# SIMULTANEOUS WIRELESS INFORMATION AND POWER TRANSFER OVER INDUCTIVELY COUPLED CIRCUITS

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## ABSTRACT

This paper introduces a system model for simultaneous information and power transfer (SWIPT) over inductively coupled circuits from the standard communication-theoretic perspective. It is shown that the rate-energy (R-E) regions, which characterize the performance of SWIPT, for inductively coupled circuits can be obtained by circuit analysis. To evaluate the performance, the R-E regions for SISO and SIMO circuit models are calculated by numerical analysis.

***Index Terms*—** Simultaneous wireless information and power transfer, inductive coupling, energy harvesting.

## 1. INTRODUCTION

With growing interest in the Internet of Things (IoT), the need for research on wireless communication and power transfer is becoming increasingly evident [1–3]. In today's commercial systems, communication and charging take place over two very different technologies. For example, cell phones use communication channels including cellular networks, wireless LAN, and Bluetooth. Since those technologies are designed solely to send information, other techniques such as Qi and WiTricity are used for power transfer [4].

Simultaneous wireless information and power transfer (SWIPT) is a promising technology where remote receivers extract information and power from the common transmit signal [5–7]. SWIPT over RF for far-field transmission was studied in [7] which showed that its performance can be characterized by a so-called rate-energy (R-E) region. The R-E region provides feasible combinations of information rate and harvested power. Furthermore, it has been shown that the efficiency of SWIPT on RF channels can be increased by employing multiple antennas [8].

Inductive coupling is a near-field wireless transmission technique where a coil at the transmitter excites a coil at the receiver [4]. Power transfer over inductively coupled circuits has been extensively studied since it achieves higher energy transfer efficiency when compared to RF power charging [9–11]. While communication via magnetic field has been extensively researched for RFID systems [12], much remains to be studied for information transfer over inductively coupled circuits [13].

Unlike in far-field RF communications, inductive coils in the transmitter and receiver circuits are magnetically coupled [14]. Due to the interaction between the coils, the received signal at a receiver antenna is not merely a sum of the outputs from the transmit antennas but determined by the physical configuration (e.g., size and location) and currents of the transmitter and receiver coils. Therefore, SWIPT over coupled circuits must be considered as a single system to properly analyze power and information transfer.

In this paper, we show that the R-E regions for inductively coupled circuits can be numerically calculated by defining circuit models for the transmitter and receivers. The channels over the coupled circuits are defined in forms commonly seen in the analysis of communications systems. Optimization problems that can be solved to determine the capacity for wireless information transfer (WIT) and harvested power for wireless power transfer (WPT) are introduced. We then define the simple circuit models of the transmitter and receivers to show that channel realizations for inductively coupled circuits can be obtained by circuit analysis and R-E regions for the system can be numerically computed.

## 2. SYSTEM MODEL AND PROBLEM FORMULATION

### 2.1. Channels over Inductively Coupled Circuits

The information and power transfer system consists of a transmitter, power harvesting receiver, and information receiver. The power harvesting receiver extracts the power from the received signal, while the information receiver decodes information. Each circuit is inductively coupled with the other circuits and the transmission channels are formed over the coupled circuits. By formulating the loop equation for each circuit in the Laplace domain, we obtain the transfer function $G_p(s)$ which represents the input-output relationship between the transmitter and power harvesting receiver, and $G_c(s)$ which represents the relationship between the transmitter and information receiver.

While a spectrally flat channel is desirable for analysis, inductively coupled circuits form a frequency-selective channel in general [15]. For convenience, we assume that there are $K$ parallel narrowband channels where for each narrowband channel, the gain between the transmitter and receiver is a constant. Let $G_p(\omega)$ be the frequency response of the transfer function $G_p(s)$. ($G_p(\omega)$ can be obtained by substituting $s = j\omega$ into $G_p(s)$) The discrete-frequency power transfer channel for $G_p(\omega)$ is defined as

$$\mathbf{G}_p = \text{diag}\{G_p(\omega_0), G_p(\omega_1), \ldots, G_p(\omega_{K-1})\}, \quad (1)$$

where $\omega_i$ is the center frequency of $i$-th narrowband channel. Similarly, the communication channel is defined as

$$\mathbf{G}_c = \text{diag}\{G_c(\omega_0), G_c(\omega_1), \ldots, G_c(\omega_{K-1})\}, \quad (2)$$

where $G_c(\omega)$ is the frequency response of $G_c(s)$. The channel realizations for specific circuit models are further discussed in Section 3.

### 2.2. Wireless Power Transfer (WPT)

Analogous to the SWIPT on multiple-input multiple-output (MIMO) channel [7], the transmission from the transmitter to the energy harvesting receiver is defined. In the following, tr($\mathbf{A}$), $\mathbf{A}^{1/2}$, and $\mathbf{A}^H$ denote the trace, square root, and Hermitian transpose of the matrix $\mathbf{A}$, respectively. In addition, $\mathbf{A} \succeq 0$ indicates that $\mathbf{A}$ is a positive-semidefinite matrix. The received signal is represented as

$$\mathbf{y}_p = \mathbf{G}_p \mathbf{x} \quad (3)$$

where $\mathbf{x}, \mathbf{y}_p \in \mathbb{C}^{K\times 1}$, and $\mathbf{G}_p$ is a diagonal matrix as defined in (1). The transmitted signal $\mathbf{x}$ is a zero-mean random vector with covariance matrix $\mathbf{S}_x = \mathbb{E}[\mathbf{x}\mathbf{x}^H]$. Note that the noise in power transfer is negligible [7] and thus omitted in (3).

Provided that there is no loss associated with AC-DC conversion, the maximum power transferred to the receiver is the solution to the following problem [7]:

$$\begin{aligned} \max_{\mathbf{S}_x} \quad & Q := \text{tr}(\mathbf{G}_p \mathbf{S}_x \mathbf{G}_p^H) \\ \text{s.t.} \quad & \text{tr}(\mathbf{S}_x) \le P_{Tx}, \mathbf{S}_x \succeq 0, \end{aligned} \quad (4)$$

**Proposition 1.** *The solution to (4) is a single-entry matrix with $i$-th diagonal element of $P_{Tx}$, where $i$ is the subband with the largest gain.*

*Proof.* Let the singular value decomposition (SVD) of $\mathbf{G}_p$ be $\mathbf{U}_p \mathbf{\Gamma}_p^{1/2} \mathbf{V}_p^H$ where $\mathbf{U}_p, \mathbf{V}_p \in \mathbb{C}^{K\times K}$ and $\mathbf{\Gamma}_p = \text{diag}(g_0, g_1, \ldots, g_{K-1})$ with $g_0 \ge \cdots \ge g_{K-1} \ge 0$, and $\mathbf{v}_g$ be the first column of $\mathbf{V}$. Since $Q = \text{tr}(\mathbf{G}_p \mathbf{S}_x \mathbf{G}_p^H) = \text{tr}(\mathbf{S}_x \mathbf{G}_p \mathbf{G}_p^H)$ and $\mathbf{G}_p \mathbf{G}_p^H$ is a diagonal matrix, $\mathbf{S}_x = P_{Tx} \mathbf{v}_g \mathbf{v}_g^H$ maximizes $Q$. Since $\mathbf{G}_p$ is diagonal, $\mathbf{v}_g$ is a single-entry vector, and thus $\mathbf{v}_g \mathbf{v}_g^H$ is a single-entry diagonal matrix. □

### 2.3. Wireless Information Transfer (WIT)

The information is linearly modulated (e.g., using a standard constellation such as phase shift keying or quadrature amplitude multiplexing) on each subband and transferred to the information receiver. The communication channel from the transmitter to the receiver is defined as

$$\mathbf{y}_c = \mathbf{G}_c \mathbf{x} + \mathbf{w}_c \quad (5)$$

where $\mathbf{y}_c \in \mathbb{C}^{K\times 1}$ is the received signal, $\mathbf{G}_c$ is the channel of the magnetic induction channel, and $\mathbf{w}_i \in \mathbb{C}^{N\times 1} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ is the noise. The capacity of the channel is obtained by solving the following optimization problem:

$$\begin{aligned} \max_{\mathbf{S}_x} \quad & R := \log_2 |\mathbf{I} + \mathbf{G}_c \mathbf{S}_x \mathbf{G}_c^H| \\ \text{s.t.} \quad & \text{tr}(\mathbf{S}_x) \le P_{Tx}, \mathbf{S}_x \succeq 0. \end{aligned} \quad (6)$$

The optimal $\mathbf{S}_x$ for (6) can be obtained by waterfilling power allocation [16]. Note that the solution for waterfilling power allocation is well known and thus omitted for brevity.

### 2.4. Wireless Information and Power Transfer (SWIPT)

In SWIPT, the power and information are transferred to the receiver over the channel $\mathbf{G}_p$ and $\mathbf{G}_c$, respectively. The capacity under the constraint of total transmission power and harvested power is the solution to the following problem [7]:

$$\begin{aligned} \max_{\mathbf{S}_x} \quad & \log_2 |\mathbf{I} + \mathbf{G}_c \mathbf{S}_x \mathbf{G}_c^H| \\ \text{s.t.} \quad & \text{tr}(\mathbf{G}_p \mathbf{S}_x \mathbf{G}_p^H) \le \bar{Q}, \text{tr}(\mathbf{S}_x) \le P_{Tx}, \mathbf{S}_x \succeq 0. \end{aligned} \quad (7)$$

The characteristics of SWIPT over coupled circuits can be expressed by the R-E region [7], which is expressed as

$$\begin{aligned} C_{R-E}(P_{Tx}) = \{(R, Q) : & R \le \log_2 |\mathbf{I} + \mathbf{G}_c \mathbf{S}_x \mathbf{G}_c^H|, \\ & Q \le \text{tr}(\mathbf{G}_p \mathbf{S}_x \mathbf{G}_p^H), \text{tr}(\mathbf{S}_x) \le P_{Tx}, \mathbf{S}_x \succeq 0\}. \end{aligned} \quad (8)$$

## 3. CIRCUIT MODELS

The channels $\mathbf{G}_p$ and $\mathbf{G}_c$ are determined by circuit parameters of the transmitters and receivers. In order to fully analyze the performance, we introduce two simple circuit models, namely, single input single output (SISO) model and single input multiple output (SIMO) model.

### 3.1. SISO Model

For the SISO model, a single transmitter coil is used to excite a single receiver circuit as shown in Figure 1. Both information and power are simultaneously extracted through the load $R_L$ on the receiver, i.e., $\mathbf{G}_c = \mathbf{G}_p$. We assume that decoding data does not consume any energy. The transmitter and receiver are modeled as RLC series circuits. The mutual inductances between coils with inductance $L_i$ and $L_j$ can be given by $M_{ij} = k_{ij}\sqrt{L_i L_j}$ where $0 \le k_{ij} \le 1$ is the coupling coefficient [14].

Assuming that all initial conditions are zero, we can apply Kirchhoff's voltage law [14] to define the loop equation in the Laplace domain for the transmitter circuit as

$$V_i(s) = I_1(s)\left(R_1 + \frac{1}{C_1 s} + L_1 s\right) - I_2(s) M_{1,2} s. \quad (9)$$

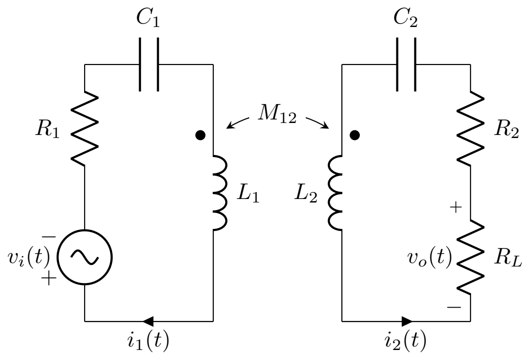


**Fig. 1**. SISO Circuit Model.

The loop equation for the receiver circuit is described as

$$I_1(s)M_{1,2}s = I_2(s)\left(R_2 + R_L + \frac{1}{C_2 s} + L_2 s\right). \quad (10)$$

Let $R_s = R_2 + R_L$. Equations (9) and (10) can be combined to form the linear relationship

$$\begin{bmatrix} V_i(s) \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 + \frac{1}{C_1 s} + L_1 s & -M_{1,2}s \\ -M_{1,2}s & R_s + \frac{1}{C_2 s} + L_2 s \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}. \quad (11)$$

When the inverse of the linear system in (11) exists, we can solve for the Laplace domain current $I_2(s)$. As such, an expression for the current $I_2(s)$ can be represented as

$$I_2(s) = V_i(s)G(s), \quad (12)$$

where the transfer function $H(s)$ is defined in Appendix A. Hence, the transfer function can be described as

$$\begin{aligned} G(s) &= V_i(s)/V_o(s) \\ &= R_L \cdot H(s). \end{aligned} \quad (13)$$

### 3.2. SIMO Model

For the system configuration that consists of separate receivers, a single transmitter coil is used to simultaneously excite two receiver circuits. The circuit model is shown in Figure 2. The energy is harvested at the load $R_p$ on energy harvesting receiver, and the information is extracted through the voltage applied to the load $R_c$ on the information receiver. In contrast to the SISO configuration, energy is consumed for data decoding since the information decoder has the resistor $R_c$. The loop equation in the Laplace domain is defined as

$$\begin{aligned} V_i(s) = I_1(s)&\left(R_i + R_1 + \frac{1}{C_1 s} + L_1 s\right) \\ &- I_2(s)M_{1,2}s - I_3(s)M_{1,3}s. \end{aligned} \quad (14)$$

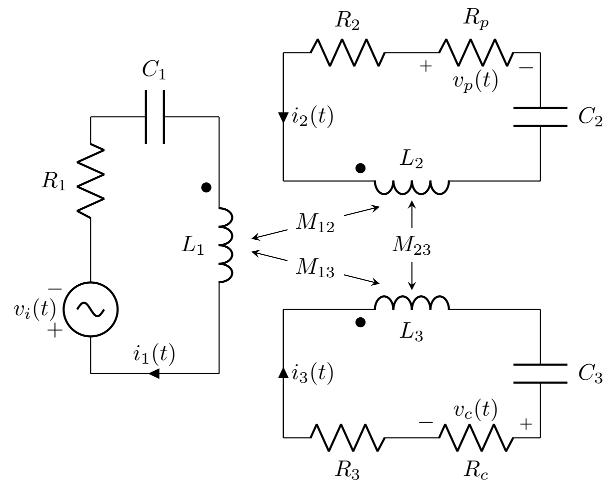


**Fig. 2**. SIMO Circuit Model.

Likewise, the equation for the communications receiver is

$$I_1(s)M_{1,2}s - I_3(s)M_{2,3}s = I_2(s)\left(R_2 + R_c + \frac{1}{C_2 s} + L_2 s\right), \quad (15)$$

and the power extraction circuit loop equation equals

$$I_1(s)M_{1,3}s - I_2(s)M_{2,3}s = I_3(s)\left(R_3 + R_p + \frac{1}{C_3 s} + L_3 s\right). \quad (16)$$

Equations (14), (15), and (16) can be combined to form the linear relationship

$$\begin{bmatrix} V_i(s) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{bmatrix}, \quad (17)$$

where

$$\begin{aligned} a_{11} &= R_1 + \tfrac{1}{C_1 s} + L_1 s \\ a_{12} &= a_{21} = -M_{1,2}s \\ a_{13} &= a_{31} = -M_{1,3}s \\ a_{22} &= R_2 + R_c + \tfrac{1}{C_2 s} + L_2 s \\ a_{23} &= a_{32} = M_{2,3}s \\ a_{33} &= R_3 + R_p + \tfrac{1}{C_3 s} + L_3 s. \end{aligned}$$

When the inverse of the linear system defined in (17) exists, we can solve for the Laplace domain currents $I_2(s)$ and $I_3(s)$. The expressions for currents $I_2(s)$ and $I_3(s)$ are defined as

$$\begin{bmatrix} I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} H_c(s) \\ H_p(s) \end{bmatrix} V_i(s) \quad (18)$$

where $H_c(s)$, $H_p(s)$ are Laplace domain transfer functions. Thus, Laplace domain power transfer channel is given by

$$\begin{aligned} G_p(s) &= V_p(s)/V_i(s) \\ &= R_p \cdot H_p(s), \end{aligned} \quad (19)$$

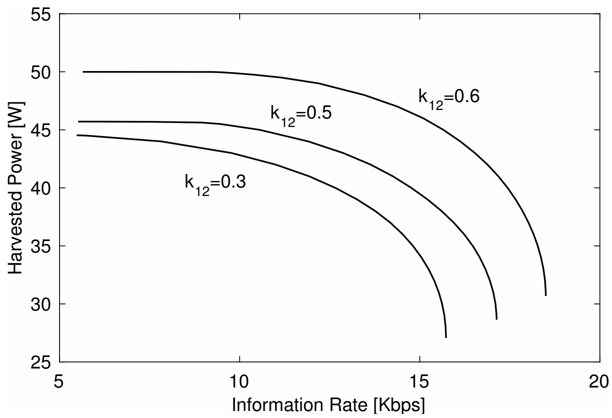


**Fig. 3**. R-E Regions for SISO Model.

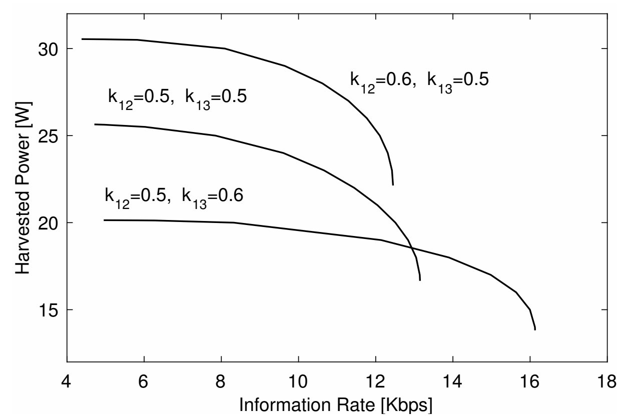


**Fig. 4**. R-E Regions for SIMO Model.

**Table 1**. Circuit Parameters for Numerical Analysis

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $R_1, R_2, R_3$ | $50\,\Omega$ | $L_1, L_2, L_3$ | $10\,\mathrm{mH}$ |
| $R_c, R_p, R_L$ | $100\,\Omega$ | $C_1, C_2, C_3$ | $1\,\mathrm{nF}$ |
| $k_{23}$ | 0.05 | | |

where $H_p(s)$ is defined in Appendix A. Similarly, the Laplace domain information transfer channel is given by

$$\begin{aligned} G_c(s) &= V_c(s)/V_i(s) \\ &= R_c \cdot H_c(s). \end{aligned} \tag{20}$$

where $H_c(s)$ is defined in Appendix A.

## 4. NUMERICAL ANALYSIS

The R-E regions for the SISO and SIMO models with parameters listed in Table 1 are calculated by Monte Carlo analysis. To solve the convex optimization problems, we used CVX, a computation package for MATLAB, with SDPT3 as a solver [17, 18]. The maximum transmit power $P_{Tx}$ is set to $200\,\mathrm{W}$, and the channel is divided into $K = 128$ subbands.

For the SISO model, the R-E regions with coupling coefficients $k = 0.3$, $0.5$, and $0.6$ are calculated. From the result shown in Figure 1, the channel with higher coupling coefficient achieves higher rate and larger power transfer in general. Note that the minimum harvested power does not go to zero since each subband has a non-zero bandwidth.

For the SIMO model, the R-E regions of three different combinations of $k_{12}$ and $k_{13}$ are calculated. The numerical analysis result shown in Figure 2, the R-E region curve with $k_{12} = 0.6$ and $k_{13} = 0.5$ transfers larger power compared to the curve with $k_{12} = 0.5$ and $k_{13} = 0.5$ at the same rate. While the R-E region with $k_{12} = 0.5$ and $k_{13} = 0.6$ achieves higher data rate, the maximum harvested power is less than the others.

## 5. CONCLUSION

In this work, the information and power transfer channels over inductively coupled circuits are defined. The optimum power allocation for maximizing the rate is obtained by waterfilling while the maximum harvested power is achieved by allocation all transmission power to the subband with maximum channel gain. The SISO and SIMO circuit models were introduced, and it was shown that the channels can be calculated using the circuit parameters. The R-E regions for the SISO and SIMO models with different sets of coupling coefficients are calculated by numerical analysis.

## A. TRANSFER FUNCTIONS

The transfer function for SISO model is given by

$$H(s) = \frac{M_{12}s}{(R_1 + \frac{1}{C_1 s}L_1 s)(R_s + \frac{1}{C_2 s + L_2 s}) - M_{12}^2 s} \tag{21}$$

Both of the transfer functions for the SIMO model share the same denominator polynomial given by

$$\begin{aligned} D(s) &= (R_3 + R_p + \tfrac{1}{C_3 s} + L_3 s)(-M_{12}^2 s^2 + (R_1 + \tfrac{1}{C_1 s} + L_1 s) \\ &(R_2 + R_c + \tfrac{1}{C_2 s})) - M_{23}s(\tfrac{M_{23}}{C_1} + s(-M_{12}M_{13}s + M_{23}(R_1 + L_1 s))) \\ &- \frac{M_{13}s}{C_2}(-C_2 M_{12} M_{23} s^2 + M_{13}(1 + C_2 s(R_2 + R_c + L_2 s))). \end{aligned} \tag{22}$$

The transfer functions can be written as

$$\begin{aligned} H_p(s) = \tfrac{1}{D(s)} \Big( &\tfrac{M_{12}}{C_3} + M_{12}R_3 s + M_{12}R_p s \\ &+ L_3 M_{12} s^2 - M_{13}M_{23}s^2 \Big) \end{aligned} \tag{23}$$

$$\begin{aligned} H_c(s) = \tfrac{1}{D(s)} \Big( &\tfrac{M_{13}}{C_2} + M_{13}R_2 s + M_{13}R_c s \\ &+ L_2 M_{13} s^2 - M_{12}M_{23}s^2 \Big). \end{aligned} \tag{24}$$

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