COPULA APPLICATION IN NONLINEAR/NON-GAUSSIAN BAYESIAN TRACKING IN THE CASE OF CORRELATED SENSORS

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ABSTRACT

One of the most important challenges in target tracking is the modeling of correlated and non-Gaussian random processes. In this paper, a new target tracking approach by means of particle filtering in environments with highly correlated sensors, is discussed. The goal is to provide an accurate model of dependency structure in multivariate observation likelihood function, with non-Gaussian marginals to obtain a new algorithm in tracking problems. The main novelty of our method, termed as Copula-based Sequential Importance Resampling particle filter (CSIR-PF), is an application of the copula theory which is a powerful tool in correlation modeling in statistical theory. The precise model obtained by copulas makes a great improvement in particles' weightings. The performance of our proposed method is evaluated through the simulations of target tracking problems with highly correlated sensors. Results clearly indicate the acceptable performance of CSIR-PF.

Index Terms— Target tracking, copula theory, particle filter, correlated sensors, multisensor

1. INTRODUCTION

Very often in many applications recently, researchers ought to model the underlying dynamics of a physical system accurately by including the elements of nonlinearity and non-Gaussianity. Many problems in engineering require estimation of time varying state of the system under investigation, through the noisy observations. Discrete-time approach is widespread and appropriate in dynamic state estimation. To model a time-series in state-space approach, we focus our attention on the state vector, which contains all relevant information required to describe the underlying system. In many target tracking applications, the kinematic characteristics of the target motion is considered as this information. In practice, the measurements of the state vector are corrupted with environment noise. State space approach is a suitable tool for handling multivariate and nonlinear/non-Gaussian processes [1]. More information on the application of nonlinear/non-Gaussian state space models can be found in [2].

At least two models are required to analyze aforementioned systems, the system(dynamic) model and the measurement model. The former describes the time evolution of states and the later one represents the relation between observations with states of target. As stated at [1], given probabilistic state-space formulation of these models makes the Bayesian framework a suitable tool for the corresponding system analysis. In on-line tracking scenarios in which state update is necessary, recursive filtering is an appropriate solution and contains two steps: prediction and update. In prediction step, PDF of target states is predicted using system model and forward from one measurement time to the next. In other words, since the state is disturbed by unknown perturbations, the prediction procedure translates, deforms, and spreads the state PDF. In the update step by means of the recent measurement, the predicted PDF is improved. Since the posterior probability density function in the Bayesian approach contains all available statistical information, it might be the complete solution to the estimation problem [1]. In many target tracking problems, optimal solution does not exist because some linear/Gaussian assumptions [3] are not held, therefor applying approximation methods seems to be necessary. Generally in practical nonlinear/non-Gaussian tracking problems, it is necessary to apply more complex and sophisticated nonlinear filtering methods [4]. The standard nonlinear filtering techniques can not model all of the salient features of the PDFs accurately [4]. Recently the set of Monte Carlo filtering techniques known as particle filtering, found numerous applications in problems wherein the PDFs are modeled as general as possible without any limitative assumptions on their forms. Particle filtering is a suboptimal solution to such problems, also is known variously as bootstrap filtering [5].

In practical radar and sonar applications [6], [7], [8] correlated sensor measurements might be engaged with the scenario; for example in Shen [6] due to the usage of the same jammer signal, the noise of sensors are correlated. Considering dependency, which involves statistical information about PDFs, improve the estimation states in the filtering methods that deal directly with PDFs. Another example is considered by Liao and Wu in [9] in which they proposed the algorithm for two dimensional angle estimation in united circular arrays. Such a measurement model is used in bearing only tracking (BOT) problems. In this paper, we propose a new method using copula embedded in for nonlinear and non-Gaussian correlated target particle filtering. The paper is structured as follows: Section (2) contains the problem statement followed by a brief summary of the copula theory and particle filtering method that the proposed approach is based upon. Finally at the end of this section, proposed method explained in details for target tracking. Some simulation results are depicted in Section (3) to approve our proposed method.

2. PROBLEM STATEMENT

2.1. Copula

Quantitative finance and tail risk minimization are recent fields in which the copula concept have found many applications [10]. In addition with these fields, target tracking and communications are another grounds that they are playing an important role in [11], [12]. Nowadays, copula has taken the place of classical correlation modeling tools such as Pearson correlation coefficient in describing dependency structure among random variables [11]. According to Nelsen [13] and Skalar's theorem [14], copulas are functions that describe the joint distribution of multivariate random variables through the following theorem:

Theorem 1. let F be an n-dimensional cumulative distribution function (CDF) with margins F_1, F_2, \dots, F_n . Then, there exists a function $C : [0, 1]^n \to [0, 1]$ such that:

$$F(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n))$$
(1)

Conversely, if C is a copula and $F_1, F_2, ..., F_n$ are CDFs, then the function F defined by (1) is an n-dimensional CDF with margins $F_1, F_2, ..., F_n$. so F is uniquely decomposed.

The proof of this theorem and more properties of C can be found in [13]. Function $C(\cdot)$ has a useful property that helps find its closed form formula: a copula is itself a CDF, defined on $[0, 1]^n$, with uniform margins. The description of multivariate CDFs by employing the copula function provides a suitable flexibility, since the margins and their dependences structure can be selected independently [11]. As stated in [15], for any copula function, there is a corresponding copula density function $c(\cdot)$ that is derived from the derivative of the function C in (1) as follows:

$$f(x_1, \ldots, x_n) = \frac{\partial^n C(F_1(x_1), \ldots, F_n(x_n))}{\partial F_1(x_1) \ldots \partial F_n(x_n)} \times \prod_{i=1}^n f_i\left(\frac{dF_i(x_i)}{\partial F_1(x_1) \ldots \partial F_n(x_n)}\right)$$

where $f_1(x_1), \ldots, f_n(x_n)$ are the marginal PDFs, so the multivariate joint PDF is derived as follows:

$$f(x_1, \ldots, x_n) = c(F_1(x_1), \ldots, F_n(x_n)) \times \prod_{i=1}^n f_i(x_i)$$
(3)

As stated before, the copula density function $c(\cdot)$ can be selected independent from the marginal PDFs, so copula function allows a consistent and flexible modeling of the dependence structure. In such a filtering methods that is dealing with multivariate PDFs directly, more accurately modeling of the dependence structure causes correction in tracking of states. In one point of view, copulas can be grouped into two classes; elliptical and Archimedean copulas [13] [16]. The most prominent elliptical copulas are normal and Students t. Therein class different levels of dependency between the margins can be specified which is a useful property. These are implicit copulas that do not have a simple closed form, but are implied by well-known multivariate distribution functions [17]. There are also a number of copulas, explicit copulas, which are not derived from multivariate distribution functions, but do have simple closed forms [17] like Archimedean copula family. Archimedean copulas are a popular class, because of their easy implementation. Many copulas as Clayton, Frank, Gumbel, ... belong to this class [13]. In this paper, three kinds of copulas, are applied for modeling the joint PDF of measurement model; Gaussian, Clayton and Student's t. The mathematical relationships for these copula family are mentioned in Table (1). That $\mathbf{x} = [x_1, x_2]$, $x_1 = \Phi^{-1}(u)$ and $x_2 = \Phi^{-1}(v)$. The function Φ^{-1} is the inverse cumulative distribution function of a standard normal distribution. \mathbf{R} is the correlation matrix of vector x was introduced. Theoretically, depending on the application, copula function can be introduced in an arbitrary formulation. This powerful tool is used in updating the state of proposed method, that will be explained in details in the next subsections.

2.2. Particle Filtering

Particle filters perform sequential Monte Carlo (SMC) estimation based on point mass (or "particle") representation of probability den-

Table 1: Some copula distribution

copula Name	Bivariate copula density $c(u, v)$			
Gaussian [◊]	$\frac{1}{ \mathbf{R} ^{\frac{1}{2}}} \exp\left\{-\frac{1}{2}\mathbf{x}^{T} \left(\mathbf{R}^{-1} - \mathbf{I}\right) \mathbf{x}\right\}$			
Students t †	$\frac{\Gamma\left(\frac{\nu+2}{2}\right)\Gamma\left(\frac{\nu}{2}\right)\left(1+\frac{\mathbf{x}^{T}\mathbf{R}^{-1}\mathbf{x}}{\nu}\right)^{-\frac{\nu+2}{2}}}{\sqrt{ \mathbf{R} }\Gamma\left(\frac{\nu+1}{2}\right)^{2}\prod_{i=1}^{2}\left(1+\frac{x_{i}^{2}}{\nu}\right)^{-\frac{\nu+1}{2}}}$			
Clayton *	$\left (1+\alpha) (uv)^{-\alpha-1} \left[-1 + u^{-\alpha} + v^{-\alpha} \right]^{-2-\frac{1}{\alpha}} \right $			
with parameters : ${}^{\diamond} \mathbf{B} \in [-1, 1]^{2 \times 2} {}^{\dagger} \mathbf{B} \in [-1, 1]^{2 \times 2} \& u > 0 \ {}^{*} \alpha \in [-1, +\infty] \setminus \{0\}$				

sities [3]. There are several variants of the particle filter that are explained in details in [1]. In this article, we use some of them such as sequential importance sampling (SIS), sampling importance resampling (SIR). The sequential importance sampling (SIS) algorithm is a Monte Carlo (MC) method that forms the basis for most sequential MC filters developed over the past decades [1]; see [18] for more details. The basic idea is to use a set of random samples with corresponding weights to describe the desired posterior density function and compute estimates based on these samples and weights. To define the problem of nonlinear filtering, the target state vector $\{\mathbf{x}_k \in \mathbb{R}^{n_x}\}$, where n_x is the dimension of the state vector; \mathbb{R} is a set of real numbers; $\{k \in \mathbb{N}\}$ is the time index; and \mathbb{N} is the set of natural numbers must be introduced. Here index k is assigned to a continuous-time instant t_k . The target state evolves according to the following discrete-time stochastic model:

$$\mathbf{x}_{k} = f_{k-1} \left(\mathbf{x}_{k-1}, \mathbf{v}_{k-1} \right) \tag{4}$$

where f_{k-1} is a known, possibly nonlinear function of the state \mathbf{x}_{k-1} and \mathbf{v}_{k-1} which is referred to as a process noise sequence. The objective of nonlinear filtering is to recursively estimate \mathbf{x}_k from measurements $\{\mathbf{z}_k \in \mathbb{R}^{n_z}\}$. These measurements are related to the target state via the measurement equation:

$$\mathbf{z}_k = h_k \left(\mathbf{x}_k, \mathbf{w}_k \right) \tag{5}$$

where h_k is a known, possibly nonlinear function and \mathbf{w}_k is a measurement noise sequence, in general non-Gaussian. For the given random measure set $\{\mathbf{x}_{0:k}^i, \omega_k^i\}_{i=1}^{N_s}$, the posteriori PDF approximation at time k is as follows:

$$p\left(\mathbf{x}_{0:k}|\mathbf{z}_{1:k}\right) \approx \sum_{i=1}^{N_s} \omega_k^i \delta\left(\mathbf{x}_{0:k} - \mathbf{x}_{0:k}^i\right)$$
(6)

wherein $\{\mathbf{x}_{0:k}^i\}_{i=1}^{N_s}$ is a set of support points with their corresponding weights $\{\omega_k^i\}_{i=1}^{N_s}$ that $\sum_i \omega_k^i = 1$, and $\mathbf{x}_{0:k} = \{\mathbf{x}_j, j = 0, \dots, k\}$ denotes all states until time k. By applying importance sampling method the samples are drawn and the corresponding weights are computed [19], [18]. Accordingly to an importance density function $q(\cdot)$ the weights in (6) are:

$$\omega_k^i \propto \frac{p(\mathbf{x}_{0:k}^i | \mathbf{z}_{1:k})}{q(\mathbf{x}_{0:k}^i | \mathbf{z}_{1:k})} \tag{7}$$

Returning to the sequential case, as stated in [3] the weight updating equation are as follows:

$$\omega_k^i \propto \omega_{k-1}^i \frac{p\left(\mathbf{z}_k | \mathbf{x}_k^i\right) p\left(\mathbf{x}_k^i | \mathbf{x}_{k-1}^i\right)}{q\left(\mathbf{x}_k^i | \mathbf{x}_{k-1}^i, \mathbf{z}_k\right)}$$
(8)

and the posterior filtered density $p(\mathbf{x}_k | \mathbf{z}_{1:k})$ can be approximated as:

$$p\left(\mathbf{x}_{k}|\mathbf{z}_{1:k}\right) \approx \sum_{i=1}^{N_{s}} \omega_{k}^{i} \delta\left(\mathbf{x}_{k} - \mathbf{x}_{k}^{i}\right)$$
(9)

It can be shown that as $N_s \rightarrow \infty$, (9) converge to the true posterior density that is not feasible.

As the degeneracy phenomenon is presented in [18], the common problem with the SIS, is inevitable that occurs after a few iterations, when all except one particle will have negligible weight [1]. A parameter was presented in [19] and [20] to detect the degeneracy, termed effective sample size N_{eff} :

$$\widehat{N_{eff}} = \frac{1}{\sum_{i=1}^{N_s} \left(\omega_k^i\right)^2} \tag{10}$$

However, the first impractical way to reduce this effect requires a very large N_s , but with good choice of importance density or applying suitable resampling methods [1] this effect can be diminished. In the resampling method, low weighted samples were eliminated and their high importance wights counterparts were multiplied. The resampling step is a crucial and computationally expensive part in a particle filter [18]. The most frequently encountered algorithms are multinomial resampling [21], stratified resampling [18], [22], systematic resampling [22], [1] and residual resampling [20].

2.3. Copula-based SIR-Particle Filtering

Because the problem which is under investigation has nonlinear/non-Gaussian conditions, in this paper, it's impossible to use optimal choice of importance density [3]. Since the prior density is the most common choice and often convenient for being the importance density, we use it in our method because of its simple implementation [1].

$$q\left(\mathbf{x}_{k}|\mathbf{x}_{k-1}^{i},\mathbf{z}_{k}\right) = p(\mathbf{x}_{k}|\mathbf{x}_{k-1}^{i})$$
(11)

hence, (8) is modified as follows:

$$\omega_k^i \propto \omega_{k-1}^i p\left(\mathbf{z}_k | \mathbf{x}_k^i\right) \tag{12}$$

where $p(\mathbf{z}_k | \mathbf{x}_k^i)$ is conditional joint probability density function named likelihood function. Accurate determination of this PDF achieves careful weight updating. In the case of correlated sensors, except a few PDF like Normal, it is so hard to describe arbitrary correlations in general joint PDFs. Pearson correlation coefficient, is a measure of the linear dependency between two stochastic variables. But it cannot model nonlinear dependency among non-Gaussian random variables well. Clearly when the noise of sensors are independent, the likelihood function is as follows:

$$p\left(\mathbf{z}_{k}|\mathbf{x}_{k}^{i}\right) = \prod_{j} p_{j}(n_{j})_{|_{\mathbf{n}=\mathbf{z}_{k}-\mathbf{g}_{k}(\mathbf{x}_{k})}$$
(13)

where $p_j(n_j)$ is the PDF of noise of the j^{th} sensor, n_j is the j^{th} element of noise vector **n**. But in the case of correlated sensors with non-Gaussian marginals, an accurate structure is necessary to

describe the correlations precisely, which is done by the copula concept. Accordingly and as what stated before:

$$p\left(\mathbf{z}_{k}|\mathbf{x}_{k}^{i}\right) = c\left(\mathbf{u}\right)\prod_{j} p_{j}(n_{j})_{|_{\mathbf{n}=\mathbf{z}_{k}-\mathbf{g}_{k}}(\mathbf{x}_{k})}$$
(14)

where c(.) is copula density function and **u** is:

$$\mathbf{u} = [P_1(n_1), P_2(n_2), \dots, P_M(n_M)]$$
(15)

that is the CDF values vector and P_j (.) is the CDF of the noise of j^{th} sensor, then in problem with M sensors, (12) becomes:

$$\omega_k^i \propto \omega_{k-1}^i c(\mathbf{u}) \prod_j p_j(n_j)_{|_{\mathbf{n}=\mathbf{z}_k-\mathbf{g}_k(\mathbf{x}_k)}}$$
(16)

So it means that accurate weights updating are done by copula density, exactly because of precise modeling of likelihood function. As stated earlier the degeneracy problem in SIS algorithm is common and inevitable, hence, the resampling method is used to reduce its bad effects on weighting particles. In resampling step, new set of random samples $\{\mathbf{x}_{k}^{i*}\}_{i=1}^{N_s}$ is generated that will be used in next iteration. Finally to compute state estimate at time k, we should take a criterion [23], which is here, the minimum mean-square error (MMSE), and its estimate is the conditional mean of \mathbf{x}_k :

$$\hat{\mathbf{x}}_{k}^{MMSE} = \int \mathbf{x}_{k} p\left(\mathbf{x}_{k} | \mathbf{z}_{1:k}\right) d\mathbf{x}_{k}$$
(17)

So with discrete approximation of $p(\mathbf{x}_k | \mathbf{z}_{1:k})$ and according to (9), the final estimation is obtained as follows:

$$\hat{\mathbf{x}}_{k}^{MMSE} \approx \sum_{i=1}^{N_{s}} \omega_{k}^{i} \mathbf{x}_{k}^{i}$$
(18)

According to what have been declared, the pseudo code of our proposed method, the Copula-based Sequential Importance Resampling particle filter (CSIR-PF) becomes as Algorithm(1).

Algorithm 1 Copula-based SIR-PF

1: $\begin{bmatrix} \hat{\mathbf{x}}_k, \{\mathbf{x}_k^i, \omega_k^i\} \end{bmatrix}$ = CSIR $\begin{bmatrix} \frac{1}{\{\mathbf{x}_{k-1}^i, \omega_{k-1}^i\}_{i=1}^{N_S}, \mathbf{z}_k \end{bmatrix}$ 2: for $i = 1 : N_s$ do 3: Draw a $\mathbf{x}_k^i \sim p(\mathbf{x}_k | \mathbf{x}_{k-1}^i)$ Compute \mathbf{u} (16) 4: Assign a weight ω_k^i (15) 5: 6: Compute total weight: $t = \text{SUM}\left[\left\{\omega_k^i\right\}_{i=1}^{N_s}\right]$ 7: for $i = 1 : N_s$ do Normalize: $\omega_k^i = t^{-1} \omega_k^i$ 8: 9: Compute $\widehat{N_{eff}}$ (10) 10: if $\widehat{N_{eff}} < N_T$ then 11: Resampling procedure [$\left\{\mathbf{x}_{k}^{j}, \omega_{k}^{j}\right\}$]=resampl[[$\left\{\mathbf{x}_{k}^{i}, \omega_{k}^{i}\right\}_{i=1}^{N_{s}}$] 12: 13: for $i = 1 : N_s$ do Normalize: $\omega_k^i = t^{-1} \omega_k^i$ 14: 15: Compute MMSE estimate: $\hat{\mathbf{x}}_k$ (18) 16: for $i = 1 : N_s$ do $\mathbf{\hat{x}}_k = \mathbf{\hat{x}}_k + \omega_k^i \mathbf{x}_k^i$ 17:

The assumptions required for applying this method are very weak. Firstly, the dynamics and measurement models, f_{k-1} and h_k in (4) and (5), respectively, have to be known; and secondly it is required to be able to do sample realizations from the process



Fig. 1: 2D Tracking in the case of Clayton copula, target starts moving with initial state $[x, y, v_x, v_y, a_x, a_y] = [-100, 50, 200, 10, 20, -30]$

noise distribution of $\mathbf{v}_{\mathbf{k}-1}$ and from the prior. Finally, the likelihood function $p(\mathbf{z}_k|\mathbf{x}_k)$ needs to be available for pointwise evaluation. Noticeably, when in such cases, we face to highly correlated sensors, it causes the abundance of samples in joint PDF density $p(\mathbf{z}_k|\mathbf{x}_k)$ to grow up, so the probability amount at points that are slightly away from the peak/peaks becomes negligible because of inertia and discipline which occurred by correlation. This event will increase the probability of happening degeneracy problem. Therefor it is so important to use appropriate resampling method. Systematic Resampling [22] is an efficient scheme, and so simple to implement, its computational complexity is O(N) and it minimizes the MC variation. Complete pseudo code can be found in [3].

3. SIMULATION AND RESULTS

Although our proposed method can be applied in sonar tracking problems, here a synthetic radar scenario is considered, in order to qualitatively gauge performance. Assume a moving target with constant acceleration¹ motion model in x - y plan. Measurements are obtained by two correlated sensors, range and bearing. These measured values are mixed with additive noises. The noise of range measurements are correlated with the noise of other sensor, and also it has non-Gaussian marginals and nonlinear non-Gaussian correlation structure. Range sensor's noise has Rayleigh distribution with var = 10 m and other one has Generalized Extreme Value Distribution (GEV) with var = .4 rad. Eventually the estimated states by CSIR-PF are compared to known and general PFs [1] such as generic particle filter (GPF) that the correlation structure is not considered therein. In both algorithms $N_s = 10^4$. The traditional measure of performance, i.e. Root Mean Squared Error (RMSE) is selected to compare final results. In some scenarios with different correlation structures, we model the correlation via these copulas: Gaussian, Studens' t and Clayton. Assuming the copula distributions with their parameters: Gaussian ($\rho = .98$), Student's t ($\rho = .98$ & $\nu = 1$) and Clayton ($\alpha = 25$), the sensors noise are highly correlated; they are named in the tables by a_H , b_H and c_H scenarios respectively. Note that ρ is the off-diagonal element of the correlation matrix **R**. Also a_L , b_L and c_L indicate the scenarios with lowly correlated sensors that are correspond to the Gaussian copula ($\rho = .1$), Student's t $(\rho = .1 \& \nu = 1)$ and Clayton $(\alpha = 2)$ distributions. Also the correlation in a_M , b_M and c_M has these distributions: Gaussian ($\rho = .5$), Student's t ($\rho = .5$ & $\nu = 1$) and Clayton ($\alpha = 15$) respectively. As it was shown in Table(2), for scenarios with more highly correlated sensors, the proposed method clearly performs better than GPF. The results obtained through independent Mont Carlo simulations. The

tracking results in x-y plain for two scenarios with Clayton copula are depicted if Fig(1). As it's foresighted, the improvement per-

		Mean		Variance
Scenario	GPF	CSIR-PF	$\operatorname{Imp}(\%)^{\diamondsuit}$	Imp(%)
a _H	0.10243	0.0735	+28.2	+61.2
b_H	0.10418	0.06757	+35.1	+20.1
c_H	0.11911	0.058382	+50.1	+50.8
a_M	0.1142	0.10603	+7.1	+16.3
b_M	0.10589	0.08848	+16.4	+30.7
c_M	0.10736	0.073692	+31.4	+07.6
a_L	0.13269	0.13341	-00.5	+01.0
b_L	0.1167	0.097385	+16.6	+04.7
c_L	0.10741	0.087706	+18.3	-13.0

Table 2: \overline{RMSE} s (m) of 10 Mont-Calro simulations

[♦] is improvement percentage of CSIR-PF in comparison to GPF.

centage has been decreased in a_L , b_L and c_L scenarios because the correlation between sensors noise is so low.

4. CONCLUSION

In this paper a new approach is proposed for nonlinear and non-Gaussian moving target tracking problems with correlated sensors. The proposed method is based on the particle filtering and the copula theory. We have detailed the algorithm that explicitly takes into account the correlation of the sensors noise in the filtering equations. This method uses the copula concept to improve weighting particles by accurate modeling of the correlation structure. By use of the copula, there is no limitation to model any arbitrary correlation between sensors even in high-dimensional state spaces. Hence, the combination of particle filtering technique and the copula concept builds a novel method to improve target tracking in indicated problems; with increasing the correlation, the performance has been improved. The performance of method was compared to the former method, the GPF, that the correlation of sensors noise is not considered therein. Precise estimated results confirm the validity of our proposed method.

¹also called Continuous/Discrete Wiener process acceleration

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