JOINT CHANNEL AND CARRIER FREQUENCY ESTIMATION FOR M-ARY CPM OVER FREQUENCY-SELECTIVE CHANNEL USING PAM DECOMPOSITION

Romain Chayot[⋆†], *Marie-Laure Boucheret*[†], *Charly Poulliat*[†], *Nathalie Thomas*[†], *Nicolas Van Wambeke* [§], *Guy Lesthievent* [¶]

[⋆] TéSA, 7 Boulevard de la Gare, 31000 Toulouse
[†]University of Toulouse INPT-IRIT, 2 rue Charles Camichel, 31000 Toulouse, France
[§]Thales Alenia Space, 26 Avenue Jean François Champollion, 31100 Toulouse, France
[¶] CNES, 18 avenue Edouard Belin, 31400 Toulouse, France

ABSTRACT

In this paper, we present a new data-aided carrier-recovery method for Continuous Phase Modulation (CPM) signals over frequency-selective channels. We first present a linear model of the received signal based on Mengali representation over selective channels and show how to use it to perform joint channel and carrier-frequency estimation. We also derive a low-complexity version of the estimator. Simulation results show that this method performs better than the optimal method suited to the Additive White Gaussian Noise (AWGN) channels.

Index Terms— Continuous Phase Modulation, carrier recovery, channel estimation, frequency-selective channel

1. INTRODUCTION

CPM signals are commonly known for their good spectral efficiency and for their constant envelope, useful with embedded amplifiers thanks to their robustness to non-linearities. They are considered for a wide range of applications as military communication, aeronautical communication by satellite or M2M applications.
In last decade, several papers deals with equalization for the CPM in selective channels. [1] presents several equalization schemes in the frequency domain for CPM using the Laurent decomposition. [2] presents a frequency-domain Minimum Mean Squared Error (MMSE) equalizer using an orthogonal filter bank. Most of these works have been conducted under the assumption of perfect synchronization and perfect channel knowledge and surprisingly few works are related to synchronization or carrier-recovery for M-ary CPM signals in frequency selective channels. In [3], a new optimal synchronization scheme has been developed for AWGN channels (including preamble design) has been proposed to achieve the Cramer Rao Bound. In its equalization scheme, [4] uses a method suited to AWGN channel for carrier recovery [5].
In the context of equalization, channel estimation has been studied in [6, 7].
[8] presents several carrier-recovery methods for CPM but extensions to frequency selective channels are not discussed. [9, 10, 11] also present carrier frequency estimation for CPM over AGWN channel.

To our knowledge, joint carrier-frequency recovery and channel estimation for M-ary CPM signals has not been done for transmission over multi-path channels. Only, the case of binary CPM (Gaussian Minimum-Shift Keying GMSK) has been dealt with in [12] as binary GMSK can be seen as a linear modulation.
In this paper, we develop a linear system model of M-ary CPMs over frequency-selective channel in order to perform a joint data-aided Maximum Likelihood (ML) estimation similar to [12]. It is organized as follows.The received signal model is presented in section 2. Section 3 describes the proposed algorithm which performs a joint ML channel and carrier-frequency estimation. Some simulation results are given in section 4. Conclusions and perspectives are reported in Section 6.

2. SYSTEM MODEL

We consider a sequence of N symbols taken from the M-ary alphabet $\{\alpha_n\}_{0\leq n\leq N-1} \in \{\pm1, \pm3, \dots, \pm M-1\}^N$. The complex envelope $s_b(t)$ associated with the transmitted CPM signal is written

$$s_b(t) = \sqrt{\frac{2E_s}{T}} \exp\left(j\theta(t, \boldsymbol{\alpha})\right) \tag{1}$$

where

$$\theta(t, \boldsymbol{\alpha}) = \pi h \sum_{i=0}^{N-1} \alpha_i q(t - iT)$$

Thanks to Thales Alenia Space and the CNES agency (French Space Agency) for funding

and

$$q(t) = \begin{cases} \int_0^t g(\tau)d\tau, t \leq L_{\text{cpm}} \\ 1/2, t > L_{\text{cpm}} \end{cases}$$

E_s is the symbol energy, T is the symbol period, $\theta(t, \boldsymbol{\alpha})$ is the information phase, $g(t)$ is the pulse response, $h = k/p$ is the modulation index where k and p are relatively prime integer and L_{cpm} is the CPM memory.
Let us consider a transmission over a frequency-selective channel. The complex envelope associated to the received signal is

$$r(t) = e^{j2\pi f_d t}(h_c * s)(t) + w(t) \tag{2}$$

with

$$h_c(t) \triangleq \sum_{l=0}^{L_c-1} A_l \delta(t - \tau_l). \tag{3}$$

where f_d is the carrier frequency offset, A_l and τ_l are respectively the complex channel coefficients and the delay of the l^{th} path. L_c is the number of paths and w represents a complex Gaussian noise with spectral density N_0.
Let us now consider the Pulse Amplitude Modulation (PAM) representation of the CPM signal introduced by [13] for binary CPMs and extended by [14] for M-ary CPMs. The transmitted signal is a sum of linear PAM modulations of pseudo-symbols $a_{k,n}$

$$s(t) = \sum_{n=0}^{N-1} \sum_{k=0}^{K-1} a_{k,n} g_k(t - nT) \tag{4}$$

where $g_k(t)$ are the components of this decomposition and K is the number of components. More details on those pseudo-symbols and components can be found in [13, 14]. It is well-known that only a few PAMs concentrate most of the signal energy, and thus we can consider an approximated signal by only considering $K' \triangleq M - 1$ components.

Hence, by using Eq.(4) in Eq.(2), our received signal can now be written as

$$r(t) = e^{j2\pi f_d t} h_c(t) * (\sum_{n=0}^{N-1} \sum_{k=0}^{K'-1} a_{k,n} g_k(t - nT)) + w(t) \tag{5}$$

$$= e^{j2\pi f_d t}(\sum_{k=0}^{K'-1} \sum_{n=0}^{N-1} a_{k,n} \underbrace{(h_c * g_k)(t - nT))}_{\triangleq h_k(t)} + w(t) \tag{6}$$

In this paper, perfect timing and frame synchronization is assumed. Consequently, the sampled received signal at $t = mT$ is given by

$$r[m] = e^{j2\pi f_d mT}(\sum_{k=0}^{K'-1} \sum_{n=0}^{N-1} a_{k,n} h_k((m - n)T)) + w[m] \tag{7}$$

We can see from the previous equation that our received signal can be considered as the sum of linear modulations with some equivalent multi-path channels $h_k(t)$. Rewriting Eq.(7) in vector notations we have

$$\boldsymbol{r} = \boldsymbol{\Gamma}(f_d) \sum_{k=0}^{K'-1} \boldsymbol{A_k h_k} + \boldsymbol{w} \tag{8}$$

where

$$\begin{aligned} \boldsymbol{r} &= \{r[0], r[1], ..., r[N-1]\}^T \\ \boldsymbol{\Gamma}(f_d) &= \text{diag}(1, e^{j2\pi f_d}, ..., e^{j2\pi f_d(N-1)}) \\ \boldsymbol{h_k} &= \{h_k(0), h_k(T), ..., h_k((L-1)T)\} \\ \boldsymbol{w} &= \{w[0], ..., w[N-1]\} \end{aligned}$$

and $\boldsymbol{A_k}$ is a (N-L)*L matrix with entries $[\boldsymbol{A_k}]_{i,j} = a_{k,i-j}$.
In this paper, we consider a Data-Aided algorithm so $\{\boldsymbol{A_k}\}$ are known matrix. Using notations $\boldsymbol{A} = [\boldsymbol{A_0}, \dots, \boldsymbol{A_{K'-1}}]$ and $\boldsymbol{h}_{\text{eq}} = [\boldsymbol{h_0}, \dots, \boldsymbol{h_{K'-1}}]^T$, our system becomes:

$$\boldsymbol{r} = \boldsymbol{\Gamma}(f_d)\boldsymbol{A}\boldsymbol{h}_{\text{eq}} + \boldsymbol{w} \tag{9}$$

3. JOINT ML FREQUENCY AND CHANNEL ESTIMATION

Similarly to [12], for a fixed $(f_d, \boldsymbol{h})$, $\boldsymbol{r}$ is a Gaussian vector with mean $\boldsymbol{\Gamma}(f_d)\boldsymbol{A}\boldsymbol{h}_{\text{eq}}$ and covariance matrix $N_0\boldsymbol{I}$, where $\boldsymbol{I}$ is the identity matrix. Hence, the likelihood function for the parameters $(f_d, \boldsymbol{h}_{\text{eq}})$ to maximize is

$$\Delta(\boldsymbol{r}; \widetilde{\boldsymbol{h}}_{\text{eq}}, \widetilde{f}_d) = \frac{1}{(\pi\sigma_n^2)^N} \cdot \exp\Big\{\frac{-1}{\sigma_n^2}[\boldsymbol{r} - \boldsymbol{\Gamma}(\widetilde{f}_d)\boldsymbol{A}\widetilde{\boldsymbol{h}}_{\text{eq}}][\boldsymbol{r} - \boldsymbol{\Gamma}(\widetilde{f}_d)\boldsymbol{A}\widetilde{\boldsymbol{h}}_{\text{eq}}]^H\Big\} \tag{10}$$

where $.^H$ stands for the hermitian transposition.
We choose to maximize Δ over $\widetilde{\boldsymbol{h}}_{\text{eq}}$ and $\widetilde{f}_d$ to obtain the joint ML estimates of $\boldsymbol{h}_{\text{eq}}$ and f_d. The estimate of $\boldsymbol{h}_{\text{eq}}$ for a given $\widetilde{f}_d$ is

$$\widehat{\boldsymbol{h}}_{\text{eq}}(\widetilde{f}_d) = (\boldsymbol{A}^H\boldsymbol{A})^{-1}\boldsymbol{A}^H\boldsymbol{\Gamma}^H(\widetilde{f}_d)\boldsymbol{r}. \tag{11}$$

Using this estimate in Eq.(10), we obtain the following carrier-frequency estimator:

$$\widehat{f}_d = \arg\max_{\widetilde{f}_d} g(\widetilde{f}_d) \tag{12}$$

where

$$g(\widetilde{f}_d) = \boldsymbol{r}^H \boldsymbol{\Gamma}(\widetilde{f}_d) \boldsymbol{B} \boldsymbol{\Gamma}^H(\widetilde{f}_d)\boldsymbol{r} \quad (13)$$

and

$$\boldsymbol{B} \triangleq \boldsymbol{A}(\boldsymbol{A}^H\boldsymbol{A})^{-1}\boldsymbol{A}^H \quad (14)$$

$g(\widetilde{f}_d)$ can be written as:

$$g(\widetilde{f}_d) = -\rho(0) + 2\Re\Big\{\sum_{m=0}^{N-1}\rho(m)e^{-j2\pi m\widetilde{f}_d}\Big\} \quad (15)$$

$$\text{with } \rho(m) = \sum_{k=m}^{N-1}[\boldsymbol{B}]_{k-m,k}r(k)r^*(k-m) \quad (16)$$

$\Re(.)$ is the real part operator and $[\boldsymbol{B}]_{k-m,k}$ is the entries $(k-m, m)$ of the matrix $\boldsymbol{B}$. This matrix can be pre-computed and the grid search of Eq.(13) can be performed by the mean of a Fast Fourier Transform (FFT) followed by a parabolic interpolation.

Thus, the following procedure can be applied at the receiver:

- Compute $\widehat{f}_d$ using Eq.(13)
- Conter-rotate the received signal $\boldsymbol{r}$ according to $\widehat{f}_d$
- Compute $\widehat{\boldsymbol{h}}_{\text{eq}}$ using the LS estimate

It can be shown that our frequency estimator is unbiased with an analysis similar to the one made in [12].

4. SIMULATION RESULTS

4.1. Simulation Parameters

We first consider a preamble of 64 symbols modulated by a GMSK with $M = 4$, $h = 1/4$, $L_{\text{cpm}} = 3$ and $BT = 0.3$. We consider a random preamble taken in the M-ary alphabet defined in section 2.
The true normalized frequency offset $\frac{f_d}{R_s}$, where R_s is the symbol rate, is taken as an uniformly random variable between -0.5 and 0.5. The SNR is defined as the average received energy per transmitted symbols over the noise variance.

We consider the following frequency-selective channel:

$$h_c(t) = \sum_{l=0}^{5} A_l \delta(t-\tau_l) \quad (17)$$

The normalized delays $\{\tau_l/T\}$ are $\{0, 0.054, 0.135, 0.432, 0.621, 1.351\}$.
The attenuation $\{A_l\}$ are complex random gaussian variables with zero mean and variances (in decibel (dB)) $\{-3, 0, -2, -6, -8, -10\}$. For those simulations, we consider that all equivalent discrete channel have a length $L = 8$.

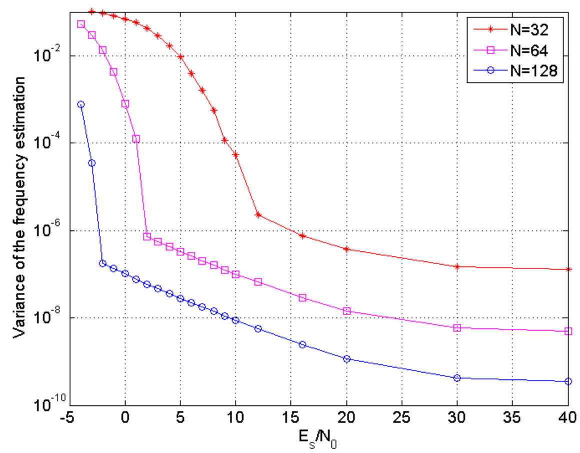

Fig. 1. Performance of the carrier recovery

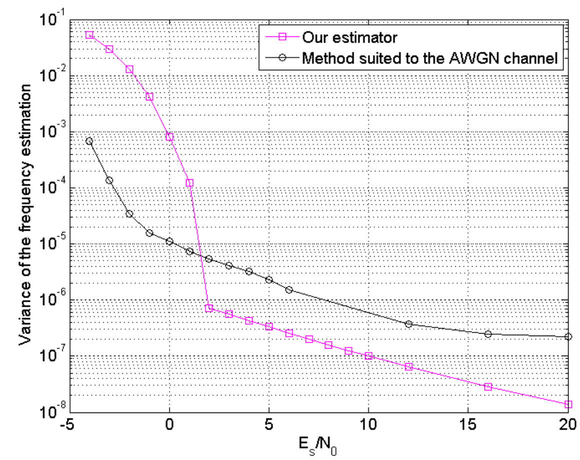

Fig. 2. Comparaison with the method suited to the AWGN channel

4.2. Performance of the carrier-frequency recovery

Fig.1. illustrates the influence of the size of the preamble for carrier-frequency recovery. We choose three training sequences of size N=32, N=64 and N=128. In this case, the selected FFT sizes are respectively 512, 1024 and 2048. We can see that when the size of the preamble increases, the threshold gets lower and lower. For our estimator, the thresholds are respectively around 12dB, 2dB and -2dB for N=32, N=64 and N=128.

Fig.2. compares the estimation developed in this article with the method suited to the AWGN channel presented in [3]. We can see that after the estimator threshold (around 2 dB), our estimator outperforms the other one, which is logical as it does not take into account the channel.

Fig.3. shows the value of the variance for frequency re-

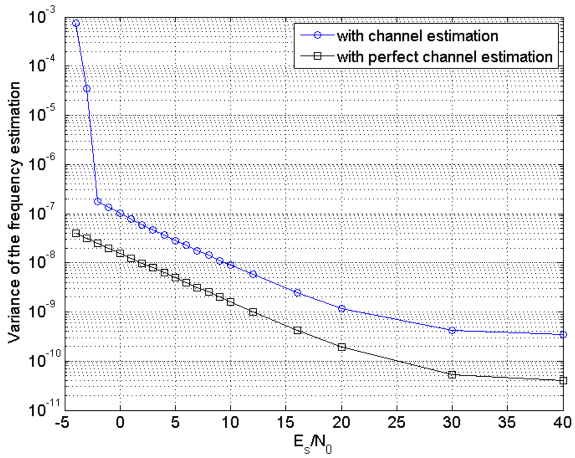

Fig. 3. Performance of the carrier recovery with perfect channel knowledge

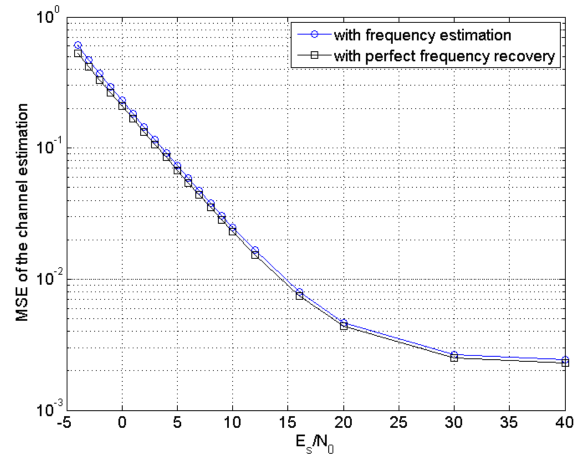

Fig. 5. MSE of the channel estimate with perfect carrier recovery

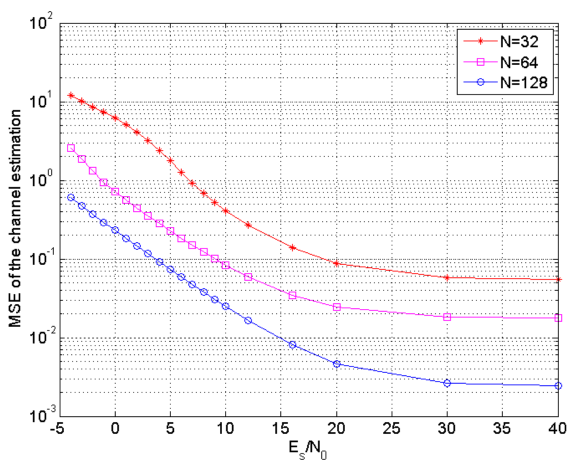

Fig. 4. MSE of the channel estimate

covery in the case of a perfect channel estimation for $N = 128$.

4.3. Performance of the channel estimation

Fig.4. shows the variance of the estimation of the discrete equivalent channel $\boldsymbol{h}_0$ for the same set of parameters. The curve for $N = 32$ shows poor accuracy as we estimate three discrete channels of length $L = 8$, with only 32 observations.

In Fig.5. we plot the ideal Mean Square Error (MSE) value for channel estimation in case of perfect frequency recovery for $N = 128$. We can see that the loss is due to the frequency estimation is less than 1dB. Hence, the overall performance of those estimates are mainly due to the channel estimation residual error. This subject is a perspective of research in the area.

5. RELATIONS TO PRIOR WORK

The method presented here is based on a linear representation of M-ary CPM signals, thanks to the PAM decomposition of CPM in [13, 14]. The PAM representation has been already used for estimation of timing [15], channel estimation [16, 17]. To our knowledge, previous works on carrier-recovery have focused on transmission over the AWGN channel expect for specific CPM signal like the binary GMSK [18, 19]. The transmission over frequency-selective channels was not considered in these previous studies.

6. CONCLUSION

In this paper, we have presented a new joint channel and carrier frequency estimation scheme suited to M-ary CPM modulation over multi-path channels. This scheme is based on a linear representation of CPM signals over frequency-selective channels. The performance is significantly improved compared to the optimal method suited to the AWGN channel.
Future studies may focus on channel estimation, on Cramer Rao bounds computation for CPM over selective channel and also on the preamble design.

7. REFERENCES

[1] F. Pancaldi and G. M. Vitetta, "Equalization algorithms in the frequency domain for continuous phase modulations," *IEEE Transactions on Communications*, vol. 54, no. 4, pp. 648–658, April 2006.

[2] Jun Tan and Gordon L Stuber, "Frequency-domain equalization for continuous phase modulation," in *IEEE Trans. Wireless Commun.*, 2005, vol. 4, pp. 2479–2490.

[3] Ehsan Hosseini and Erik Perrins, "Timing, carrier, and frame synchronization of burst-mode cpm," *IEEE Transactions on Communications*, vol. 61, no. 12, pp. 5125–5138, 2013.

[4] C Brown and PJ Vigneron, "Equalisation and iterative reception for spectrally efficient cpm in multipath environments," in *MILITARY COMMUNICATIONS CONFERENCE, 2010-MILCOM 2010*. IEEE, 2010, pp. 1719–1724.

[5] Jeff Pugh, Colin Brown, and Phil Vigneron, "Preamble design and acquisition for cpm," in *SPIE Defense, Security, and Sensing*. International Society for Optics and Photonics, 2010, pp. 77060O–77060O.

[6] Cheol Hee Park, Robert W HeathJr, and Theodore S Rappaport, "Frequency-domain channel estimation and equalization for continuous-phase modulations with superimposed pilot sequences," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 9, pp. 4903–4908, 2009.

[7] C Brown and PJ Vigneron, "Channel estimation and equalisation for single carrier continuous phase modulation," in *2011-MILCOM 2011 Military Communications Conference*, 2011.

[8] Umberto Mengali and Aldo N. D'Andrea, *Synchronization Techniques for Digital Receivers*, Springer Science & Business Media, 1997.

[9] AN D'Andrea, Alberto Ginesi, and Umberto Mengali, "Frequency detectors for cpm signals," *IEEE transactions on communications*, vol. 43, no. 2/3/4, pp. 1828–1837, 1995.

[10] AN D'Andrea, A Ginesi, and U Mengali, "Digital carrier frequency estimation for multilevel cpm signals," in *Communications, 1995. ICC'95 Seattle,'Gateway to Globalization', 1995 IEEE International Conference on*. IEEE, 1995, vol. 2, pp. 1041–1045.

[11] Johannes Huber and Weilin Liu, "Data-aided synchronization of coherent cpm-receivers," *IEEE Transactions on Communications*, vol. 40, no. 1, pp. 178–189, 1992.

[12] M. Morelli and U. Mengali, "Carrier-frequency estimation for transmissions over selective channels," *IEEE Transactions on Communications*, vol. 48, no. 9, pp. 1580–1589, Sep 2000.

[13] Pierre Laurent, "Exact and approximate construction of digital phase modulations by superposition of amplitude modulated pulses (amp)," *IEEE Trans. Commun.*, vol. 34, no. 2, pp. 150–160, 1986.

[14] Umberto Mengali and Michele Morelli, "Decomposition of m-ary cpm signals into pam waveforms," *IEEE Trans. Inf. Theory*, vol. 41, no. 5, pp. 1265–1275, 1995.

[15] Erik Perrins, Sayak Bose, and Marilynn P Wylie-Green, "Timing recovery based on the pam representation of cpm," in *MILCOM 2008-2008 IEEE Military Communications Conference*. IEEE, 2008, pp. 1–8.

[16] Shawn P Neugebauer and Zhi Ding, "Blind simo channel estimation for cpm using the laurent approximation," in *Circuits and Systems, 2004. ISCAS'04. Proceedings of the 2004 International Symposium on*. IEEE, 2004, vol. 5, pp. V–676.

[17] Jyh-Hau Chen and Yumin Lee, "Joint synchronization, channel length estimation, and channel estimation for the maximum likelihood sequence estimator for high speed wireless communications," in *Vehicular Technology Conference, 2002. Proceedings. VTC 2002-Fall. 2002 IEEE 56th*. IEEE, 2002, vol. 3, pp. 1535–1539.

[18] P Pragadeeshwaran and L Borman, "Single pulse approximation of laurent amp representation of gmsk for map motivated carrier recovery," in *Emerging Trends in Communication, Control, Signal Processing & Computing Applications (C2SPCA), 2013 International Conference on*. IEEE, 2013, pp. 1–4.

[19] Mario E Magana and Ajay Kandukuri, "Non-data-aided parametric-and nonparametric-based carrier frequency estimators for burst gmsk communication systems," *IEEE Transactions on Instrumentation and Measurement*, vol. 59, no. 7, pp. 1783–1792, 2010.