling. Noticing the fact that the data of different range cells may share the same patterns of missing data, we divide the observed data matrix $\boldsymbol{R}$ into G groups along range cells. Thus each group exhibits the same pattern of channel index data present or missing. The range cell index set and channel index set of the g-th group ($g = 1, 2, \ldots, G$) are denoted by Ψ_g ($\Psi_g \subset \{1, 2, \ldots, K\}$) and Ω_g ($\Omega_g \subset \{1, 2, \ldots, N\}$) respectively, with cardinality $|\Psi_g| = K_g$ and $|\Omega_g| = N_g$. Specifically, for the g-th group, the detection problem can be written as

$$\begin{cases} H_0 : \boldsymbol{R}_g = \boldsymbol{W}_g \\ H_1 : \boldsymbol{R}_g = \boldsymbol{p}_{t_g} \boldsymbol{\alpha}_g^T + \boldsymbol{W}_g, \end{cases} \tag{2}$$

where $\boldsymbol{R}_g$, $\boldsymbol{W}_g$, $\boldsymbol{p}_{t_g}$ and $\boldsymbol{\alpha}_g$ denote the partial form of the corresponding parameters. In this scenario, $\boldsymbol{B}_g \in \mathbb{Z}^{K_g \times K}$ and $\boldsymbol{D}_g \in \mathbb{Z}^{N_g \times N}$ denote the selection matrix of range cell index and channel index, respectively. The elements in $\boldsymbol{B}_g$ and $\boldsymbol{D}_g$ are composed of 1's or 0's, indicating the observations on the corresponding indices are present or missing. To illustrate, $\boldsymbol{R}_g = \boldsymbol{D}_g \boldsymbol{R} \boldsymbol{B}_g^T$, $\boldsymbol{p}_{t_g} = \boldsymbol{D}_g \boldsymbol{p}_t$ and $\boldsymbol{\alpha}_g = \boldsymbol{B}_g \boldsymbol{\alpha}$.

It is necessary to note that the grouping does not affect the detection result, since α_ks are assumed to be uncorrelated. If α_ks are dependent, the expression can be obtained by regarding each range cell as a group.

With the above assumptions, the probability density function (PDF) under H_1 can be formulated as

$$\begin{aligned} & f_{\boldsymbol{R}}\left(\boldsymbol{R} | \boldsymbol{\alpha}_{H_1}, \boldsymbol{X}_{H_1}, H_1\right) \\ & \quad = \prod_{g=1}^{G} f_{\boldsymbol{R}}\left(\boldsymbol{R}_g | \boldsymbol{\alpha}_{g,H_1}, \boldsymbol{X}_{g,H_1}, H_1\right) \\ & \quad = \prod_{g=1}^{G} \frac{\left[\det\left(\boldsymbol{X}_{g,H_1}\right)\right]^{-K_g}}{\pi^{K_g N_g}} \exp\left[-\operatorname{tr}\left(\boldsymbol{X}_{g,H_1}^{-1} \boldsymbol{M}_{g,H_1}\right)\right], \end{aligned} \tag{3}$$

where $\boldsymbol{M}_{g,H_1}$ denotes

$$\boldsymbol{M}_{g,H_1} = \left(\boldsymbol{R}_g - \boldsymbol{p}_{t_g} \boldsymbol{\alpha}_g^T\right) \left(\boldsymbol{R}_g - \boldsymbol{p}_{t_g} \boldsymbol{\alpha}_g^T\right)^H, \tag{4}$$

and the partial form of covariance matrix $\boldsymbol{X}_{g,H_1}$ is given by

$$\boldsymbol{X}_{g,H_1} = \boldsymbol{D}_g \boldsymbol{X}_{H_1} \boldsymbol{D}_g^T. \tag{5}$$

The PDF under H_0 can be obtained by replacing $\boldsymbol{\alpha}_{H_1}$ with $\boldsymbol{0}$ and $\boldsymbol{X}_{H_1}$ with $\boldsymbol{X}_{H_0}$ in (3)$\sim$(5).

According to the Neyman-Pearson criterion, the Likelihood Ratio Test (LRT) enables the maximum detection probability with the false alarm probability fixed. However, for the case under consideration, $\boldsymbol{\alpha}$ and $\boldsymbol{X}$ are unknown. A classic approach is to replace the unknown parameter with its maximum-likelihood estimate (MLE) under each hypothesis, leading to GLRT [14]. We resort to this strategy herein. Then the GLRT decision rule in the partial observation scenario is formulated as

$$\frac{\max\limits_{\boldsymbol{\alpha}_{H_1}, \boldsymbol{X}_{H_1}} f_{\boldsymbol{R}}\left(\boldsymbol{R} | \boldsymbol{\alpha}_{H_1}, \boldsymbol{X}_{H_1}, H_1\right)}{\max\limits_{\boldsymbol{X}_{H_0}} f_{\boldsymbol{R}}\left(\boldsymbol{R} | \boldsymbol{X}_{H_0}, H_0\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{6}$$

Thus, the key issue to devise the GLRT detector is the estimation of unknown parameters under each hypothesis, i.e., $\{\boldsymbol{\alpha}_{H_1}, \boldsymbol{X}_{H_1}, \boldsymbol{X}_{H_0}\}$, and then substitute the estimate into (6).

3. ESTIMATION OF UNKNOWN PARAMETERS

3.1. MLE expression of the unknown parameters

The MLE of the unknown parameters associated with each hypothesis can be obtained by solving the maximizing problem in the numerator and the denominator of (6) respectively.

As to $\boldsymbol{\alpha}_{H_1}$, with the logarithmic form of the PDF under H_1 given by (3), we can get the MLE of $\boldsymbol{\alpha}_{H_1}$ ($\hat{\boldsymbol{\alpha}}_{H_1}$) via complex matrix partial differential. Specifically, let $\frac{\partial}{\partial \boldsymbol{\alpha}}(\log f_{\boldsymbol{R}}\left(\boldsymbol{R} | \boldsymbol{\alpha}_{H_1}, \boldsymbol{X}_{H_1}, H_1\right)) = 0$, we obtain

$$\sum_{g=1}^{G} m_g \boldsymbol{I}_{\boldsymbol{B}_g} \hat{\boldsymbol{\alpha}}_{H_1}^* = \sum_{g=1}^{G} \boldsymbol{d}_g, \tag{7}$$

where $m_g = \boldsymbol{p}_t^H \boldsymbol{D}_g^T \boldsymbol{X}_g^{-1} \boldsymbol{D}_g \boldsymbol{p}_t$, $\boldsymbol{I}_{\boldsymbol{B}_g} = \boldsymbol{B}_g^T \boldsymbol{B}_g$, and $\boldsymbol{d}_g = \boldsymbol{B}_g^T \boldsymbol{B}_g \boldsymbol{R}^H \boldsymbol{D}_g^T \boldsymbol{X}_g^{-1} \boldsymbol{D}_g \boldsymbol{p}_t$. Note that $\boldsymbol{I}_{\boldsymbol{B}_g}$ represents a partial identical matrix with main diagonal elements 1's in those rows specified in the set of Ψ_g and 0's for others. Since every range cell has at least one channel observed (i.e., missing an entire column is not permitted in the observed data matrix), it suggests that $\sum_{g=1}^{G} m_g \boldsymbol{I}_{\boldsymbol{B}_g}$ is certainly invertible. Therefore, the MLE of $\boldsymbol{\alpha}_{H_1}$ can be given as

$$\hat{\boldsymbol{\alpha}}_{H_1} = \Big[\Big(\sum_{g=1}^{G} m_g \boldsymbol{I}_{\boldsymbol{B}_g} \Big)^{-1} \Big(\sum_{g=1}^{G} \boldsymbol{d}_g \Big) \Big]^*. \tag{8}$$

Similarly, it can be shown that the MLE of $\boldsymbol{X}$ (refers to $\boldsymbol{X}_{H_0}$ or $\boldsymbol{X}_{H_1}$) satisfies

$$\sum_{g=1}^{G} K_g \boldsymbol{D}_g^T \hat{\boldsymbol{X}}_g^{-1} \boldsymbol{D}_g = \sum_{g=1}^{G} \boldsymbol{D}_g^T \hat{\boldsymbol{X}}_g^{-1} \boldsymbol{M}_g \hat{\boldsymbol{X}}_g^{-1} \boldsymbol{D}_g. \tag{9}$$

Though the implicit MLE is obtained, there is still a challenge that it is difficult to get the explicit expression of $\boldsymbol{X}$ from (9), due to the presence of the summation term and only partial observation is available. Moreover, it can be seen that $\hat{\boldsymbol{\alpha}}_{H_1}$ and $\hat{\boldsymbol{X}}_{H_1}$ are interdependent. Therefore, we would adopt more powerful tools to acquire the solution of $\boldsymbol{X}$, which will be shown in the following.

3.2. Estimation of the Disturbance Covariance Matrix via Matrix Completion

Consider the estimation of unknown parameters under H_1 first. Since the analytical expression of $\hat{\boldsymbol{\alpha}}_{H_1}$ is already given by (8), we focus on the estimation of $\boldsymbol{X}_{H_1}$ when $\hat{\boldsymbol{\alpha}}_{H_1}$ is known. Once the analytical expressions are acquired, an alternating iterative method can be used to achieve the final estimate of $\boldsymbol{\alpha}_{H_1}$ and $\boldsymbol{X}_{H_1}$. In the following of this subsection, the subscript H_1 is omitted in all the expressions for brevity.

Suppose that the received disturbance signal $\boldsymbol{w}_k = \boldsymbol{c}_k + \boldsymbol{n}_k$, and the interference term $\boldsymbol{c}_k$ is considered as a contribution of N_i sources, which can be expressed as

$$\boldsymbol{c}_k = \sum_{j=1}^{N_i} \beta_j \boldsymbol{p}_{i_j} = \boldsymbol{P}\boldsymbol{\beta}, \tag{10}$$

where $\boldsymbol{P} = [\boldsymbol{p}_{i_1}, \boldsymbol{p}_{i_2}, \ldots, \boldsymbol{p}_{i_{N_i}}]$ and $\boldsymbol{\beta} = [\beta_1, \beta_2, \ldots, \beta_{N_i}]^T$. β_j and $\boldsymbol{p}_{i_j}$ are the complex amplitude and the deterministic steering vector of the j-th interference ($j = 1, 2, \ldots, N_i$), respectively. Assuming the interference term $\boldsymbol{c}_k$ and the white noise term $\boldsymbol{n}_k$ are mutually uncorrelated and $\boldsymbol{n}_k$s are i.i.d. zero-mean complex circular Gaussian vectors, the covariance matrix of the disturbance is given as

$$\boldsymbol{X} = \boldsymbol{X}_i + \sigma_0^2 \boldsymbol{I}_N, \tag{11}$$

where $\boldsymbol{X}_i$ and $\sigma_0^2 \boldsymbol{I}_N$ represent the covariance matrix of $\boldsymbol{c}_k$ and $\boldsymbol{n}_k$, respectively. σ_0^2 is the known power level of the white noise term. Furthermore, $\boldsymbol{X}_i = E\{\boldsymbol{c}_k \boldsymbol{c}_k^H\} = \boldsymbol{P} E\{\boldsymbol{\beta}\boldsymbol{\beta}^H\} \boldsymbol{P}^H$. It is reasonable to assume that $N_i \ll N$ (see, e.g., [4, 15]), then $\mathrm{Rank}(\boldsymbol{X}_i) \leq N_i$. Namely, the rank of $\boldsymbol{X}_i$ is much smaller than its dimension. To conclude, $\boldsymbol{X}_i$ is low rank.

Consider a single item of the summation in (9) and inspired by the sample covariance matrix, we notice that $\check{\boldsymbol{X}}_g = \boldsymbol{M}_g / K_g$ meets (9) when $\boldsymbol{M}_g$ is invertible. In the case where $\boldsymbol{M}_g$ is not invertible, a generalized inverse $\boldsymbol{M}_g^\dagger$ via singular value decomposition(SVD) [16] can be adopted and equation (9) still holds. Since $\check{\boldsymbol{X}}_g = \boldsymbol{M}_g / K_g$, referring to (5) and considering all the groups (equivalent to all the entries of the covariance matrix), we have

$$\sum_{g=1}^{G} K_g \boldsymbol{D}_g^T \boldsymbol{D}_g \check{\boldsymbol{X}} \boldsymbol{D}_g^T \boldsymbol{D}_g = \sum_{g=1}^{G} \boldsymbol{D}_g^T \boldsymbol{M}_g \boldsymbol{D}_g. \tag{12}$$

The left side and right side of (12) are denoted as $\widetilde{\boldsymbol{X}}$ and $\widetilde{\boldsymbol{M}}$, respectively.

On the basis of the above considerations, our scheme aims at finding a low rank matrix $\boldsymbol{X}_i$ that approximates $\widetilde{\boldsymbol{X}}$ to $\widetilde{\boldsymbol{M}}$. As rank minimization is NP-hard, it is relaxed to a nuclear norm minimization problem [17]. The optimization problem can be formulated as

$$\begin{aligned} &\hat{\boldsymbol{X}}_i = \underset{\boldsymbol{X}_i}{\arg\min} \, \|\widetilde{\boldsymbol{X}} - \widetilde{\boldsymbol{M}}\|_{F_w}^2 + \gamma \|\boldsymbol{X}_i\|_* \\ &s.t. \quad \boldsymbol{X}_i \succeq \boldsymbol{0}, \boldsymbol{X} = \boldsymbol{X}_i + \sigma_0^2 \boldsymbol{I}_N \succ \boldsymbol{0}, \end{aligned} \tag{13}$$

where γ is a regularization coefficient balancing the nuclear norm term and the data approximation term. It is worth noting that we define a weighted Frobenius norm $||\cdot||_{F_w}$ to replace the Frobenius norm in the data approximation term in (13). Specifically, the weighted Frobenius norm of $A \in \mathbb{C}^{m\times n}$ can be written as

$$||A||_{F_w} \triangleq \Big(\sum_{i=1}^{m} \sum_{j=1}^{n} w_{ij} |a_{ij}|^2 \Big)^{1/2}. \tag{14}$$

In $||\cdot||_{F_w}$, w_{ij} is introduced to control the effects of matrix entries on the data approximation caused by the difference of observation number. It can be easily verified that $||\cdot||_{F_w}$ is a norm.

In conclusion, $\hat{\boldsymbol{X}}_i$ can be obtained via solving the optimization problem in (13), $\hat{\boldsymbol{X}}$ is then estimated by $\hat{\boldsymbol{X}} = \hat{\boldsymbol{X}}_i + \sigma_0^2 \boldsymbol{I}$. An alternating iterative algorithm is used to achieve the final estimate of $\boldsymbol{\alpha}_{H_1}$ and $\boldsymbol{X}_{H_1}$. As to the estimation of unknown parameter under H_0, namely $\hat{\boldsymbol{X}}_{H_0}$, it can be obtained by replacing $\hat{\boldsymbol{\alpha}}_{H_1}$ with $\boldsymbol{0}$ in the expression of $\hat{\boldsymbol{X}}_{H_1}$. Finally, once the estimates of unknown parameters under both hypotheses are derived, the GLRT detector is achieved referring to (6).

3.3. Relation to Prior Work

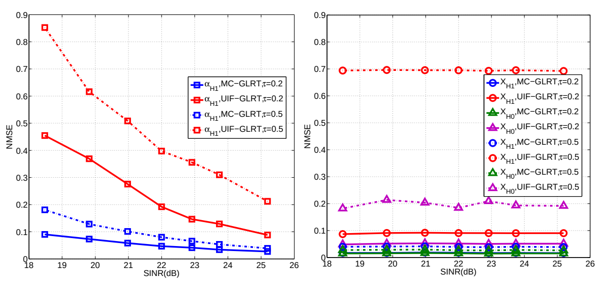
The practical and complicated case where only partial observation is available for distributed target is firstly addressed in this work. Meanwhile, the existing works just address one side of the issue, i.e., point-target with partial observation or distributed target with complete observation.

Besides, the GLRT approach for distributed target with complete observation can be verified to be a degenerate case of our approach. Specifically, when $\boldsymbol{B}_g = \boldsymbol{I}_K, \boldsymbol{D}_g = \boldsymbol{I}_N$ and $G = 1$, i.e., there is no data missing, the MLE obtained in (9) can be verified to be consistent with the full observation scenario in [4, 18].

We firstly utilize the low rank property and adopt MC to estimate the covariance matrix of interference. Though MLE are used for parameter estimation in all the approaches, our method use MC uniquely. While in the existing cases, the M-LE can be obtained directly with an analytical solution, which is not achievable in our case.

4. NUMERICAL RESULTS

In this section, we compare the performance of proposed method denoted as MC-GLRT, with the method presented in [4], which is referred to as UIF-GLRT. In the simulations, a uniform linear array radar with N elements and half-wavelength spacing is considered, where the possible target is sought within K range cells. The configuration of major parameters in the simulations are listed as follows: $N = 20, K = 50$, the probability of false alarm $P_F = 10^{-4}$, interference number $N_i = 3$, power of the interference $\sigma_i^2 = 30$dB, power level of the white noise $\sigma_0^2 = 0$dB, target phase angle $\phi_t = 0$, and interference phase angle $\phi_i = [20°, 40°, 60°]$. Moreover, we set $w_{ij} = K_{o_{ij}}/K$ in (13), where $K_{o_{ij}}$ denotes the observation number of the (i, j)-th entry of the covariance matrix $\boldsymbol{X}$. Additionally, the missing positions of $\boldsymbol{R}$ is adopted with zero-padding in the UIF-GLRT algorithm.



(a) (b)

Fig. 1. The NMSE vs SINR of unknown parameters with different missing rates (τ). (a) The NMSE of $\boldsymbol{\alpha}_{H_1}$; (b) The NMSE of $\boldsymbol{X}_{H_1}$ and $\boldsymbol{X}_{H_0}$

Since the detection performance of GLRT highly depends on the estimation accuracy of the unknown parameters $\boldsymbol{\alpha}_{H_1}, \boldsymbol{X}_{H_1}$, and $\boldsymbol{X}_{H_0}$, we record the normalized mean squared error (NMSE) with different missing rates (τ) firstly. The NMSE is defined as $E(\|\hat{\boldsymbol{Z}} - \boldsymbol{Z}\|_F^2/\|\boldsymbol{Z}\|_F^2)$, where

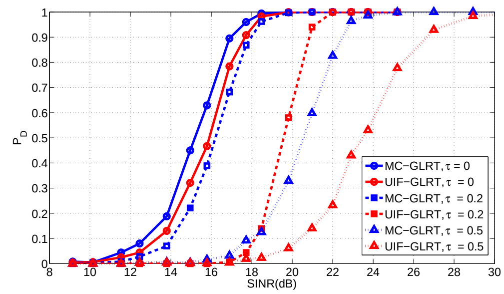


Fig. 2. Performance comparison with different missing rates.

$\hat{\boldsymbol{Z}}$ is the estimate obtained from (8) or (13) while $\boldsymbol{Z}$ is the corresponding true value. The result is shown in Fig.1, in which each point represents the average of 1000 Monte Carlo trials. It can be seen that as SINR increases, the estimation error of $\boldsymbol{\alpha}_{H_1}$ reduces, and the NMSE of $\boldsymbol{X}_{H_1}$ and $\boldsymbol{X}_{H_0}$ of the proposed approach maintains in a low level. Meanwhile, the estimate of all the unknown parameters in MC-GLRT are significantly better than UIF-GLRT with the same τ. Moreover, as τ increases, namely available data decreases, the estimation accuracy of UIF-GLRT deteriorates faster than MC-GLRT. The curves of P_D versus SINR with different τ are shown in Fig.2. We can see that the detection performance of MC-GLRT outperforms UIF-GLRT when the same observations are available. Furthermore, the performance advantage is particularly noticeable when τ changes from 0 to 0.2. The improvement is achieved by utilizing the specific structure of disturbance and recasting the estimation of $\boldsymbol{X}$ as the recovery of $\boldsymbol{X}_i$. Specifically, we assign larger weights to the entries with more observations in the reconstruction process and consider the properties of the full matrix (positive semi-definite and low rank) as well, which improves the estimation accuracy and further the detection performance.

5. CONCLUSION

In this paper, we propose a GLRT detector to deal with the detection of distributed target with partial observation. Though point-target model with partial observation and distributed target with complete observation are all considered in literature, the more complicated issue of distributed target with partial observation is firstly addressed in this work. With the hypothesis test model built by multiple groups of data missing, the GLRT detector is derived. The key issue of estimating the unknown parameters in GLRT detector is accomplished by using MC and the alternating iterative algorithm. The existing GLRT detectors are verified to be special cases of our approach. Numerical results verify the improvement of the estimation accuracy and furthermore the detection performance.

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