CROSS-CORRELATIONS OF ZERO CROSSINGS IN JOINTLY GAUSSIAN AND STATIONARY PROCESSES WITH ZERO MEANS

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ABSTRACT

Zero crossing data contain information of a process in compact form and is therefore of interest in wireless sensor networks where only reduced amounts of data can be transmitted. When analyzing the properties of certain algorithms using zero crossing data, the cross-covariance between the zero crossing rates of two jointly Gaussian and stationary processes is needed. The evaluation of such a cross-covariance is considered in the paper and an exact numerical expression as well as an asymptotic expression are presented.

***Index Terms*—** Gaussian processes, zero crossings, cross-correlations

1. INTRODUCTION

The theory of zero crossings is a mathematical tool used primarily in information theory, mathematical statistics and statistical signal processing for describing and analyzing the properties of stochastic processes. Some early results on zero crossings appear in the information theory literature [1, 2] and in work on radio transmission [3]. Zero crossings have continued to gain interest in information theory [4–6], but also in mathematical statistics [7–9] and statistical signal processing [10–13].

The number of zero crossings of a process contains information of the process properties in a compact form. Zero crossing data can therefore be of interest in information transmission with strong requirements on available bandwidth where only reduced amounts of data from a process can be sent. Furthermore, zero crossing data appear when using sensors that can only decide if a quantity is above or below a certain level. One example of the use of zero crossing data of current interest within statistical signal processing is wireless sensor networks where reduced amounts of information are sent from simple sensors to a node center for further processing [13].

When analyzing the statistical properties of Gaussian process parameter estimators using zero crossing data, expressions for the variance of the zero crossing rate of a process are needed. The results in [9] are useful in this respect, allowing for the exact numerical evaluation of the variance of the zero crossing rate and also giving its asymptotics. A way to compute the cross-covariance between the zero crossing rates of two different processes is also needed in the analysis. The topic of the current paper is therefore to evaluate such a cross-covariance based on the results in [9]. To the best of the authors' knowledge, such a cross-covariance has not been evaluated before. The relation to prior work is mainly through [9] as the current work is built upon the results presented there.

The rest of the paper is organized as follows. Some general setting and definitions are given in Section 2, together with the problem definition. The cross-covariance of zero crossing rates and the cross-covariances of zero crossings for two different processes are considered in Section 3. The cross-covariance of zero crossing rates depends on cross-covariances of zero crossings, and exact numerical evaluations of as well as asymptotical expressions for the cross-covariances of zero crossings are considered in Sections 3.1 and 3.2, respectively. A summary of the evaluation of the cross-covariance of zero crossing rates is given in algorithmic form in Section 4. Numerical illustrations are given in Section 5 and conclusions are drawn in Section 6.

2. SETTING

Here, $\mathbb{R}$ is used to denote real numbers, $\mathbb{Z}$ to denote integers, and $\mathbb{N}$ to denote natural numbers excluding zero. Let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space with real-valued stochastic processes $\boldsymbol{X} = (X_t)_{t\in\mathbb{Z}}$ and $\boldsymbol{Y} = (Y_t)_{t\in\mathbb{Z}}$. It is assumed that $\boldsymbol{X}$ and $\boldsymbol{Y}$ are jointly Gaussian and stationary, i.e., for all $m, n \in \mathbb{N}$ and $s_1, s_2, \ldots, s_m, t_1, t_2, \ldots t_n \in \mathbb{N}$,

- the random vector

$$(X_{s_1}, X_{s_2}, \ldots, X_{s_m}, Y_{t_1}, Y_{t_2}, \ldots, Y_{t_n})$$

has a multivariate Gaussian distribution,

- the random vector

$$(X_{s_1}, X_{s_2}, \ldots, X_{s_m}, Y_{t_1}, Y_{t_2}, \ldots, Y_{t_n})$$

has the same distribution as

$$(X_{s_1+k}, X_{s_2+k}, \ldots, X_{s_m+k}, Y_{t_1+k}, Y_{t_2+k}, \ldots, Y_{t_n+k})$$

for each $k \in \mathbb{N}$.

It is also assumed that $\boldsymbol{X}$ and $\boldsymbol{Y}$ are non-degenerate with zero means, i.e.,

- $\mathrm{Var}(X_s) > 0$ and $\mathrm{Var}(Y_t) > 0$ for all $s, t \in \mathbb{N}$,
- $\mathbb{E}[X_s] = 0$ and $\mathbb{E}[Y_t] = 0$ for all $s, t \in \mathbb{N}$.

Furthermore, ρ_k^x, ρ_k^y, ρ_k^{xy}, and ρ_k^{yx} are used to denote the correlation functions of $\boldsymbol{X}$, $\boldsymbol{Y}$, and the functions of cross-correlations among $\boldsymbol{X}$ and $\boldsymbol{Y}$, respectively, i.e.,

$$\begin{aligned}\rho_k^x &= \mathrm{Corr}(X_0, X_k) \text{ for all } k \in \mathbb{Z},\\ \rho_k^y &= \mathrm{Corr}(Y_0, Y_k) \text{ for all } k \in \mathbb{Z},\\ \rho_k^{xy} &= \mathrm{Corr}(X_0, Y_k) \text{ for all } k \in \mathbb{Z},\\ \rho_k^{yx} &= \mathrm{Corr}(Y_0, X_k) \text{ for all } k \in \mathbb{Z}.\end{aligned}$$

The indicators of zero crossings in $\boldsymbol{X}$ and $\boldsymbol{Y}$ are considered as

$$\begin{aligned}C_k^x &:= \mathbf{1}(X_{k-1} \cdot X_k < 0) \text{ for } k \in \mathbb{Z},\\ C_k^y &:= \mathbf{1}(Y_{k-1} \cdot Y_k < 0) \text{ for } k \in \mathbb{Z}.\end{aligned}$$

Moreover, the sample frequencies of zero crossings are introduced as

$$\begin{aligned}S_n^x &:= \frac{1}{n}\sum_{k=1}^{n} C_k^x \text{ for } n \in \mathbb{N},\\ S_n^y &:= \frac{1}{n}\sum_{k=1}^{n} C_k^y \text{ for } n \in \mathbb{N}.\end{aligned}$$

The covariance $\mathrm{Cov}(C_0^x, C_k^x)$ of the zero crossings C_0^x and C_k^x for a single stochastic process $\boldsymbol{X}$ is studied by Sinn and Keller [9], allowing for the exact numerical evaluation of and asymptotic expressions for the variance $\mathrm{Var}(S_n^x)$ of the zero crossing rate S_n^x. Here, the cross-covariance $\mathrm{Cov}(C_0^x, C_k^y)$ of zero crossings C_0^x and C_k^y and the cross-covariance $\mathrm{Cov}(C_0^y, C_k^x)$ of zero crossings C_0^y and C_k^x for two stochastic processes $\boldsymbol{X}$ and $\boldsymbol{Y}$ are studied, allowing for the exact numerical evaluation of as well as an asymptotic expression for the cross-covariance $\mathrm{Cov}(S_n^x, S_n^y)$ of the zero crossing rates S_n^x and S_n^y.

3. COVARIANCES

First, we note that

$$\begin{aligned}\mathrm{Cov}(S_n^x, S_n^y) &= \frac{1}{n^2}\sum_{i,j=1}^{n} \mathrm{Cov}(C_i^x, C_j^y)\\ &= \frac{1}{n^2}\Big(n \cdot \mathrm{Cov}(C_0^x, C_0^y)\\ &\quad + \sum_{k=1}^{n-1}(n-k)\mathrm{Cov}(C_0^x, C_k^y)\\ &\quad + \sum_{k=1}^{n-1}(n-k)\mathrm{Cov}(C_0^y, C_k^x)\Big).\end{aligned} \tag{1}$$

Moreover, for any $k \in \mathbb{N} \cup \{0\}$,

$$\begin{aligned}\mathrm{Cov}(C_0^x, C_k^y) &= \mathbb{E}[C_0^x \cdot C_k^y] - \mathbb{E}[C_0^x] \cdot \mathbb{E}[C_k^y]\\ &= \mathbb{P}(X_{-1} \cdot X_0 < 0, Y_{k-1} \cdot Y_k < 0)\\ &\quad - \mathbb{P}(X_{-1} \cdot X_0 < 0) \cdot \mathbb{P}(Y_{-1} \cdot Y_0 < 0)\\ &= 2\,\mathbb{P}(X_{-1} > 0, -X_0 > 0, Y_{k-1} > 0, -Y_k > 0)\\ &\quad + 2\,\mathbb{P}(X_{-1} > 0, -X_0 > 0, -Y_{k-1} > 0, Y_k > 0)\\ &\quad - 4\,\mathbb{P}(X_{-1} > 0, -X_0 > 0)\mathbb{P}(Y_{-1} > 0, -Y_0 > 0)\end{aligned}$$

and analogously for $\mathrm{Cov}(C_0^y, C_k^x)$. Next, exact numerical evaluations of $\mathrm{Cov}(C_0^x, C_k^y)$ and $\mathrm{Cov}(C_0^y, C_k^x)$ are considered in Section 3.1 and asymptotical expressions are given in Section 3.2.

3.1. Exact numerical evaluations

Using the notation introduced in [9, Section 2],

$$\mathrm{Cov}(C_0^x, C_k^y) = \Psi(\boldsymbol{r}), \tag{2}$$

with

$$\boldsymbol{r} = (r_1, r_2, r_3, r_4, r_5, r_6)$$

and $r_1 = \rho_1^x$, $r_2 = r_5 = \rho_k^{xy}$, $r_3 = \rho_{k+1}^{xy}$, $r_4 = \rho_{k-1}^{xy}$, and $r_6 = \rho_1^y$. In general, no closed-form expressions exist for $\Psi(\boldsymbol{r})$, but using the results in [9, Section 3], $\Psi(\boldsymbol{r})$ can be efficiently evaluated numerically. In [9, Section 3], it is stated that

$$\Psi(\boldsymbol{r}) = \sum_{i=2}^{5} r_i \int_0^1 \psi_i(\mathbf{I}_H \boldsymbol{r})\, \mathrm{d}H, \tag{3}$$

where

$$\begin{aligned}\mathbf{I}_H &= \mathrm{diag}(1, H, H, H, H, 1),\\ \mathbf{I}_H \boldsymbol{r} &= (r_1, Hr_2, Hr_3, Hr_4, Hr_5, r_6),\\ \boldsymbol{r} &= (r_1, r_2, r_3, r_4, r_5, r_6),\end{aligned}$$

and where

$$\psi_2(\boldsymbol{r}) = \frac{1}{\pi^2\sqrt{1-r_2^2}} \arcsin\left(\frac{\sigma_{24}(\boldsymbol{r})}{\sqrt{\sigma_{22}(\boldsymbol{r})\sigma_{44}(\boldsymbol{r})}}\right),$$
$$\psi_3(\boldsymbol{r}) = \frac{1}{\pi^2\sqrt{1-r_3^2}} \arcsin\left(\frac{\sigma_{23}(\boldsymbol{r})}{\sqrt{\sigma_{22}(\boldsymbol{r})\sigma_{33}(\boldsymbol{r})}}\right),$$
$$\psi_4(\boldsymbol{r}) = \frac{1}{\pi^2\sqrt{1-r_4^2}} \arcsin\left(\frac{\sigma_{14}(\boldsymbol{r})}{\sqrt{\sigma_{11}(\boldsymbol{r})\sigma_{44}(\boldsymbol{r})}}\right),$$
$$\psi_5(\boldsymbol{r}) = \frac{1}{\pi^2\sqrt{1-r_5^2}} \arcsin\left(\frac{\sigma_{13}(\boldsymbol{r})}{\sqrt{\sigma_{11}(\boldsymbol{r})\sigma_{33}(\boldsymbol{r})}}\right),$$

with

$$\begin{aligned}
\sigma_{11}(\boldsymbol{r}) &= 1 - r_4^2 - r_5^2 - r_6^2 + 2r_4r_5r_6,\\
\sigma_{22}(\boldsymbol{r}) &= 1 - r_2^2 - r_3^2 - r_6^2 + 2r_2r_3r_6,\\
\sigma_{33}(\boldsymbol{r}) &= 1 - r_1^2 - r_3^2 - r_5^2 + 2r_1r_3r_5,\\
\sigma_{44}(\boldsymbol{r}) &= 1 - r_1^2 - r_2^2 - r_4^2 + 2r_1r_2r_4,\\
\sigma_{13}(\boldsymbol{r}) &= r_2 - r_1r_4 + r_3r_4r_5 - r_2r_5^2 - r_3r_6 + r_1r_5r_6,\\
\sigma_{14}(\boldsymbol{r}) &= r_3 - r_1r_5 + r_2r_4r_5 - r_3r_4^2 - r_2r_6 + r_1r_4r_6,\\
\sigma_{23}(\boldsymbol{r}) &= r_4 - r_1r_2 + r_2r_3r_5 - r_4r_3^2 - r_5r_6 + r_1r_3r_6,\\
\sigma_{24}(\boldsymbol{r}) &= r_5 - r_1r_3 + r_2r_3r_4 - r_5r_2^2 - r_4r_6 + r_1r_2r_6.
\end{aligned}$$

In summary, the covariance in (2) can be evaluated numerically using (3).

Moreover,

$$\mathrm{Cov}(C_0^y, C_k^x) = \Psi(\bar{\boldsymbol{r}}), \tag{4}$$

with

$$\bar{\boldsymbol{r}} = (\bar{r}_1, \bar{r}_2, \bar{r}_3, \bar{r}_4, \bar{r}_5, \bar{r}_6)$$

and $\bar{r}_1 = \rho_1^y$, $\bar{r}_2 = \bar{r}_5 = \rho_k^{yx}$, $\bar{r}_3 = \rho_{k+1}^{yx}$, $\bar{r}_4 = \rho_{k-1}^{yx}$, and $\bar{r}_6 = \rho_1^x$. Here, $\Psi(\bar{\boldsymbol{r}})$ can be efficiently evaluated numerically using, yet again, (3).

3.2. Asymptotical expressions

Using the results in [9, Theorem 4.1], an asymptotically equivalent expression is obtained. For example, in the case $\rho_k^{xy} \sim \rho_{k+1}^{xy} \sim \rho_{k-1}^{xy} \sim f(k)$, where $\sim$ denotes asymptotic equivalence and where $f(k)$ is a function $\mathbb{N} \to \mathbb{R}$ with $f(k) \to 0$ as $k \to \infty$, it is obtained that

$$\begin{aligned}\mathrm{Cov}(C_0^x, C_k^y) \sim{}& \frac{2(1-\rho_1^x-\rho_1^y+\rho_1^x\rho_1^y)}{\pi^2\sqrt{(1-(\rho_1^x)^2)(1-(\rho_1^y)^2)}}\big(f(k)\big)^2\\ &+ \mathcal{O}\Big(\big(f(k)\big)^4\Big).\end{aligned} \tag{5}$$

Furthermore, in the case $\rho_k^{yx} \sim \rho_{k+1}^{yx} \sim \rho_{k-1}^{yx} \sim g(k)$, the covariance $\mathrm{Cov}(C_0^y, C_k^x)$ is computed as

$$\begin{aligned}\mathrm{Cov}(C_0^y, C_k^x) \sim{}& \frac{2(1-\rho_1^x-\rho_1^y+\rho_1^x\rho_1^y)}{\pi^2\sqrt{(1-(\rho_1^x)^2)(1-(\rho_1^y)^2)}}\big(g(k)\big)^2\\ &+ \mathcal{O}\Big(\big(g(k)\big)^4\Big).\end{aligned} \tag{6}$$

4. ALGORITHMIC SUMMARY

The evaluation of $\mathrm{Cov}(S_n^x, S_n^y)$ is summarized next in algorithmic form.

1. Consider expression (1) for $\mathrm{Cov}(S_n^x, S_n^y)$.
2. Evaluate $\mathrm{Cov}(C_0^x, C_0^y)$ that appears in (1) using the exact numerical expression (2) or the asymptotical expression (5).
3. For $k = 1, \ldots, n-1$, evaluate $\mathrm{Cov}(C_0^x, C_k^y)$ that appear in (1) using the exact numerical expression (2) or the asymptotical expression (5).
4. For $k = 1, \ldots, n-1$, evaluate $\mathrm{Cov}(C_0^y, C_k^x)$ that appear in (1) using the exact numerical expression (4) or the asymptotical expression (6).
5. Use the results from steps 2–4 in expression (1) for $\mathrm{Cov}(S_n^x, S_n^y)$.

It is suggested to compute the covariances in steps 2–4 in both ways for small values of k and to use the results from the asymptotical expressions only after that the relative error between the results is less than a certain acceptable limit.

5. NUMERICAL ILLUSTRATIONS

A diffusion process, whose properties are described in Section 5.1, is considered in some numerical examples, where illustrations of the theoretical results in Section 3 are given in Section 5.2 and where empirical results from a Monte Carlo study are given in Section 5.3. Furthermore, comparisons between theoretical and empirical results are made.

5.1. Processes

Consider the diffusion process

$$\mathrm{d}X(T) = -a_0X(T)\mathrm{d}T + \mathrm{d}W(T)$$

as an example of a Gaussian process, where $\mathrm{d}W(T)$ is the increment of a Wiener process $W(T)$ with incremental variance σ^2. Consider the sampled process $X_i := X(ih)$, where h is the sampling interval, which is a first order discrete-time autoregressive process. The correlation function ρ_k^x of the sampled process X_i is given as $\rho_k^x = \mathrm{e}^{-a_0|k|h}$. Also consider the differenced process $Y_i := X_i - X_{i-1}$, whose correlation function ρ_k^y is given as

$$\rho_k^y = \frac{2\rho_k^x - \rho_{k+1}^x - \rho_{k-1}^x}{2(\rho_0^x - \rho_1^x)}.$$

The cross-correlation functions $\rho_k^{xy} = \mathrm{Corr}(X_0, Y_k)$ and $\rho_k^{yx} = \mathrm{Corr}(Y_0, X_k)$ are expressed as

$$\rho_k^{xy} = \frac{\rho_k^x - \rho_{k-1}^x}{\sqrt{2(1-\rho_1^x)}} \text{ and } \rho_k^{yx} = \frac{\rho_k^x - \rho_{k+1}^x}{\sqrt{2(1-\rho_1^x)}},$$

respectively, and it holds that $\rho_k^{xy} = \rho_{-k}^{yx}$.

5.2. Theoretical results

The covariances $\mathrm{Cov}(C_0^x, C_k^y)$ and $\mathrm{Cov}(C_0^y, C_k^x)$ are computed for the case $a_0 = 2$, $\sigma^2 = 1$, and $h = 0.1$. The covariances are computed in two different ways, first using the exact numerical expressions (2) and (4), and then using the asymptotical expressions (5) and (6). In the asymptotical expressions, $f(k) = \rho_k^{xy}$ in (5) and $g(k) = \rho_k^{yx}$ in (6). The results are illustrated in Figures 1 and 2 for $k = 0, \ldots, 10$, where it is seen how well the asymptotical expression approximates the exact numerical expression.

The covariance $\mathrm{Cov}(S_{n-2}^x, S_{n-2}^y)$ between the zero crossing rates S_{n-2}^x and S_{n-2}^y in (1) is computed for $n = 10000$, first using covariances $\mathrm{Cov}(C_0^x, C_k^y)$ and $\mathrm{Cov}(C_0^y, C_k^x)$ computed with the exact numerical expressions (2) and (4), and then using covariances computed with the asymptotical expressions (5) and (6). The results are presented in Table 1, where it is seen how much the results based on the two different computations differ.

5.3. Empirical results

Data X_i and Y_i are generated for the case $a_0 = 2$, $\sigma^2 = 1$, $h = 0.1$, and $n = 10000$. The zero crossings C_k^x and C_k^y as well as the zero crossing rates S_{n-1}^x and S_{n-2}^y are registered. In order to get empirical results for $\mathrm{Cov}(C_0^x, C_k^y)$ and $\mathrm{Cov}(C_0^y, C_k^x)$ as well as for $\mathrm{Cov}(S_{n-2}^x, S_{n-2}^y)$, the data generation and the registration of zero crossing rates are repeated 10^6 times in a Monte Carlo study.

The results for $\mathrm{Cov}(C_0^x, C_k^y)$ and $\mathrm{Cov}(C_0^y, C_k^x)$ are illustrated in Figures 1 and 2, respectively, for $k = 0, \ldots, 10$. It is seen how well the empirical covariances are described by the exact numerical expressions. The curves are indeed indistinguishable.

In Table 1, the result for $\mathrm{Cov}(S_{n-2}^x, S_{n-2}^y)$ is presented. It is clear that the empirical covariance between the zero crossing rates is described by the theoretical expression based on covariances computed with the exact numerical expression.

6. CONCLUSIONS

An exact numerical expression as well as an asymptotic expression for the cross-covariance between the zero crossing rates of two jointly Gaussian and stationary processes have been presented. The evaluation of the cross-covariance has been summarized in algorithmic form and the expressions have been illustrated numerically. The expressions are useful when analyzing the statistical properties of Gaussian process parameter estimators using zero crossing data.

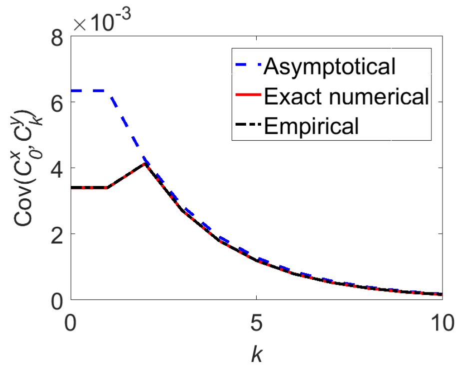

Fig. 1. Asymptotical, exact numerical, and empirical covariances $\mathrm{Cov}(C_0^x, C_k^y)$, $k = 0, \ldots, 10$.

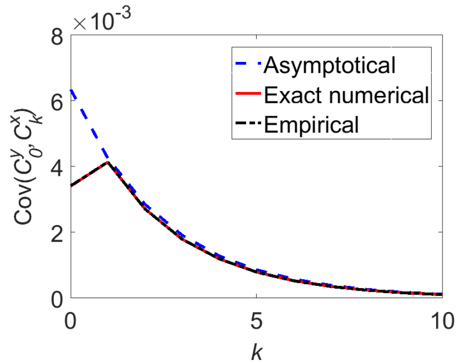

Fig. 2. Asymptotical, exact numerical, and empirical covariances $\mathrm{Cov}(C_0^y, C_k^x)$, $k = 0, \ldots, 10$.

Table 1. Theoretical and empirical covariances $\mathrm{Cov}(S_{n-2}^x, S_{n-2}^y)$ between zero crossing rates S_{n-2}^x and S_{n-2}^y. Theoretical results are obtained by using the exact numerical (e.n.) expressions in (2) and (4) as well as the asymptotical (a.) expressions in (5) and (6).

$\mathrm{Cov}(S_{n-2}^x, S_{n-2}^y)$	
Theoretical	Empirical
$3.13 \cdot 10^{-6}$ (e.n.) $3.85 \cdot 10^{-6}$ (a.)	$3.12 \cdot 10^{-6}$

7. ACKNOWLEDGMENT

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