

SINGLE-CHANNEL WIENER FILTERING OF DETERMINISTIC SIGNALS IN STOCHASTIC NOISE USING THE PANORAMA

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ABSTRACT

The Wiener filter is a well-known signal processing method for improving a noisy signal's quality. The Wiener filter requires either knowledge of or estimates of the power spectra of the signal-of-interest and of the undesired noise, leading to implementation challenges. In this paper, we show how a recently-developed second-order signal quantity termed the *panorama* can be employed to compute the Wiener filter for deterministic signals – containing nearly-constant frequency and phase components – in additive stochastic noise. We first show how the Wiener filter transfer function is related to the absolute value of the panorama and the power spectrum of the noisy measured signal. We then provide a practical procedure for estimating the absolute value of the panorama across frequency for single-channel sampled recordings. Numerical examples show the ability of the proposed procedure for estimating the panorama and for computing the Wiener filter for noisy deterministic signals.

Index Terms— autocorrelation, convolution, frequency estimation, spectral analysis, Wiener filtering.

1. INTRODUCTION

Noise reduction is at the heart of many applications involving recorded sensor signals. In such applications, perhaps the most well-known approach for reducing noise is the Wiener filter, which tries to maximize the signal-to-noise ratio (SNR) at the output of the filter. Consider the continuous-time signal

$$x(t) = s(t) + \eta(t), \quad (1)$$

where $s(t)$ is an unknown signal of interest and $\eta(t)$ is an additive noise signal. Then, a classic form of the Wiener filter is defined in the frequency domain as

$$H(\omega) = \frac{R_{ss}(\omega)}{R_{ss}(\omega) + R_{\eta\eta}(\omega)} = \frac{R_{ss}(\omega)}{R_{xx}(\omega)}, \quad (2)$$

where $R_{ss}(\omega)$ and $R_{\eta\eta}(\omega)$ are the power spectra of the signal of interest and the additive noise signal, respectively, and $R_{xx}(\omega) = R_{ss}(\omega) + R_{\eta\eta}(\omega)$. Implementation of the linear filter specified by (2) can employ either time-domain or

frequency-domain processing to the measured signal $x(t)$ and is usually performed using discrete-time processing. Such a solution assumes that both $s(t)$ and $\eta(t)$ have well-defined power spectra over some time period of interest.

The primary challenge in the implementation of the Wiener filter is the estimation of two distinct power spectra from a single signal $x(t)$. Various methods can be employed to extract information about either $s(t)$ or $\eta(t)$ within $x(t)$, including activity detection for intermittent signals, sparsity across some signal basis, statistical relationships between signal components in time or frequency, and the like [1]–[8]. The key issue is tailoring knowledge about signal properties to the estimation of the signal spectra.

Recently, two novel mathematical tools for relating the second-order statistical properties of mixtures of deterministic and stochastic finite-power signals have been introduced [9]. The *autoconvolution* is a time-domain quantity computed from $x(t)$ that maintains the phase information of the deterministic components within $x(t)$, while the *panorama* is the Fourier transform of the autoconvolution. Using the panorama, it has been shown that deterministic sinusoidal components can be detected in correlated stochastic noise without knowledge of the sinusoidal frequencies or amplitudes, even when the stochastic noise has peaks in its power spectrum. The main challenge so far in using the autoconvolution and panorama in practical scenarios is stated in [9]: Simple time averages cannot be used in place of ensemble averages to compute the autoconvolution and/or panorama. Thus, one of the most-common estimation tools cannot be easily applied. In addition, a rigorous connection between the panorama and the Wiener filter in (2) is an open issue.

In this paper, we describe a strategy for constructing a Wiener filter for enhancing a deterministic signal $s(t)$ observed through a signal $x(t)$ containing additive stochastic noise $\eta(t)$ using concepts originally developed for the panorama. The method relies only on the nearly-constant nature of the amplitudes and phases of the frequency components of the signal-of-interest, and no other information about the deterministic signal is used. To this end, we first relate the power spectrum and panorama of $x(t)$ to (2), indicating

that the phase information contained within the panorama is not required. We then describe a technique for estimating the absolute value of the panorama of a single-channel signal in the discrete-time domain. Block processing is used, in which time-domain averages of processed versions of the discrete Fourier transform are employed to estimate $R_{ss}(\omega)$, after which the Wiener filter solution is constructed. Applications of the technique on both synthetic data and on noisy speech signals indicate the usefulness of the approach.

2. WIENER FILTER AND THE PANORAMA

As first described in [9], the autoconvolution $p_{xx}(t)$ of a time-domain signal $x(t)$ containing both deterministic and stochastic components is defined as

$$p_{xx}(t) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T E\{x(\tau)x(t-\tau)\}d\tau, \quad (3)$$

where $E\{\cdot\}$ denotes statistical expectation. The autoconvolution is different from the autocorrelation $r_{xx}(t)$, defined as

$$r_{xx}(t) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T E\{x(\tau)x(t+\tau)\}d\tau, \quad (4)$$

due to the appearance of the minus sign on the integral argument τ in (3). The panorama $P_{xx}(\omega)$ as a function of frequency ω is defined in [9] as the Fourier transform of $p_{xx}(t)$:

$$P_{xx}(\omega) = \int_{-\infty}^{\infty} p_{xx}(t)e^{-j\omega t}dt. \quad (5)$$

The relationship between the panorama of $x(t)$ and the power spectrum of $s(t)$, denoted as

$$R_{ss}(\omega) = \int_{-\infty}^{\infty} r_{ss}(t)e^{-j\omega t}dt, \quad (6)$$

where $r_{ss}(t)$ is the autocorrelation of the deterministic signal $s(t)$, is key to understanding the utility of the panorama for enhancing deterministic signals in stochastic noise. First, consider a signal $s(t)$ of the form

$$s(t) = \sum_{n=1}^N A_n \cos(\omega_n t + \phi_n). \quad (7)$$

It is easy to show via Fourier transforms of Eqs. (5) and (6) of [9] that the power spectrum and panorama of $s(t)$ are

$$R_{ss}(\omega) = \sum_{n=1}^N \frac{A_n^2}{4} [\delta(\omega - \omega_n) + \delta(\omega + \omega_n)] \quad (8)$$

$$P_{ss}(\omega) = \sum_{n=1}^N \frac{A_n^2}{4} [\delta(\omega - \omega_n)e^{j2\phi_n} + \delta(\omega + \omega_n)e^{-j2\phi_n}]. \quad (9)$$

Thus, we see for deterministic signals that

$$R_{ss}(\omega) = |P_{ss}(\omega)|. \quad (10)$$

The panorama of a deterministic signal can be used to construct its power spectrum using the absolute value operator.

Now, consider the general case in which $x(t) = s(t) + \eta(t)$ contains both deterministic and stochastic components. Then, due to the fact that the autoconvolution of a wide-sense stationary stochastic signal is zero [9], we have the result that

$$P_{ss}(\omega) = P_{xx}(\omega). \quad (11)$$

Combining (11) with (10), we obtain

$$R_{ss}(\omega) = |P_{xx}(\omega)| \quad (12)$$

when $x(t)$ contains stochastic noise.

The usefulness of the result in (12) for constructing the Wiener filter solution in (2) is readily seen. If the absolute value $|P_{xx}(\omega)|$ of the panorama of $x(t)$ can be estimated, we can express (2) as

$$H(\omega) = \frac{|P_{xx}(\omega)|}{R_{xx}(\omega)}, \quad (13)$$

which depends only on quantities related to the measured signal $x(t)$. This result is important, as joint amplitude-and-phase estimation of a signal's frequency components is in practice a more challenging task than amplitude estimation alone. Thus, estimation of $|P_{xx}(\omega)|$ should be easier than estimation of $P_{xx}(\omega)$.

The main difficulty in using the filter solution in (13) is the challenge in estimating the absolute value of the panorama of a signal $x(t)$ from a single recording as opposed to ensemble averages from multiple recordings, as indicated in [9]. In the next section, we describe a strategy for this estimation task.

3. PRACTICAL ESTIMATION PROCEDURE

Our proposed strategy for estimating $|P_{xx}(\omega_i)|$ is a novel modification of the well-known spectral estimation technique of periodogram averaging [10]. To introduce the modification, consider first the problem of estimating the ordinary power spectrum $R_{xx}(\omega_i)$ at frequency ω_i from fast Fourier transform (FFT) bin values $X_k(\omega_i)$, computed across different block indices k . For both deterministic signals and for wide-sense stationary signals, a viable strategy is simply to use time-averaging across blocks, such that

$$\hat{R}_{xx}(\omega_i) = \sum_k \alpha_k |X_k(\omega_i)|^2, \quad (14)$$

where $\{\alpha_k\}$ are real-valued non-negative weights that sum to unity across some range of k values. Depending on how the weights are chosen, this method could be used for signals whose characteristics change with or are constant over time.

While it might seem reasonable to use a similar technique for estimating the panorama $P_{xx}(\omega_i)$ by weighted-time averages of $[X(\omega_i)]^2$, the difficulty with such a method is the lack of time synchronization between the blocks used to form the

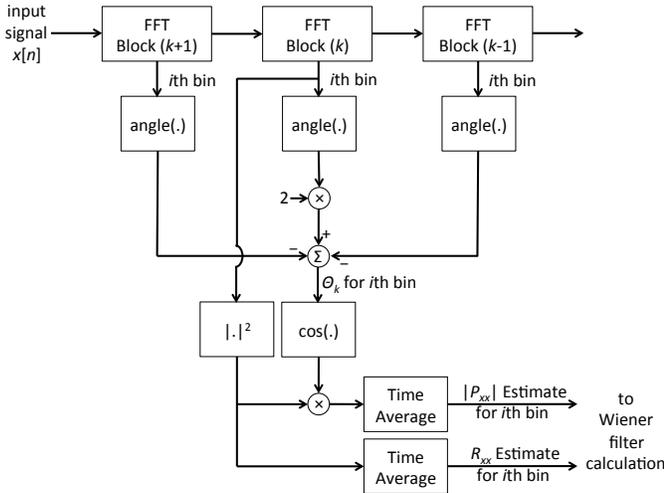


Fig. 1. Block diagram of a general estimation procedure for $|P_{xx}(\omega_i)|$ and $R_{xx}(\omega_i)$.

averages. For estimating the Wiener filter function, however, only an estimate of the absolute value $|P_{xx}(\omega_i)|$ is needed. Moreover, for deterministic signals, the relations in (11)–(12) tell us that appropriate weighted time averages of $|X_k(\omega_i)|^2$ can be used if we can account for the varying bin-value phases across the data blocks. We can leverage the constant time extent of the data blocks to account for this phase variation.

The proposed joint strategy for estimating both $R_{xx}(\omega_i)$ and $|P_{xx}(\omega_i)|$ is shown in Figure 1. In this diagram, the FFTs of three successive data blocks at times $(k+1)$, k , and $(k-1)$, respectively, are used, thus introducing a single block time delay. The phases $\angle X_k(\omega_i)$ of the i th bin values from each of these blocks are used to compute a compensation angle for the i th bin value at block k as

$$\theta_k(\omega_i) = 2\angle X_k(\omega_i) - \angle X_{k+1}(\omega_i) - \angle X_{k-1}(\omega_i) \quad (15)$$

Then, the estimate of the absolute value of the panorama for the i th bin is computed as

$$|\hat{P}_{xx}(\omega_i)| = \left| \sum_k \alpha_k |X_k(\omega_i)|^2 \cos(\theta_k(\omega_i)) \right|, \quad (16)$$

where the same weighting sequence $\{\alpha_k\}$ for computing $\hat{R}_{xx}(\omega_i)$ in (14) is used. For any one frequency bin dominated by deterministic components at or near its center frequency ω_i , it is easily seen that $\theta_k(\omega_i)$ will be small in amplitude, and thus $\cos(\theta_k(\omega_i)) \approx 1$ in such frequency bins. For frequency bins dominated by stochastic components, the average value of $\cos(\theta_k(\omega_i))$ will vary such that the block time averaging used in (16) will lead to small amplitudes for the panorama estimates in such frequency bins. The use of the factor of 2 and the cos function itself is motivated by the facts that (a) $P_{xx}(\omega_i)$ is related to the square of $X(\omega_i)$ for deterministic

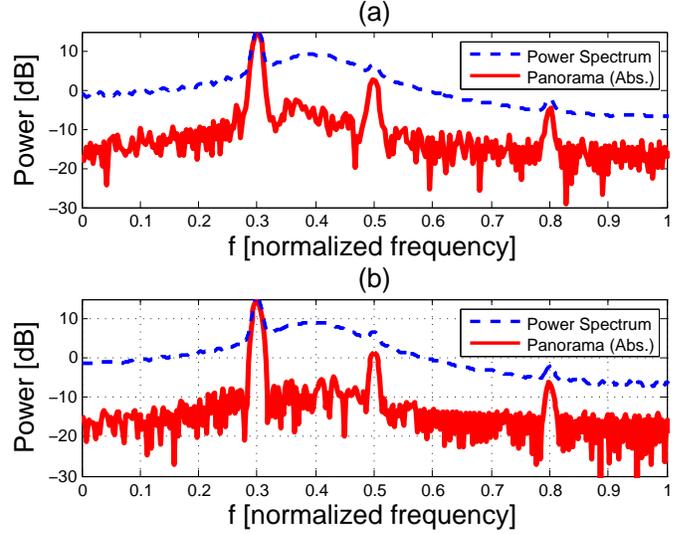


Fig. 2. Numerical estimates of $R_{xx}(\omega_k)$ and $|P_{xx}(\omega)|$ using (a) the ensemble-averaging technique described in [9] over 100 different realizations and (b) the time-averaging technique described in this paper over a single realization with the same number of total data points.

signals and (b) $e^{2j\delta\theta} = \cos(2\delta\theta) + j \sin(2\delta\theta) \approx \cos(2\delta\theta)$ for small $\delta\theta$ when the imaginary part is negligible.

Remark #1: The approach used to compute the compensation angle $\theta_k(\omega_i)$ employs successive blocks that ideally are statistically-independent of each other, so that both $\angle X_{k+1}(\omega_i)$ and $\angle X_{k-1}(\omega_i)$ within $\theta_k(\omega_i)$ are uncorrelated with $|X_k(\omega_i)|^2$ in (16). While such a condition cannot be guaranteed, it is desirable to choose *non-overlapping* blocks to minimize any effects of such signal correlation. The effects of this signal correlation are currently under investigation.

Remark #2: The joint strategy described above does not address the tradeoff between frequency resolution of the FFT and the statistical variability of the estimates across frequency bins. It is useful to employ some form of frequency smoothing or, equivalently, an inverse FFT/windowing/FFT processing sequence to the power spectrum and panorama estimates to address these issues. Numerical examples in the next section employ the latter processing steps.

4. NUMERICAL EVALUATIONS

The ability of the proposed methods to estimate the panorama and the Wiener filter is illustrated via numerical evaluations.

We first explore the ability of the approach illustrated in the last section to accurately estimate $|P_{xx}(\omega_i)|$ via simulations. The example chosen in this first case is similar to that described in [9], in which

$$x(n) = \cos(2\pi 0.15n - \pi/6) + 0.25 \cos(2\pi 0.25n + \pi/3) + 0.1 \cos(2\pi 0.4n + \pi/8) + \eta(n), \quad (17)$$

where $\eta(n)$ is a zero-mean Gaussian random process gener-

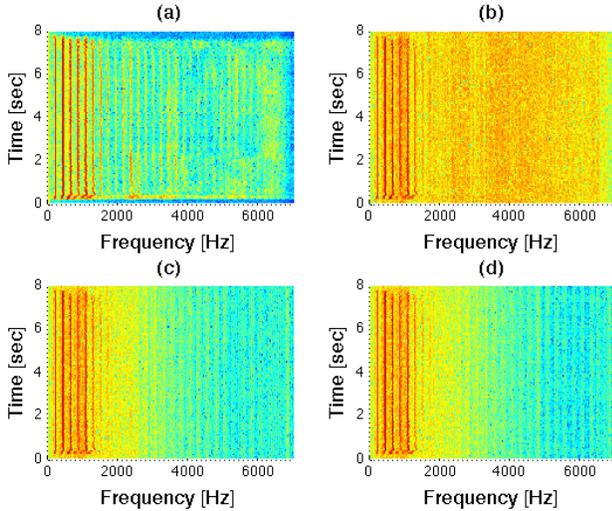


Fig. 3. Spectrograms of (a) original noiseless speech signal, (b) noisy speech signal, (c) Wiener filtered output signal estimated using both signals, and (d) Wiener filtered output estimated using only the noisy speech signal via the panorama.

ated by filtering a zero-mean uncorrelated Gaussian random process with a digital filter whose system function is

$$H(z) = \frac{1}{1 - 1.6 \cos(2\pi 0.2)z^{-1} + 0.64z^{-2}} \quad (18)$$

We generated two data sets of this signal: (a) 100 different 1000-sample realizations in which the phase of the deterministic signal in $x(n)$ is identical across each realization and (b) a single 100000-sample realization. The approach for estimating the power spectrum and panorama employing ensemble averages of the autocorrelation and the autoconvolution as described in [9] followed by FFT with a window size of $N = 256$ and a Hamming window was used on the first data set. Figure 2(a) shows the power spectrum and the absolute value of the panorama produced by this technique. We then employed the method illustrated in Figure 1 to estimate the power spectrum and panorama from the second data set, with identical window sizes and windowing choices. Figure 2(b) shows the power spectrum and the absolute value of the panorama produced by the proposed method with constant α_k values. Comparing (a) with (b), we find that the two $|P_{xx}(\omega_i)|$ estimates are quite similar to each other across all frequencies, both in overall level and in frequency resolution. This result illustrates that we have achieved the desired goal of the accurate estimation of (the absolute value of) the panorama using time-averaging, such that the ensemble averaging limitation stated in [9] has been removed.

We now illustrate the ability of the method illustrated in Figure 1 to estimate the Wiener filter function for a semi-deterministic signal in stochastic noise. Figure 3(a) shows the spectrogram of an eight-second recording of the first author singing an “ah” vowel sound with a fundamental frequency of about 220 Hz, sampled at 16kHz. Figure 3(b) shows the

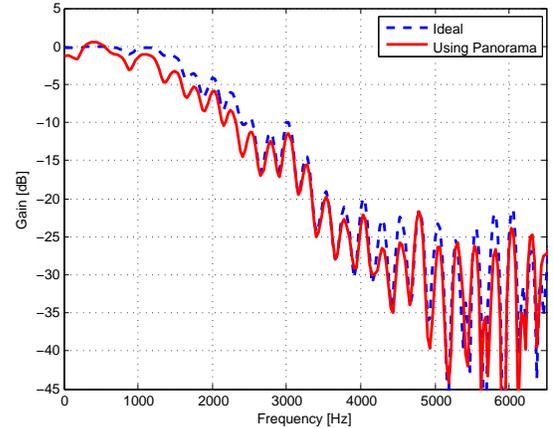


Fig. 4. Wiener filter functions for the ideal and panorama-based estimates in the speech enhancement example.

spectrogram of a noisy version of this eight-second recording, in which the filter in (18) has been used to generate random Gaussian noise that is played out of a loudspeaker in the same room, and this signal is then added to the first signal, simulating the effect of simultaneous recording. The average SNR of the noisy signal was 10 dB. The signals in Figures 3(a) and (b) are used to estimate $R_{ss}(\omega_i)$ and $R_{xx}(\omega_i)$ for a standard Wiener filter and are then applied to the signal in Figure 3(b), resulting in the “ground truth” spectrogram in Figure 3(c), in which $N = 256$ -sample data blocks and Bartlett windowing are used. Finally, the signal in Figure 3(b) alone is used to estimate both $|P_{xx}(\omega_i)|$ and $R_{xx}(\omega_i)$ using the method described in this paper, from which the Wiener filter is estimated and applied to the signal. Figure 3(d) shows the corresponding spectrogram. Comparing Figures 3(c) and (d), we see that they are quite similar, indicating that the proposed method is capable of estimating a Wiener filter function in this situation. The proposed Wiener filtering method achieved an average SNR of 15.2 dB, which is close to the 15.6 dB achieved by the optimal Wiener filter of identical structure. The two estimated Wiener filter functions are shown in Figure 4 and are found to be extremely similar. Both filters capture the general characteristics of the harmonics of the quasi-periodic signal and have nearly-identical frequency rolloffs. This result shows that our proposed Wiener filtering technique employing the panorama can be estimated from a single-channel recording without activity detection, noise identification, or training data.

5. CONCLUSIONS

In this paper, we have described a procedure for estimating a Wiener filter function for an unknown deterministic signal in unknown additive stochastic noise. The recently-developed second-order quantity of a signal, called the panorama, is employed to construct the Wiener filter function from a single realization of the signal. A practical estimation procedure is given, and numerical evaluations shows the usefulness of the technique for both spectral estimation and noise suppression.

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