SPARSE ERROR CORRECTION WITH MULTIPLE MEASUREMENT VECTORS: OBSERVABILITY-AWARE APPROACH

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ABSTRACT

We study sparse gross error correction for state estimation in a nonlinear sensing system. We consider a practical assumption that gross errors are sparse, and their locations tend to be invariant over a few consecutive measurement periods. Under the assumption, a robust state estimation and error correction algorithm using multiple measurement vectors is proposed based on local linear approximation of the nonlinear measurement model. Unlike existing approaches in the literature, the proposed method ensures that the estimated gross error locations are such that system is observable, *i.e.*, the system state is uniquely identifiable. The proposed method was applied for power system AC state estimation of the IEEE 14-bus network and outperformed benchmark techniques.

Index Terms— Sparse signal recovery, gross error correction, power system state estimation

1. INTRODUCTION

A large-scale sensing system such as a national-scale power grid relies on state estimation for monitoring the operational conditions of the system. In state estimation, we aim to estimate the system state based on real time sensor measurements. The state estimate is commonly used as an input to important decision making processes such as system control, and thus it is crucial to ensure that the estimation error is within a tolerable range. In the absence of gross errors in the measurements, the typical state estimation process computes maximum likelihood (ML) or maximum a posteriori (MAP) estimates [1, 2]. However, if a portion of the measurements is corrupted by gross errors, Gaussian model ML or MAP estimates can be significantly biased; a proper robust state estimation mechanism is necessary.

In practice, gross errors can originate from sensor failures, calibration errors [3], or data falsification by a cyber attack [4]. Correction of gross errors introduced by an adversary of a cyber attack can be particularly challenging as the gross errors can be designed in an elaborate manner by the adversary. If the attacker can manipulate a proper subset of measurements, he or she can even introduce gross errors that are fundamentally not identifiable [5, 6]. In order to prevent such a scenario, several strategies have been proposed to restrict locations of gross errors such that the errors are detectable and identifiable [7, 8]. In this paper, we study robust state estimation and gross errors correction assuming that the gross errors are identifiable.

In most practical sensing systems, gross errors exhibit some common properties. First, only a small number of sensor measurements are affected by gross errors, *i.e.*, gross errors are *sparse*. The sparsity assumption has been popularly exploited to enhance the performance of gross error correction (see [9-12]). Another common property of gross errors, which has not been popularly explored, is that potential gross error locations remain invariant over multiple measurement periods. For instance, if gross errors originate from sensor failure, the gross error locations remain the same until more sensors fail, or some failed sensors are repaired [13]. On the other hand, if the gross errors are due to a cyber attack, the subset of sensors that are compromised by the attacker remains the same over multiple measurement periods [8]. This invariance property results in temporal correlation among gross error locations over multiple measurement periods. The temporal correlation of error locations was recently exploited in [14] to improve robust state estimation performance, but the presentation therein is limited to linear measurement systems. Here, we present a robust state estimation framework that exploits the temporal correlation of gross error locations for generic nonlinear sensing system.

Our contributions in the paper are as follows. We present a sparse error correction and robust state estimation approach for nonlinear sensing systems that exploits short-term invariance of potential error locations. Another novel aspect of our approach is that the system observability is preserved in the error localization procedure. It has been shown in [5,7] that if gross errors are fundamentally identifiable, the system should remain observable even after removing the sensors that are corrupted by the gross errors. Therefore, assuming that the underlying gross errors are identifiable, it is reasonable to restrict the estimate of error locations such that the system will remain observable after removal of the corresponding corrupt sensors. The proposed robust state estimator adopts an observability-aware error localization step to improve the estimation accuracy. Lastly, we demonstrate that the proposed method can be successfully applied for power system AC state estimation of the IEEE 14-bus network and outperform benchmark techniques.

1.1. Related works and organization

Gross error correction problem has been studied for decades in various contexts, *e.g.*, power system state estimation [2], error control coding [15]. The sparse nature of gross errors has been exploited in several existing approaches [9–12, 15, 16], but most of them have not exploited the short-term invariance of potential error locations. Recently, a sparse error correction method exploiting short-term invariance of potential error locations was proposed [14], but the approach therein is limited to linear sensing systems.

In the current paper, we are extending the framework in [14] to make it applicable to generic nonlinear sensing systems, and the observability-aware error localization is adopted to improve accuracy of robust state estimation.

The organization of the paper is as follows. In Section 2, we

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present the mathematical formulation of robust state estimation in the presence of gross errors. Section 3 presents our sparse error correction and robust state estimation approach. In Section 4, we present the result of applying the proposed estimator to power system AC state estimation. Section 5 contains some concluding remarks.

2. PROBLEM FORMULATION

2.1. Notations

Throughout this paper, vectors and matrices are denoted using bold lower case letters (e.g., \mathbf{x}) and bold upper case letters (e.g., \mathbf{X}) respectively. The i^{th} entry of a vector \mathbf{x} is denoted by $\mathbf{x}[i]$ and the i^{th} row of the matrix \mathbf{X} is denoted by $\mathbf{X}_{i,:}$. We use \mathbf{I}_m to denote the $m \times m$ identity matrix and $\mathbf{0}$ to denote a vector or a matrix of all zeros with appropriate dimensions.

Here, $supp(\mathbf{x})$ denotes the support of \mathbf{x} , which is defined as the set of indices of nonzero entries of \mathbf{x} . On the other hand, the set of row indices of nonzero rows of a matrix \mathbf{X} is defined as the support of \mathbf{X} and is denoted by $supp(\mathbf{X})$. Note that if $\mathbf{X} = [\mathbf{x}_1 \cdots \mathbf{x}_L]$ then $supp(\mathbf{X}) = \bigcup_{t=1}^L supp(\mathbf{x}_t)$. Furthermore, the Frobenius norm of \mathbf{X} is denoted by $\|\mathbf{X}\|_F \triangleq \sqrt{\sum_{i,j} \mathbf{X}_{ij}^2}$.

2.2. Nonlinear measurement model with gross errors

In this paper, we consider a generic nonlinear sensing system, in which measurements are related to the system state by a nonlinear measurement function. Specifically, at a given measurement period t, the measurement vector $\mathbf{z}_t \in \mathbb{R}^m$ is related to the system state $\mathbf{x}_t \in \mathbb{R}^n$ as follows:

$$\mathbf{z}_t = h(\mathbf{x}_t) + \mathbf{e}_t + \mathbf{a}_t, \quad t = 1, \dots, L.$$
(1)

where $h(\cdot)$ is a nonlinear function of the system state. We assume that $h(\cdot)$ is differentiable and locally observable at every \mathbf{x}_t , *i.e.*, the Jacobian of $h(\cdot)$ has full rank. This condition is necessary to guarantee that the system state is uniquely identifiable based on noiseless measurements. The measurement noise vector is denoted as $\mathbf{e}_t \in \mathbb{R}^m$. We assume that the entries of \mathbf{e}_t have an independent and identically distributed (i.i.d.) Gaussian distribution with zero mean and variance σ^2 . Finally, $\mathbf{a}_t \in \mathbb{R}^m$ denotes the unknown deterministic gross error vector. The model described in (1) can also be written in the following matrix form.

$$\mathbf{Z} = \mathbf{h}(\mathbf{X}) + \mathbf{E} + \mathbf{A},\tag{2}$$

where

$$\mathbf{Z} \triangleq [\mathbf{z}_1 \dots \mathbf{z}_L], \quad \mathbf{X} \triangleq [\mathbf{x}_1 \dots \mathbf{x}_L], \quad \mathbf{E} \triangleq [\mathbf{e}_1 \dots \mathbf{e}_L], \\ \mathbf{A} \triangleq [\mathbf{a}_1 \dots \mathbf{a}_L], \quad \mathbf{h}(\mathbf{X}) \triangleq [h(\mathbf{x}_1) \dots h(\mathbf{x}_L)].$$

Our aim is to exploit the sparsity and temporal correlation property of gross errors for error correction. To this end, we impose the following conditions: there exists an unknown set of potential gross error locations $S \subset \{1, \ldots, m\}$ such that $|S| \ll m$ and $supp(\mathbf{a}_t) \subset S, t = 1, \ldots, L$. As $supp(\mathbf{A}) = \bigcup_{t=1}^{L} supp(\mathbf{a}_t)$, the aforementioned conditions imply that $supp(\mathbf{A}) \subset S$, and thus \mathbf{A} is a row-sparse matrix.

2.3. State estimation with multiple measurement vectors

In state estimation, we aim to estimate the states $\{\mathbf{x}_t\}_{t=1}^L$, based on the observed sensor measurements $\{\mathbf{z}_t\}_{t=1}^L$. When gross errors are present, we essentially need to estimate the pairs $\{(\mathbf{x}_t, \mathbf{a}_t)\}_{t=1}^L$ using the sensor measurements. Conventional robust estimation approaches estimate the pair $(\mathbf{x}_t, \mathbf{a}_t)$ by observing the sensor measurement \mathbf{z}_t only [2, 9, 15]. In contrast, in order to exploit the temporal correlation of gross error locations, we utilize multiple measurement vectors to simultaneously obtain state estimates for multiple periods. In other words, we aim at estimating the pair (\mathbf{X}, \mathbf{A}) based on \mathbf{Z} .

2.4. Gross error identifiability

To be able to obtain an accurate estimate of (\mathbf{X}, \mathbf{A}) , the underlying gross errors have to be fundamentally *identifiable* based on the sparsity assumption. The concept of identifiability is easier to understand from the following noiseless measurement model:

$$\mathbf{Z} = \mathbf{h}(\mathbf{X}) + \mathbf{A}.$$
 (3)

If the true gross error matrix is the unique sparsest row-sparse matrix that is consistent with \mathbf{Z} , then \mathbf{A} can be identified as the sparsest row-sparse matrix that is consistent with the measurements. Otherwise, \mathbf{A} cannot be uniquely identified from \mathbf{Z} based on the row-sparsity assumption. We can formally define identifiability as follows.

Definition 2.1 The pair (\mathbf{X}, \mathbf{A}) is said to be identifiable if it is the unique solution to the following equation with a row-sparsity constraint: "We solve for $(\mathbf{\tilde{X}}, \mathbf{\tilde{A}})$:

$$\mathbf{h}(\tilde{\mathbf{X}}) + \tilde{\mathbf{A}} = \mathbf{h}(\mathbf{X}) + \mathbf{A}, \quad |supp(\tilde{\mathbf{A}})| \le |supp(\mathbf{A})|^{\circ}.$$
 (4)

The gross error matrix \mathbf{A} is said to be identifiable if (\mathbf{X}, \mathbf{A}) is identifiable for all $\mathbf{X} \in \mathbb{R}^{n \times L}$.

In this paper, we consider robust state estimation and error correction in the presence of identifiable gross errors. Gross errors that are not identifiable according to Definition 2.1 cannot be corrected based on the sparsity assumption, and dealing with such cases is out of scope of this paper.

2.5. Observability condition for identifiability

If the pair $(\mathbf{x}_t, \mathbf{a}_t)$ is identifiable, then after removing the measurements that are corrupted by nonzero gross errors (*i.e.*, those indexed by $supp(\mathbf{a})$,) we should be able to estimate the states uniquely from the remaining measurements. In other words, the system should be *observable* even after removing the corrupted measurements. To illustrate this, we use the first order approximation of (1) at a nominal operating point \mathbf{x}_o :

$$\mathbf{z}_t = h(\mathbf{x}_t) + \mathbf{a}_t \approx h(\mathbf{x}_o) + \mathbf{H}_o \cdot (\mathbf{x}_t - \mathbf{x}_o) + \mathbf{a}_t.$$
 (5)

where \mathbf{H}_o denotes the Jacobian of $h(\cdot)$ evaluated at \mathbf{x}_o , which has full column rank.

Now, suppose that \mathbf{H}_o becomes rank-deficient if we remove the rows that are indexed by $supp(\mathbf{a}_t)$: *i.e.*, the system becomes unobservable after removal of the corrupted measurements. Then, it implies that there exists a nonzero $\mathbf{u} \in \mathbb{R}^n$ such that $supp(\mathbf{H}_o\mathbf{u}) \subset supp(\mathbf{a}_t)$ (*e.g.*, see Theorem 1 in [5].) Then, we have

$$h(\mathbf{x}_o) + \mathbf{H}_o \cdot (\mathbf{x}_t - \mathbf{x}_o) + \mathbf{a}_t = h(\mathbf{x}_o) + \mathbf{H}_o \cdot (\bar{\mathbf{x}}_t - \mathbf{x}_o) + \bar{\mathbf{a}}_t.$$
 (6)

where $\bar{\mathbf{x}}_t \triangleq \mathbf{x}_t + \mathbf{u}$, and $\bar{\mathbf{a}}_t \triangleq \mathbf{a}_t - \mathbf{H}_o \mathbf{u}$. Note that $supp(\bar{\mathbf{a}}_t)$ is included in $supp(\mathbf{a}_t)$. Therefore, $(\bar{\mathbf{x}}_t, \bar{\mathbf{a}}_t)$ is also consistent with \mathbf{z}_t with $|supp(\bar{\mathbf{a}}_t)| \leq |supp(\mathbf{a}_t)|$, and thus $(\mathbf{x}_t, \mathbf{a}_t)$ is not identifiable.

The above arguments imply that for identifiable gross errors, the Jacobian \mathbf{H}_o should remain full rank after its rows indexed by $supp(\mathbf{a}_t)$ are removed. When the measured components of a sensing system are sparsely connected or locally coupled, as in power systems [2], removing only few sensors from the system can cause the Jacobian to become rank-deficient. In such cases, we can exploit the observability property of identifiable gross errors to enhance accuracy of gross error localization. In the next section, we will present an observability-aware approach of gross error localization.

3. ROBUST STATE ESTIMATION ALGORITHM: NONLINEAR MEASUREMENT SYSTEM

In this section, we present our robust state estimator. In our approach, we first estimate the gross error matrix \mathbf{A} by promoting the row-sparsity of \mathbf{A} while restricting it to be consistent with the measurement model and the measurements \mathbf{Z} . Then, the gross error estimate is used to identify gross error locations by taking into account the observability condition described in Section 2.5. Finally, assuming that the estimate of the system state.

3.1. Estimation of row-sparse gross error matrix

For an identifiable gross error matrix \mathbf{A} , the true gross error matrix is the unique, row-sparsest gross error matrix that is consistent with the measurements, as stated in Definition 2.1. To promote the rowsparsity of gross error estimates while avoiding combinatorial complexity, we solve the following optimization to obtain our estimate of \mathbf{A} :

$$\min_{\tilde{\mathbf{X}}, \tilde{\mathbf{E}}, \tilde{\mathbf{A}}} \|\tilde{\mathbf{A}}\|_{1,2},$$
subject to $\tilde{\mathbf{A}} = \mathbf{Z} - \mathbf{h}(\tilde{\mathbf{X}}) - \tilde{\mathbf{E}};$

$$\|\tilde{\mathbf{E}}\|_{F} \le \epsilon.$$

$$(7)$$

where $\|\tilde{\mathbf{A}}\|_{1,2} \triangleq \sum_{i=1}^{m} \|\tilde{\mathbf{A}}_{i,:}\|_2$. For most nonlinear systems, the above problem is non-convex, and thus finding a global optimum is computationally intractable. However, if the system state is known to be near certain nominal operating point, which we denote by \mathbf{x}_o , then we can resort to finding a local optimum in a neighborhood of \mathbf{x}_o . To this end, we employ an iterative linear approximation method to find a local optimum. The concrete steps are as follows:

- 1. Initialization: Set $\tilde{\mathbf{x}}_t^0 = \mathbf{x}_o, t = 1, \dots, L$, and k = 1.
- 2. Iteration: let $\Delta \mathbf{z}_t^k = \mathbf{z}_t h(\tilde{\mathbf{x}}_t^{k-1})$ and solve the following convex optimization problem:

$$\min_{\substack{\Delta \tilde{\mathbf{x}}_{t}^{k}, \tilde{\mathbf{e}}_{t}^{k}, \tilde{\mathbf{a}}_{t}^{k} \\ \text{subj. to}} \quad \|\tilde{\mathbf{A}}^{k}\|_{1,2}, \\ \tilde{\mathbf{a}}_{t}^{k} = \Delta \mathbf{z}_{t}^{k} - \mathbf{H}_{t}^{k-1} \Delta \tilde{\mathbf{x}}_{t}^{k} - \tilde{\mathbf{e}}_{t}^{k}, \\ t = 1, \dots, L; \\ \|\tilde{\mathbf{E}}^{k}\|_{F} \leq \epsilon.$$
(8)

where \mathbf{H}_t^{k-1} is the Jacobian of $h(\cdot)$ evaluated at $\tilde{\mathbf{x}}_t^{k-1}$. The optimal solution $\Delta \tilde{\mathbf{x}}_t^k$ to (8) is then used to update the state estimation by

$$\tilde{\mathbf{x}}_t^k = \tilde{\mathbf{x}}_t^{k-1} + \Delta \tilde{\mathbf{x}}_t^k.$$
(9)

If ||Δx̃_t^k||₂ becomes less than a predetermined threshold ε or k reaches the maximum number of iterations, then return Ã^k. Otherwise, increment k by one and go to Step 2.

3.2. Observability-aware gross error localization and state estimation

The estimate $\tilde{\mathbf{A}}^*$, obtained as an outcome of the above iterative algorithm, tends to be biased due to the use of $\|\cdot\|_{1,2}$ norm. Further, the support of $\tilde{\mathbf{A}}^*$ might not satisfy the observability property of identifiable gross errors that we discussed in Section 2.5. Therefore, rather than making a direct use of $\tilde{\mathbf{A}}^*$, we use it to estimate the locations of gross errors and then compute the ML estimate of the system states. For gross error localization, we first compare each $\|\tilde{\mathbf{A}}^*_{i,:}\|_2$ to a threshold to determine a candidate set of gross error locations. Then, among the candidate locations, we chose the ones with largest $\|\tilde{\mathbf{A}}^*_{i,:}\|_2$ values while satisfying the observability condition. Once the gross error locations are estimated, we compute the ML estimates of the system states. The concrete steps are as follows:

1. Obtain a candidate set of gross error locations, denoted by 3:

$$\mathfrak{I} \triangleq \{i: \|\tilde{\mathbf{A}}_{i,:}^*\|_2 > \sigma \sqrt{L}\}.$$
(10)

- 2. *Observability-aware error localization*: compute K, the set of estimated error locations, as follows.
 - (a) Initialize $\mathcal{K} = \emptyset$.
 - (b) Find $i^* = \arg \max_{i \in \mathcal{I}} \|\tilde{\mathbf{A}}_{i,:}^*\|_2$.

 - (e) Remove *i* from J. If J becomes empty, go to Step 3. Otherwise, go to Step 2-(b).
- Remove the rows of z₁,..., z_L, and h(·) that are indexed by K. Based on the remaining system, compute the nonlinear least squares estimates of states:

$$\tilde{\mathbf{x}}_t = \arg\min_{\mathbf{x}_t} \|\mathbf{z}_t - h(\mathbf{x}_t)\|_2^2, \ t = 1, \dots, L.$$
(11)

4. EXPERIMENTAL RESULTS: POWER SYSTEM AC STATE ESTIMATION

We tested the proposed estimator for power system AC state estimation of the IEEE 14-bus network [17]. In power system AC state estimation, the system state is defined as the vector of bus voltage magnitudes and phase angles. There are various types of sensors in power systems, but for simplicity, we considered only two types of sensors: line flow and bus injection sensors. A line flow sensor measures a power flow from certain bus to its neighboring bus, and a bus injection sensor measures the amount of power injection at a certain bus. These sensor measurements are related to the state by the nonlinear measurement function that can be simply derived by using Kirchhoff's laws (see Chapter 2 of [2] for details.)

In our simulations, we assumed that every line in the 14-bus network has line flow sensors for both directions, and every bus has a bus injection sensor. This setting results in total 108 sensor measurements per measurement period and 27 state variables. At each Monte Carlo run, state vectors were sampled from the i.i.d. Gaussian distribution with the mean equal to the operating point given in the IEEE 14-bus data [17] and standard deviation $\sigma_x = 0.02$. The measurement noise vectors were sampled from the i.i.d. Gaussian distribution with zero mean and standard deviation $\sigma_e = 0.005$. For

generating gross errors, we first picked 12 random locations of gross errors. Specifically, the network was partitioned into four parts, and within each part, three locations were selected uniformly at random¹. Then, for t = 1, ..., L, the gross error locations were fixed to the selected locations. We considered two different types of gross errors: random Gaussian gross errors and state-dependent gross errors that reverse the sign of the affected measurements. For random Gaussian gross errors, gross error entries were sampled from the i.i.d. Gaussian distribution with zero mean and standard deviation $\sigma_{\mathbf{a}} = 0.1$ p.u. For the sign reversal case, we simply flipped the signs of the measurement entries that correspond to the selected gross error locations. In the initialization step of the gross error matrix estimation algorithm, we set \mathbf{x}_o to flat start, *i.e.*, we set all the voltage magnitudes to 1 p.u., and all the phase angles to 0. This is a reasonable assumption, as under normal operating conditions, the bus voltage magnitudes tend to be close to the nominal voltage, and the phase angle differences between buses tend to be very small (see [2]).

For performance evaluation, we considered two performance metrics. The first one is the normalized mean squared error (MSE):

$$\rho_{l_2} = \frac{\mathbb{E}[\frac{1}{L}\sum_{t=1}^{L}(\|\tilde{\mathbf{x}}_t - \mathbf{x}_t\|_2^2)]}{\mathbb{E}[\frac{1}{L}\sum_{t=1}^{L}(\|\tilde{\mathbf{x}}_t^{oracle} - \mathbf{x}_t\|_2^2)]}.$$
(12)

where $\tilde{\mathbf{x}}_t^{oracle}$ is the oracle estimate of \mathbf{x}_t , which is the ML estimate computed after all corrupt measurements are correctly removed. The second metric is the probability of detection, denoted by p_{det} , which is the probability of the event that all gross error locations are identified as error locations by our method. This probability quantifies how well our method can filter out corrupt measurements.

4.1. Performance of the proposed robust estimator

Fig. 1 shows the plots of the normalized MSE versus the number of measurement vectors used by our method.



Fig. 1. Normalized MSE for proposed estimator versus measurement period L. The results are based on 500 Monte Carlo runs.

By utilizing more number of measurement vectors, our method was able to decrease the normalized MSE very close to 1. For comparison, we also tested two benchmark robust estimators for nonlinear systems: the iterative $J(\hat{\mathbf{x}})$ test [2] and the l_1 norm minimization method [9]. In estimating \mathbf{x}_t , both methods utilize only the measurements at time t, *i.e.*, \mathbf{z}_t . Table 1 shows the performance of the benchmark techniques. Comparing with Fig. 1, one can see that our method outperformed the benchmarks. In particular, the l_1 norm minimization approach in [9] is equivalent to our approach without the observability-aware error localization step, if our method uses a single measurement vector (L = 1). Therefore, the results suggest that incorporating the observability-aware error localization step led to a significant improvement in robust estimation performance.

	Gaussian gross error	Sign reversal
$J(\hat{\mathbf{x}})$ [2]	3.6656	3.1752
l_1 min. [9]	4.5500	10.4502

Table 1. Normalized MSE of benchmark techniques.

The probability of detection also increased as L increased. For example, in the case of Gaussian gross errors, p_{det} increased from 0.8194 to 1 when L increased from 1 to 5. In the case of sign reversal errors, p_{det} increased from 0.9552 to 1 as L increased from 1 to 3. For both types of gross errors, the error localization accuracy improved as we utilized more measurement vectors.

5. CONCLUSION

We presented a new approach of sparse gross error correction for nonlinear sensing systems that exploits both the temporal correlation of gross error locations and the observability property of identifiable gross errors. The proposed estimator was applied for power system AC state estimation and outperformed benchmark techniques. While our experimental results are promising, theoretical analysis of the proposed framework is needed. In the future work, we will examine the convergence of the proposed iterative method of gross error estimation and derive performance guarantees for our robust estimator.

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¹This way of selecting error locations can increase the chance that the resulting gross errors are identifiable (see [5] for the rationale behind this.)

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