EXTENDED KALMAN FILTER FOR EXTENDED OBJECT TRACKING

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ABSTRACT

In this work, we present a novel method for tracking an elliptical shape approximation of an extended object based on a varying number of spatially distributed measurements. For this purpose, an explicit nonlinear measurement equation is formulated that relates the kinematic and shape parameters to a measurement by means of a multiplicative noise term. Based on the measurement equation, we derive an extended Kalman filter (EKF) for a closed-form recursive measurement update. The performance of the proposed method is demonstrated with simulations.

Index Terms— Extended object tracking, multiplicative noise, Kalman filter, extended Kalman filter

1. INTRODUCTION

In contrast to point target tracking, the extended object tracking problem deals with multiple spatially distributed measurements per target object. The extended object tracking problem is becoming increasingly important in applications such as autonomous driving. For a detailed recent overview, we refer to [1, 2].

In case of high sensor noise and a few measurements per scan, one is usually bound to work with simple shape approximations such as ellipses. One of the most common approaches for this purpose is the random matrix approach [3, 4, 5, 6], which characterizes the object extent by a random matrix. Another approach is the Random Hypersurface Model (RHM) [7, 8, 9], which assumes that measurements originate from randomly scaled versions of the object contour. In this manner, extended object tracking is reduced to a curve fitting problem. This approach is tailored to complex, i.e., star-convex, shapes and (rather) low measurement noise.

This paper builds upon our idea to use a multiplicative noise term in the measurement equation [10] to model the spatial distribution of the measurements. In order to estimate the shape with a linear estimator, a pseudo-measurement must be constructed based on the original measurements [10, 11]. While the original work [10] was restricted to axis-aligned ellipses, we developed in [11] a Second Order Extended Kalman filter (SOEKF) [12, 13] that is capable of



Fig. 1: Illustration of the measurement model. The location of the object is $\boldsymbol{m}_k = [m_{k,1}, m_{k,2}]^T \in \mathbb{R}^2$ at time k. The ellipse parameters for the shape $\boldsymbol{p}_k = [\alpha_k, l_{k,1}, l_{k,2}]^T \in \mathbb{R}^3$ consist of the orientation and semi-axes lengths. Measurements are spatially distributed on the object and additionally corrupted by additive sensor noise. The *i*-th measurement at time k is denoted as $\boldsymbol{y}_k^i \in \mathbb{R}^2$.

estimating the parameters of arbitrary aligned ellipses. Unfortunately, the SOEKF [11] requires tedious calculations of several Hessians.

In this work, we solve this issue by developing an extended Kalman filter [14, 15] that does not involve any Hessians at all. For this purpose, the fundamental insight is that the expectation and covariance of the pseudo-measurement can be approximated directly from the original measurement covariance matrix. Compared to the random matrix approach [3, 4, 5, 6], our approach explicitly works with the orientation and lengths of the semi-axes of an ellipse. A major advantage is that one can easily formulate process models for the individual shape parameters, e.g., orientation changes can be more significant than changes of the semi-axes.

2. PROBLEM FORMULATION

The kinematic parameters of the extended object at time k

$$\boldsymbol{r}_{k} = \left[\boldsymbol{m}_{k}^{T}, \boldsymbol{\dot{m}}_{k}^{T}, \dots\right]^{T}$$
(1)

consist of the object position $m_k \in \mathbb{R}^2$, velocity $\dot{m}_k \in \mathbb{R}^2$, and possible further quantities. Following the same parameterization for ellipses as in [11], the shape parameters are

$$\boldsymbol{p}_{k} = \left[\alpha_{k}, l_{k,1}, l_{k,2}\right]^{T} \in \mathbb{R}^{3} , \qquad (2)$$

where $\alpha_k \in [-\pi, \pi)$ is the orientation, and $l_{k,1}$ and $l_{k,2} \in \mathbb{R}^+$ indicate the lengths of the semi-axes, see Fig. 1.

2.1. Measurement Model

A fluctuating number of two-dimensional Cartesian measurements $\mathcal{Y}_k = \{y_k^i\}_{i=1}^{n_k}$ is available at each time k. Each individual measurement $y_k^i \in \mathbb{R}^2$ is related to the kinematic and shape parameters according to the multiplicative noise model [10, 11] for extended objects. The basic idea is that y_k^i results from the object center plus randomly scaled semi-axes plus measurement noise (see Fig. 1), i.e.,

$$\boldsymbol{y}_{k}^{i} = \boldsymbol{m}_{k} + \begin{bmatrix} \cos \alpha_{k} \\ \sin \alpha_{k} \end{bmatrix} l_{k,1} h_{k,1}^{i} + \begin{bmatrix} -\sin \alpha_{k} \\ \cos \alpha_{k} \end{bmatrix} l_{k,2} h_{k,2}^{i} + \boldsymbol{v}_{k}^{i} \quad .$$
(3)

In compact form (3) can be written as

$$\boldsymbol{y}_{k}^{i} = \mathbf{H}\boldsymbol{r}_{k} + \underbrace{\mathbf{R}(\alpha_{k}) \begin{bmatrix} l_{k,1} & 0\\ 0 & l_{k,2} \end{bmatrix}}_{:=\mathbf{S}(\boldsymbol{p}_{k})} \begin{bmatrix} h_{k,1}^{i}\\ h_{k,2}^{i} \end{bmatrix} + \boldsymbol{v}_{k}^{i} , \quad (4)$$

with $\mathbf{H} = \begin{bmatrix} \mathbf{I}_2 & \mathbf{0} \end{bmatrix}$ (where \mathbf{I}_2 is the two-dimensional identity matrix and $\mathbf{0}$ is the null matrix with appropriate dimensions), rotation matrix $\mathbf{R}(\alpha_k) = \begin{bmatrix} \cos \alpha_k & -\sin \alpha_k \\ \sin \alpha_k & \cos \alpha_k \end{bmatrix}$, and multiplicative noise $\mathbf{h}_k^i = \begin{bmatrix} h_{k,1}^i \\ h_{k,2}^i \end{bmatrix} \in \mathbb{R}^2$. We assume both $h_{k,1}^i$ and $h_{k,2}^i$ to be mutually independent of all other random variables. Furthermore, we suggest their variances to be $\sigma_{h_1} = \sigma_{h_2} = \frac{1}{4}$ in order to match an elliptical uniform distribution, see [2].

2.2. Dynamic Model

Both the kinematic and the shape parameters of the extended object are assumed to follow a linear Markov model

$$\boldsymbol{r}_{k+1} = \boldsymbol{A}_k^{\boldsymbol{r}} \boldsymbol{r}_k + \boldsymbol{w}_k^{\boldsymbol{r}} , \qquad (5)$$

$$\boldsymbol{p}_{k+1} = \boldsymbol{A}_k^{\boldsymbol{p}} \boldsymbol{p}_k + \boldsymbol{w}_k^{\boldsymbol{p}} , \qquad (6)$$

where \mathbf{A}_{k}^{r} and \mathbf{A}_{k}^{p} are process matrices; \boldsymbol{w}_{k}^{r} and \boldsymbol{w}_{k}^{p} specify zero-mean Gaussian process noise with covariance matrices \mathbf{C}_{r}^{w} and \mathbf{C}_{p}^{w} . Note that we can model the temporal evolution of each individual shape parameter.

3. EXTENDED KALMAN FILTER

In this section, we derive an extended Kalman filter for recursively estimating both the kinematic and shape parameters of an extended object. An essential assumption that we make is that the shape and kinematic parameters are independent. By this means, we can significantly simplify the update formulas.

3.1. Measurement Update

In the measurement update step, we would like to calculate the posterior means and covariances for the kinematic and shape parameters

$$\hat{m{r}}_k^i,~\mathbf{C}_{k,i}^{m{r}}$$
 and $\hat{m{p}}_k^i,~\mathbf{C}_{k,i}^{m{p}}$,

that incorporate all measurements up to the *i*-th measurement \boldsymbol{y}_k^i based on on the previous estimate

$$\hat{m{r}}_k^{i-1},~\mathbf{C}_{k,i-1}^{m{r}}$$
 and $\hat{m{p}}_k^{i-1},~\mathbf{C}_{k,i-1}^{m{p}}$

where $i = 0, \dots, n_k$. Note that individual measurements in \mathcal{Y}_k are independent (conditioned on the state), so that they can be processed sequentially.

It is shown in [10] that the shape parameters are not observable if a linear measurement update with y_k^i is performed. Hence, we propose a two-step measurement update. First, the kinematic parameters are updated using the original measurement. Then, we create a pseudo-measurement \tilde{y}_k^i by an uncorrelated transformation [16] on y_k^i in order to update the shape variables. Note that there is no double-counting of the measurement as the shape and kinematic parameters are independent.

3.1.1. Kinematic Parameters

It is obvious that (4) is linear in the kinematic state and nonlinear in the shape parameters. By approximate p_k using the previous estimate \hat{p}_k^{i-1} , we have

$$\boldsymbol{y}_{k}^{i} \approx \mathbf{H}\boldsymbol{r}_{k} + \mathbf{S}(\hat{\boldsymbol{p}}_{k}^{i-1})\boldsymbol{h}_{k}^{i} + \boldsymbol{v}_{k}^{i}$$
 . (7)

According to the Kalman filter [15], we obtain

$$\mathsf{E}\{\boldsymbol{y}_{k}^{i}\} = \mathbf{H}\hat{\boldsymbol{r}}_{k}^{i-1} , \qquad (8)$$

$$\mathbf{C}_{k,i}^{ry} = \mathbf{C}_{k,i-1}^{r} \mathbf{H}^{T} , \qquad (9)$$

$$\mathbf{C}_{k,i}^{\boldsymbol{y}\boldsymbol{y}} = \mathbf{H}\mathbf{C}_{k,i-1}^{\boldsymbol{r}}\mathbf{H}^{T} + \mathbf{S}(\hat{\boldsymbol{p}}_{k}^{i-1})\mathbf{C}^{\boldsymbol{h}}\left(\mathbf{S}(\hat{\boldsymbol{p}}_{k}^{i-1})\right)^{T} + \mathbf{C}^{\boldsymbol{v}}.$$
(10)

The standard Kalman filter measurement update results in the equations

$$\hat{\boldsymbol{r}}_{k}^{i} = \hat{\boldsymbol{r}}_{k}^{i-1} + \mathbf{C}_{k,i}^{\boldsymbol{ry}} (\mathbf{C}_{k,i}^{\boldsymbol{yy}})^{-1} (\boldsymbol{y}_{k}^{i} - \mathbf{E} \{\boldsymbol{y}_{k}^{i}\}) \quad , \qquad (11)$$

and

$$\mathbf{C}_{k,i}^{\boldsymbol{r}} = \mathbf{C}_{k,i-1}^{\boldsymbol{r}} - \mathbf{C}_{k,i}^{\boldsymbol{ry}} (\mathbf{C}_{k,i}^{\boldsymbol{yy}})^{-1} (\mathbf{C}_{k,i}^{\boldsymbol{ry}})^{T} \quad .$$
(12)

3.1.2. Shape Parameters

To exploit the information contained in a measurement in a linear estimator, we construct a pseudo-measurement by taking the 2-fold Kronecker product of the original measurement. Furthermore, we shift the measurement by its expectation so that the expectation and covariance of the pseudomeasurement are the second and fourth central moments of the original measurement. In short, we define the pseudomeasurement

$$\tilde{\boldsymbol{y}}_{k}^{i} = \underbrace{\begin{bmatrix} (y_{k,1}^{i} - \hat{m}_{k,1}^{i-1})^{2} \\ (y_{k,2}^{i} - \hat{m}_{k,2}^{i-1})^{2} \\ (y_{k,1}^{i} - \hat{m}_{k,1}^{i-1})(y_{k,2}^{i} - \hat{m}_{k,2}^{i-1}) \end{bmatrix}}_{:=g(\boldsymbol{m}_{k}, \boldsymbol{p}_{k}, \boldsymbol{h}_{k}^{i}, \boldsymbol{v}_{k}^{i})} \qquad (13)$$

We assume the covariance we get from (10) is

$$\mathbf{C}_{k,i}^{\boldsymbol{y}\boldsymbol{y}} = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} \quad . \tag{14}$$

Then, according to the definition of the second and fourth central moments

$$\mathsf{E}\{\tilde{\boldsymbol{y}}_{k}^{i}\} = \begin{bmatrix} \sigma_{11} & \sigma_{22} & \sigma_{12} \end{bmatrix}^{T}, \tag{15}$$

$$\mathbf{C}_{k,i}^{\tilde{\boldsymbol{y}}\tilde{\boldsymbol{y}}} = \begin{bmatrix} 3\sigma_{11}^2 & \sigma_{11}\sigma_{22} + 2\sigma_{12}^2 & 3\sigma_{11}\sigma_{12} \\ \sigma_{11}\sigma_{22} + 2\sigma_{12}^2 & 3\sigma_{22}^2 & 3\sigma_{22}\sigma_{12} \\ 3\sigma_{11}\sigma_{12} & 3\sigma_{22}\sigma_{12} & \sigma_{11}\sigma_{22} + 2\sigma_{12}^2 \end{bmatrix}.$$
(16)

In order to calculate the cross-covariance $\mathbf{C}_{k,i}^{p\tilde{y}}$, the pseudomeasurement equation (13) is linearized around \hat{m}_{k}^{i-1} and \hat{p}_{k}^{i-1} according to a Taylor series expansion

$$\begin{split} \tilde{\boldsymbol{y}}_{k}^{i} &\approx g(\hat{\boldsymbol{m}}_{k}^{i-1}, \hat{\boldsymbol{p}}_{k}^{i-1}, \boldsymbol{h}_{k}^{i}, \boldsymbol{v}_{k}^{i}) \\ &+ \tilde{\boldsymbol{J}}_{\boldsymbol{m}}(\boldsymbol{h}_{k}^{i}, \boldsymbol{v}_{k}^{i})(\boldsymbol{m}_{k} - \hat{\boldsymbol{m}}_{k}^{i-1}) \\ &+ \tilde{\boldsymbol{J}}_{\boldsymbol{p}}(\boldsymbol{h}_{k}^{i}, \boldsymbol{v}_{k}^{i})(\boldsymbol{p}_{k} - \hat{\boldsymbol{p}}_{k}^{i-1}) , \end{split}$$
(17)

where $\tilde{\mathbf{J}}_{m}(\mathbf{h}_{k}^{i}, \mathbf{v}_{k}^{i})$, $\tilde{\mathbf{J}}_{p}(\mathbf{h}_{k}^{i}, \mathbf{v}_{k}^{i})$ are the Jacobians of (13) evaluated at $\hat{\mathbf{m}}_{k}^{i-1}$ and $\hat{\mathbf{p}}_{k}^{i-1}$ with respect to \mathbf{m}_{k} and \mathbf{p}_{k} . Note that the expectation of $\tilde{\mathbf{J}}_{m}(\mathbf{h}_{k}^{i}, \mathbf{v}_{k}^{i})$ is zero and it is uncorrelated to the shape parameters. As such, only $\tilde{\mathbf{J}}_{p}(\mathbf{h}_{k}^{i}, \mathbf{v}_{k}^{i})$ matters in the cross-covariance calculation. The cross-covariance can be written as

$$\mathbf{C}_{k,i}^{\boldsymbol{p}\boldsymbol{\tilde{y}}} = \mathbf{C}_{k,i-1}^{\boldsymbol{p}} \mathbf{E} \left\{ \mathbf{\tilde{J}}_{\boldsymbol{p}}(\boldsymbol{h}_{k}^{i}, \boldsymbol{v}_{k}^{i}) \right\}^{T} , \qquad (18)$$

where

$$\mathbf{E}\left\{\tilde{\mathbf{J}}_{\boldsymbol{p}}(\boldsymbol{h}_{k}^{i},\boldsymbol{v}_{k}^{i})\right\} = \begin{bmatrix} -\sin 2\alpha & \cos^{2}\alpha & \sin^{2}\alpha \\ \sin 2\alpha & \sin^{2}\alpha & \cos^{2}\alpha \\ \cos 2\alpha & \sin 2\alpha & -\sin 2\alpha \end{bmatrix}$$
$$\cdot \begin{bmatrix} (l_{1})^{2}\sigma_{h_{1}} - (l_{2})^{2}\sigma_{h_{2}} & 0 & 0 \\ 0 & 2l_{1}\sigma_{h_{1}} & 0 \\ 0 & 0 & 2l_{2}\sigma_{h_{2}} \end{bmatrix}$$
(19)

with $\left[\alpha, l_1, l_2\right]$ substituted by $\left[\hat{\alpha}_k^{i-1}, \hat{l}_{k,1}^{i-1}, \hat{l}_{k,2}^{i-1}\right]$.

Finally, with the standard $\check{K}alman$ filter update equations, we get

$$\hat{\boldsymbol{p}}_{k}^{i} = \hat{\boldsymbol{p}}_{k}^{i-1} + \mathbf{C}_{k,i}^{\boldsymbol{p}\tilde{\boldsymbol{y}}} (\mathbf{C}_{k,i}^{\tilde{\boldsymbol{y}}\tilde{\boldsymbol{y}}})^{-1} (\tilde{\boldsymbol{y}}_{k}^{i} - \mathbf{E}\{\tilde{\boldsymbol{y}}_{k}^{i}\}), \quad (20)$$

$$\mathbf{C}_{k,i}^{\boldsymbol{p}} = \mathbf{C}_{k,i-1}^{\boldsymbol{p}} - \mathbf{C}_{k,i}^{\boldsymbol{p}\tilde{\boldsymbol{y}}} (\mathbf{C}_{k,i}^{\tilde{\boldsymbol{y}}\tilde{\boldsymbol{y}}})^{-1} (\mathbf{C}_{k,i}^{\boldsymbol{p}\tilde{\boldsymbol{y}}})^{T} .$$
(21)

3.2. Time Update

Since the temporal evolution of both the kinematic and shape parameters follows a linear Markov model, the standard Kalman filter time update formulas can be used, i.e.,

$$\hat{\boldsymbol{r}}_{k+1}^0 = \boldsymbol{A}_k^{\boldsymbol{r}} \hat{\boldsymbol{r}}_k^{n_k} , \qquad (22)$$

$$\mathbf{C}_{k+1,0}^{\boldsymbol{r}} = \mathbf{A}_{k}^{\boldsymbol{r}} \mathbf{C}_{k,n_{k}}^{\boldsymbol{r}} (\mathbf{A}_{k}^{\boldsymbol{r}})^{T} + \mathbf{C}_{\boldsymbol{r}}^{\boldsymbol{w}} , \qquad (23)$$

and

$$\hat{\boldsymbol{p}}_{k+1}^0 = \mathbf{A}_k^{\boldsymbol{p}} \hat{\boldsymbol{p}}_k^{n_k} , \qquad (24)$$

$$\mathbf{C}_{k+1,0}^{\boldsymbol{p}} = \mathbf{A}_{k}^{\boldsymbol{p}} \mathbf{C}_{k,n_{k}}^{\boldsymbol{p}} (\mathbf{A}_{k}^{\boldsymbol{p}})^{T} + \mathbf{C}_{\boldsymbol{p}}^{\boldsymbol{w}} .$$
(25)

4. SIMULATION

In this section, we compare the developed extended Kalman filter with

- the Second-Order Extended Kalman Filter (SOEKF) based on the multiplicative noise model [11], and
- the random matrix approach [4].

The elliptical object we simulated has diameters of 340m and 80m. Its starting position is at the origin and then it moves with a constant speed of 50km/h. At each time step, measurements are generated from a uniform distribution on the object extent. The number of measurements per time step follows a Poisson distribution with mean 5. The variances of the measurement noise are $2000m^2$ and $80m^2$ for each dimension. The process noise covariance for the kinematic state is assumed as diag ([100, 100, 1, 1]) for all three estimators. The multiplicative noise approaches can model the temporal evolution of each individual shape parameter. For this purpose, we simply use additive process noise, where the standard deviation of the orientation noise is 0.02rad and the semi-axes variances are $0.05m^2$ for both SOEKF and EKF. The measurements, trajectory, and one exemplar run of the estimates are depicted in Fig. 2. We can observe that the random matrix approach adopts faster to the shape than the other estimators in the beginning. But both the SOEKF and EKF have a better performance when the orientation changes. Furthermore, the EKF and SOEKF almost coincide.

For calculating the mean error, we use the Gaussian Wasserstein distance for comparing ellipses as described in [17]. The Gaussian Wasserstein distance incorporates both the location and shape errors and gives a final scalar score. The result of a Monte Carlo simulation with 1000 runs is



Fig. 2: The measurements, trajectory, and one exemplar run of the three estimators.



Fig. 3: Estimation errors for 1000 Monte Carlo runs.

shown in Fig. 3(a). It confirms the observation we get from Fig. 2. This is mainly because the vanilla random matrix approach cannot differentiate if shape changes are caused by orientation or size changes. However, note that approximate predictions might be derived as described in [18].

In the beginning, the EKF has a slightly higher error than SOEKF, but less error around the third coordinate turn. Overall, EKF estimates nearly match the SOEKF estimates, even though no Hessians are needed. All three approaches estimate the velocities quite well as shown in Fig. 3(b) by means of the Root Mean Squared Error (RMSE) for the velocities.

5. CONCLUSION

In this work, the extended object tracking problem is treated in the standard Kalman filtering framework. For this purpose, we developed compact formulas for an efficient closed-form measurement update. Compared to our previous work [11], we present progress in the following aspects:

- The kinematic state is updated using the original measurement. A measurement transformation is only necessary to update the shape variables.
- The calculation of Hessians is avoided by obtaining the (approximate) expectation and covariance of the pseudo-measurement directly from the covariance matrix of the original measurement.

In the future, we will continue to explore the multiplicative noise model and apply it to real-world data sets.

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6. REFERENCES

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