ENHANCING QOS IN SPATIALLY CONTROLLED BEAMFORMING NETWORKS VIA DISTRIBUTED STOCHASTIC PROGRAMMING

Dionysios S. Kalogerias and Athina P. Petropulu

Department of Electrical & Computer Engineering
Rutgers, The State University of New Jersey, Piscataway, NJ 08854, USA

ABSTRACT

We address the problem of enhancing Quality-of-Service (QoS) in power constrained, mobile relay beamforming networks, by controlling the motion of the relaying nodes. We consider a time slotted system, where the relays update their positions before the beginning of each time slot. Adopting a spatiotemporal stochastic field model of the wireless channel, we propose a novel 2-stage stochastic programming formulation for specifying the relay positions at each time slot, such that the QoS of the network is maximized on average, based on causal Channel State Information (CSI) and under a total relay transmit power budget. Via the Method of Statistical Differentials, the motion control problem considered is shown to be approximately equivalent to a set of simple subproblems, which are solved in a distributed fashion, one at each relay. Numerical simulations are also presented, corroborating the efficacy of the proposed approach.

Index Terms— Network Mobility Control, Distributed Cooperative Networks, Spatially Controlled / Mobile Relay Beamforming, QoS Maximization, Motion Control, Stochastic Programming

1. INTRODUCTION

Typically, the literature on cooperative relay beamforming networks [1, 2, 3, 4, 5, 6, 7] does not consider optimum placement of network nodes in order to improve the quality of communications. In most cases, network nodes are either assumed to be stationary in space, or, if some of them move while communicating, their trajectories are assumed to be independent of the respective communication task.

Recently, however, relay mobility has been proposed as an effective means to further enhance performance in beamforming networks. In [8], optimal transmit Amplify-and-Forward (AF) beamforming has been combined with potential-field-based relay mobility control in multiuser cooperative networks, in order to minimize relay transmit power, while meeting certain QoS constraints. In [9], in the framework of information theoretic physical layer security, decentralized jammer motion control has been jointly combined with noise nulling and cooperative jamming, maximizing the network secrecy rate. In [10], optimal relay positioning has been studied in systems where multiple relays deliver information to a destination, in the presence of an eavesdropping node, with a goal of maximizing or achieving a target level of ergodic secrecy. In [8, 9, 10], the links among the nodes of the network (or the related statistics) are assumed to be available in the form of static channel maps, during the whole motion of the jammers/relays. However, this might be oversimplifying in scenarios where the channels change significantly in time and space [11, 12, 13]. Given a one source/destination relay beamforming network, [14] considers the problem of optimally selecting relay positions, in order to minimize their total transmit power under a certain QoS specification. Communications and relay motion are systematically scheduled via a simple time division protocol. Different from [8, 9, 10], in [14], the wireless channel is modeled as a *spatiotemporal stochastic field*, based on a realistic, commonly employed "log-normal" channel model [13].

In this paper, assuming the same channel model and scheduling protocol as in [14], we consider relay position selection under a different optimality criterion. In particular, we propose a novel 2-*stage stochastic programming* formulation of the problem of specifying the positions of the relays, such that the *Signal-to-Interference+Noise Ratio (SINR)* at the destination is maximized *on average*, based on causal CSI, and subject to a total power constraint at the relays. This objective is more well behaved as compared to that in [14], where, not only formulation of the *initial* problem was made only approximately, but also the potentially oversimplifying assumption of a high source-relay Signal-to-Noise Ratio (SNR) was imposed, to obtain a solution. Contrary to [14], in this work, it is possible to manipulate the objective, without imposing further simplifications to the beamforming problem, or trivializing the initial problem formulation, thus maintaining system robustness. Exploiting the *Method of Statistical Differentials* [15], it is shown that the aforementioned motion control problem is approximately equivalent to a set of two dimensional subproblems, solved in a *distributed fashion*, one at each relay. Numerical simulations are presented, verifying that the proposed approach indeed works, resulting in motion control policies, which yield significantly improved performance, when compared to agnostic, randomized relay motion.

2. SYSTEM & CHANNEL MODELS

2.1. System Model

On a closed planar region $\mathcal{S} \subset \mathbb{R}^2$, we consider a wireless cooperative network, as shown in Fig. 1. The network consists of one source, one destination and $R \in \mathbb{N}^+$ assistive relays, all equipped with a single antenna and being able for both information reception and broadcasting/transmission. The source and destination are fixed at $\mathbf{p}_S \in \mathcal{S}$ and $\mathbf{p}_D \in \mathcal{S}$, respectively, whereas the relays are assumed spatially controllable; each relay $i \in \mathbb{N}_R^+$ moves along a trajectory $\mathbf{p}_i(t) \in \mathcal{S}, t \in \mathbb{R}_+$. Also define $\mathbf{p}(t) \triangleq \left[\mathbf{p}_1^{\boldsymbol{T}}(t) \, \dots \, \mathbf{p}_R^{\boldsymbol{T}}(t)\right]^{\boldsymbol{T}} \in \mathcal{S}^R$. The relays can cooperate, either by local message exchange, or by communicating with a fusion center, through a dedicated channel.

Due to non-existence of a direct link between the source and the destination, a two-phase AF beamforming policy is adopted. As in [14], choose a $T > 0$, and *divide the time interval* $[0, T]$ *into* N_T *time slots*. Let $t \in \mathbb{N}_{N_T}^+$ denote the respective time slot. The symbol transmitted at time slot t is denoted as $s(t) \in \mathbb{C}$, with $\mathbb{E}\left\{|s(t)|^2\right\} \equiv 1$. Assuming a flat fading channel model, as well as channel reciprocity and quasistaticity in each time slot, let the

This work is supported by the National Science Foundation (NSF) under Grants CCF-1526908 & CNS-1239188.

sets $\{f_i(t) \equiv f(\mathbf{p}_i(t), t)\}_{i \in \mathbb{N}_R^+}$ and $\{g_i(t) \equiv g(\mathbf{p}_i(t), t)\}_{i \in \mathbb{N}_R^+}$ contain the complex, *random, spatiotemporally varying* source-relay and relay-destination channel gains, respectively. Then, if $P_0 > 0$ denotes transmission power, during AF phase 1, relay i receives the amplified symbol $f(\mathbf{p}_i(t), t)\sqrt{P_0}s(t)$, plus an additive noise component $n_i(t) \sim \mathcal{CN}\left(0, \sigma^2\right)$, $i \in \mathbb{N}_R^+$, independent across relays. During AF phase 2, all relays simultaneously retransmit the information received, each modulating their received signal by a weight $w_i(t) \in \mathbb{C}, i \in \mathbb{N}_R^+$. The signal received at the destination can be expressed as the superposition of the weighted relay signals, plus another noise component $n_D(t) \sim \mathcal{CN}\left(0, \sigma_D^2\right)$.

Hereafter, while it is assumed that the fields $f(\mathbf{p}, t)$ and $g(\mathbf{p}, t)$ may be *statistically dependent both spatially and temporally*, the processes $s(t)$, $[f(\mathbf{p}, t)\ g(\mathbf{p}, t)]$, $n_i(t)$ for all $i \in \mathbb{N}_R^+$, and $n_D(t)$ are assumed to be mutually independent. Lastly, at each time slot t, CSI $\{f_i(t)\}_{i \in \mathbb{N}_R^+}$ and $\{g_i(t)\}_{i \in \mathbb{N}_R^+}$ is assumed to be known *exactly* to all relays. This may be achieved through pilot based estimation.

2.2. Channel Model

At each time slot $t \in \mathbb{N}_{N_T}^+$, the i-th source-relay channel gain, $f_i(t)$, is assumed to be composed by three components, multiplied with each other; the deterministic path loss, the shadowing component and the multipath fading one [16]. In this work, of special importance is the magnitude of $f_i(t)$, which may be expressed as [14, 17]

$$|f_i(t)| \equiv 10^{\rho/20} \exp\left(\frac{\log(10)}{20} F_i(t)\right), \quad \text{with} \tag{1}$$

$$F_i(t) \equiv F(\mathbf{p}_i(t), t) \triangleq \alpha_S(\mathbf{p}_i(t))\,\ell + \sigma_S^i(t) + \xi_S^i(t), \tag{2}$$

for all $i \in \mathbb{N}_R^+$ and for all $t \in \mathbb{N}_{N_T}^+$, where, in the above, $\rho > 0$ denotes the mean of the fading component of the channel, $\ell > 0$ denotes the path loss exponent (both assumed to be known), $\alpha_S(\mathbf{p}_i(t)) \triangleq -10\log_{10}(d_{iS}(t))$, $d_{iS}(t) \triangleq \|\mathbf{p}_i(t) - \mathbf{p}_S\|_2$, $\xi_S^i(t) \equiv \xi_S(\mathbf{p}_i(t), t) \overset{i.i.d.}{\sim} \mathcal{N}\left(0, \sigma_\xi^2 > 0\right)$, for all $t \in \mathbb{N}_{N_T}^+$ and $i \in \mathbb{N}_R^+$ [18], and $\sigma_S^i(t) \equiv \sigma_S(\mathbf{p}_i(t), t) \sim \mathcal{N}\left(0, \eta^2 > 0\right)$, for all $(\mathbf{p}_i(t), t) \in \mathcal{S} \times \mathbb{N}_{N_T}^+$. The spatiotemporal interactions of the latter process will be specified shortly. Of course, we may stack all the $F_i(t)$'s defined in (2), resulting in the vector additive model

$$\boldsymbol{F}(t) \triangleq \boldsymbol{\alpha}_S(\mathbf{p}(t))\,\ell + \boldsymbol{\sigma}_S(t) + \boldsymbol{\xi}_S(t) \in \mathbb{R}^{R \times 1}, \tag{3}$$

where $\boldsymbol{\alpha}_S(t)$, $\boldsymbol{\sigma}_S(t)$ and $\boldsymbol{\xi}_S(t)$ are defined accordingly. We may also define $\boldsymbol{G}(t) \triangleq \boldsymbol{\alpha}_D(\mathbf{p}(t))\,\ell + \boldsymbol{\sigma}_D(t) + \boldsymbol{\xi}_D(t) \in \mathbb{R}^{R \times 1}$, with each quantity in direct correspondence with (3).

It is further assumed that for any N_T and any respective ensemble of positions of the relays in $\mathbb{N}_{N_T}^+$, the random vector

$$\left[\boldsymbol{F}^{\boldsymbol{T}}(1)\ \boldsymbol{G}^{\boldsymbol{T}}(1)\ \ldots\ \boldsymbol{F}^{\boldsymbol{T}}(N_T)\ \boldsymbol{G}^{\boldsymbol{T}}(N_T)\right]^{\boldsymbol{T}} \in \mathbb{R}^{2RN_T \times 1} \tag{4}$$

is jointly Gaussian with known means and known covariance matrix. In particular, extending Gudmundson's model [19], the spatiotemporal correlations of the $\sigma_S(\mathbf{p}_i(t), t)$'s are specified as [14, 17]

$$\mathbb{E}\left\{\sigma_S^i(k)\,\sigma_S^j(l)\right\} \triangleq \eta^2 e^{-\frac{\|\mathbf{p}_i(k) - \mathbf{p}_j(l)\|_2}{\beta} - \frac{|k-l|}{\gamma}}, \tag{5}$$

and correspondingly for the $\sigma_D^i(t)$'s, and, additionally,

$$\mathbb{E}\left\{\sigma_S^i(k)\,\sigma_D^j(l)\right\} \triangleq \mathbb{E}\left\{\sigma_S^i(k)\,\sigma_S^j(l)\right\} e^{-\frac{\|\mathbf{p}_S - \mathbf{p}_D\|_2}{\delta}}, \tag{6}$$

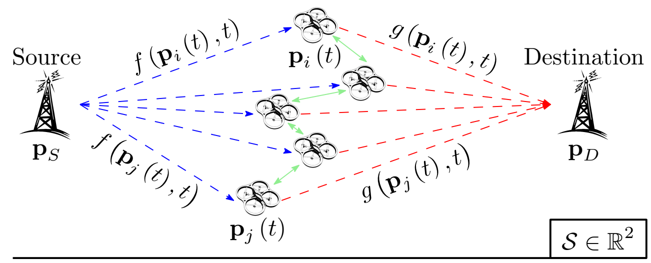

Fig. 1. A schematic of the system model considered.

for all $(i, j) \in \mathbb{N}_R^+ \times \mathbb{N}_R^+$ and all $(k, l) \in \mathbb{N}_{N_T}^+ \times \mathbb{N}_{N_T}^+$. In the above, the parameters η^2, $\beta > 0$, $\gamma > 0$ and $\delta > 0$ are called the *shadowing power*, *correlation distance*, *correlation time*, *BS (Base Station) correlation*, respectively. Gaussianity constitutes the fundamental assumption in the adopted channel model; in fact, under mild technical conditions, covariance structures other than those presented above, may be equally considered [17].

3. QOS ENHANCED BY RELAY MOTION

At each time slot $t \in \mathbb{N}_T^+$, the following joint communication / decision making TDMA-like protocol is adopted [14, 17] (see Fig. 2): **1)** The source broadcasts a pilot signal to the relays, which then estimate the channels relative to the source. **2)** The same procedure is carried out for the channels relative to the destination. **3)** Based on the estimated CSI, beamforming is implemented. **4)** Based on the CSI received *so far*, spatial controllers of the relays are determined, implementing accurate stochastic decision making.

Each relay obeys the kinematic model $\dot{\mathbf{p}}(\tau) \equiv \mathbf{u}(\tau)$, for all $\tau \in \mathbb{R}_+$. Assuming the relays move *only after their controls have been determined and up to the start of the next time slot*, we may write

$$\mathbf{p}(t) \equiv \mathbf{p}(t-1) + \int_{\Delta\tau_{t-1}} \mathbf{u}_{t-1}(\tau)\,\mathrm{d}\tau, \ \forall t-1 \in \mathbb{N}_{N_T-1}^+, \tag{7}$$

with $\mathbf{p}(1) \equiv \mathbf{p}_{init}$, and where $\Delta\tau_t \in \mathbb{R}$ denotes the time interval that the relays are allowed to move in each time slot $t \in \mathbb{N}_{N_T-1}^+$. Regarding the form of $\mathbf{u}_{t-1}(\tau), \tau \in \Delta\tau_{t-1}$, *given a goal position vector at time slot* t, $\mathbf{p}^o(t)$, it suffices to fix a path in $\mathcal{S}$, such that the points $\mathbf{p}^o(t)$ and $\mathbf{p}(t-1)$ are connected in at most time $\Delta\tau_t$. A generic choice for such a path is the straight line connecting $\mathbf{p}^o(t)$ and $\mathbf{p}(t-1)$. Therefore, at time slot $t-1 \in \mathbb{N}_{N_T-1}^+$ we may choose $\mathbf{u}_{t-1}^o(\tau) \triangleq (\Delta\tau_{t-1})^{-1}(\mathbf{p}^o(t) - \mathbf{p}(t-1))$, for all $\tau \in \Delta\tau_{t-1}$. As a result, any motion control problem can now be formulated in terms of specifying the goal relay positions at the next time slot, given current information. In the following, let $\{\mathscr{C}(\mathcal{T}_t)\}_{t \in \mathbb{N}_{N_T}^+}$ denote the set of channels observed by the relays (the filtration), *along the path of their point trajectories* $\mathcal{T}_i \triangleq \{\mathbf{p}(t)\}_{t \in \mathbb{N}_i^+}, i \in \mathbb{N}_{N_T}$.

3.1. Spatially Controlled Beamforming

The beamforming criterion considered herein is that of maximizing the SINR at the destination, subject to a total power budget at the relays. At time $t \in \mathbb{N}_{N_T}^+$, *given* CSI encoded in $\mathscr{C}(\mathcal{T}_t)$, this may be achieved by formulating the constrained optimization problem [1, 4]

$$\begin{array}{ll} \underset{\boldsymbol{w}(t)}{\text{maximize}} & \dfrac{\mathbb{E}\{P_S(t)|\mathscr{C}(\mathcal{T}_t)\}}{\mathbb{E}\{P_{I+N}(t)|\mathscr{C}(\mathcal{T}_t)\}} \\ \text{subject to} & \mathbb{E}\{P_R(t)|\mathscr{C}(\mathcal{T}_t)\} \le P_c \end{array}, \tag{8}$$

where $P_R(t)$, $P_S(t)$ and $P_{I+N}(t)$ denote the instantaneous power at the relays, that of the signal component and that of the interference plus noise component at the destination, respectively. $P_c > 0$ denotes the total available relay transmission power. Under our assumptions, presented in Section 2.1, (8) can be reexpressed as [1]

$$\begin{array}{cl} \underset{\boldsymbol{w}(t)\triangleq[w_1(t)\dots w_R(t)]^{\boldsymbol{T}}}{\text{maximize}} & \dfrac{\boldsymbol{w}^{\boldsymbol{H}}(t)\,\mathbf{R}(\mathbf{p}(t),t)\,\boldsymbol{w}(t)}{\sigma_D^2+\boldsymbol{w}^{\boldsymbol{H}}(t)\,\mathbf{Q}(\mathbf{p}(t),t)\,\boldsymbol{w}(t)} \\ \text{subject to} & \boldsymbol{w}^{\boldsymbol{H}}(t)\,\mathbf{D}(\mathbf{p}(t),t)\,\boldsymbol{w}(t)\le P_c \end{array}, \quad (9)$$

where, dropping the dependence on $(\mathbf{p}(t), t)$ or t for brevity,

$$\mathbf{D} \triangleq P_0 \mathrm{diag}\left(\left[|f_1|^2\ |f_2|^2\ \dots\ |f_R|^2\right]^{\boldsymbol{T}}\right) + \sigma^2 \mathbf{I}_R \in \mathbb{S}_{++}^R, \quad (10)$$

$$\mathbf{R} \triangleq P_0 \mathbf{h}\mathbf{h}^{\boldsymbol{H}} \in \mathbb{S}_{+}^R, \text{ with } \mathbf{h} \triangleq [f_1 g_1\ f_2 g_2\ \dots\ f_R g_R]^{\boldsymbol{T}} \text{ and} \quad (11)$$

$$\mathbf{Q} \triangleq \sigma^2 \mathrm{diag}\left(\left[|g_1|^2\ |g_2|^2\ \dots\ |g_R|^2\right]^{\boldsymbol{T}}\right) \in \mathbb{S}_{++}^R. \quad (12)$$

Note that the program (9) is *always feasible, as long as* $P_c > 0$. It is known that the optimal value of (9) is given *analytically* by [1, 4]

$$\begin{aligned} V_t &\equiv \sum_{i\in\mathbb{N}_R^+} \frac{P_c P_0 |f(\mathbf{p}_i(t),t)|^2 |g(\mathbf{p}_i(t),t)|^2}{P_0\sigma_D^2 |f(\mathbf{p}_i(t),t)|^2 + P_c\sigma^2 |g(\mathbf{p}_i(t),t)|^2 + \sigma^2\sigma_D^2} \\ &\triangleq \sum_{i\in\mathbb{N}_R^+} V_I(\mathbf{p}_i(t),t), \quad \forall t \in \mathbb{N}_{N_T}^+. \end{aligned} \quad (13)$$

Note that the constraint on the total transmission power of the relays in implicitly satisfied when considering V_t as a function of $(\mathbf{p}(t), t)$.

At time slot $t-1$, we are interested in choosing the positions of the relays at time slot t, such that V_t is maximized. However, at $t-1$, we are only given $\mathscr{C}(\mathcal{T}_{t-1})$, which does not encode future CSI, revealed at time slot t. Therefore, *exact* optimization of the relay positions at the next time slot is *impossible*. Nevertheless, it would be reasonable to search for the best decision on the positions of the relays at time slot t (as a functional of $\mathscr{C}(\mathcal{T}_{t-1})$), such that V_t is maximized *in expectation*. This results, at each time slot $t-1 \in \mathbb{N}_{N_T-1}^+$, in the 2-*stage stochastic program* [20]

$$\begin{array}{ll} \underset{\mathbf{p}(t)}{\text{maximize}} & \mathbb{E}\{V_t\} \\ \text{subject to} & \mathbf{p}(t) \equiv \mathcal{M}(\mathscr{C}(\mathcal{T}_{t-1})),\ \mathcal{M}:\mathbb{R}^{2R(t-1)} \to \mathbb{R}^{2R} \\ & \mathbf{p}(t) \in \mathcal{C}(\mathbf{p}^o(t-1)) \end{array}, \quad (14)$$

where $\mathbf{p}^o(1) \in \mathcal{S}^R$ is a known constant, representing the initial positions of the relays and $\mathcal{C}(\mathbf{p}^o(t-1)) \subseteq \mathcal{S}^R$ denotes a closed set representing a spatially feasible neighborhood around the point $\mathbf{p}^o(t-1) \in \mathcal{S}^R$, the (possibly optimal) decision vector at time $t-2$. Problems (14) and (9) are referred to as the *first-stage problem* and the *second-stage problem*, respectively [20].

The stochastic *variational* program (14) is extremely difficult to solve in its original form. Nevertheless, it can be shown that, under some generic assumptions, (14) satisfies a specific set of technical conditions (for a detailed analysis, see [17]), which allow the invocation of the *Fundamental Lemma of Stochastic Control (FLSC)* [21, 22, 23, 20, 24, 25, 17]. Actually, the FLSC refers to a *family* of technical results, providing conditions which permit interchange of expectation and max/minimization in general stochastic programs, including (14). For our purposes, the FLSC may be developed via delicate applications of the tower property of expectations [17], and

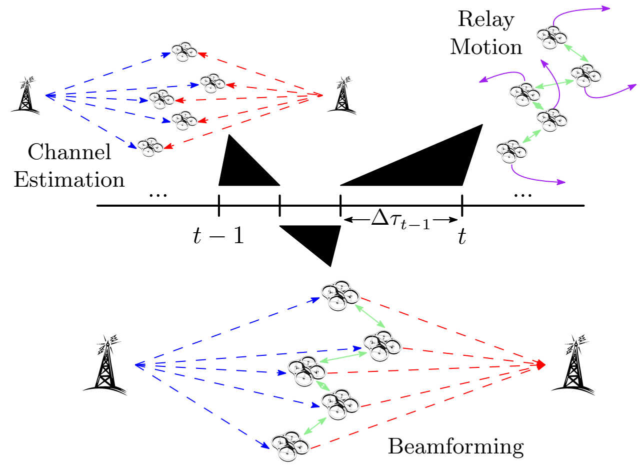

Fig. 2. TDMA-like joint scheduling of communications & controls.

implies that the first stage problem (14) is *exchangeable* by the *pointwise (over constants) problem*

$$\begin{array}{ll} \underset{\mathbf{p}(t)}{\text{maximize}} & \sum_{i\in\mathbb{N}_R^+} \mathbb{E}\{V_I(\mathbf{p}_i(t),t) | \mathscr{C}(\mathcal{T}_{t-1})\} \\ \text{subject to} & \mathbf{p}(t) \in \mathcal{C}(\mathbf{p}^o(t-1)) \end{array}, \quad (15)$$

to be solved at each $t-1 \in \mathbb{N}_{N_T-1}^+$.

We readily observe that, by definition, the problem (15) is separable. In fact, given that, *for each* $t \in \mathbb{N}_{N_T-1}^+$*, decisions taken and CSI collected so far are available to all relays*, (15) can be solved in a *completely distributed fashion at the relays*, with the i-th relay being responsible for solving the *two dimensional* program

$$\begin{array}{ll} \underset{\mathbf{p}}{\text{maximize}} & \mathbb{E}\{V_I(\mathbf{p},t) | \mathscr{C}(\mathcal{T}_{t-1})\} \\ \text{subject to} & \mathbf{p} \in \mathcal{C}_i(\mathbf{p}^o(t-1)) \end{array}, \quad (16)$$

at each $t-1 \in \mathbb{N}_{N_T-1}^+$, where $\mathcal{C}_i : \mathbb{R}^2 \rightrightarrows \mathbb{R}^2$ denotes the corresponding part of $\mathcal{C}$, for each $i \in \mathbb{N}_R^+$. Note that no local exchange of intermediate results is required among relays; given the available information, each relay independently solves its own subproblem. The problem, however, with (16), is that its objective involves the evaluation of a conditional expectation of a ratio of almost surely positive random variables, which is *impossible to perform analytically*.

3.2. Approximation via the Method of Statistical Differentials

First, V_I can be equivalently expressed as

$$\begin{aligned} V_I(\mathbf{p},t) &\equiv \frac{1}{V_{II}(\mathbf{p},t)} \\ &\triangleq \frac{1}{\frac{\sigma_D^2}{P_c}|g(\mathbf{p},t)|^{-2} + \frac{\sigma^2}{P_0}|f(\mathbf{p},t)|^{-2} + \frac{\sigma^2\sigma_D^2}{P_c P_0}|f(\mathbf{p},t)|^{-2}|g(\mathbf{p},t)|^{-2}} \end{aligned} \quad (17)$$

for all $(\mathbf{p}, t) \in \mathcal{S} \times \mathbb{N}_{N_T}^+$. Then, for $t \in \mathbb{N}_{N_T}^2$, we may locally approximate $\mathbb{E}\{V_I(\mathbf{p},t) | \mathscr{C}(\mathcal{T}_{t-1})\}$ around $\mathbb{E}\{V_{II}(\mathbf{p},t) | \mathscr{C}(\mathcal{T}_{t-1})\}$ (also known as the *Method of Statistical Differentials* [15]) via a second order Taylor expansion as

$$\mathbb{E}\{V_I(\mathbf{p},t) | \mathscr{C}(\mathcal{T}_{t-1})\} \approx \frac{\mathbb{E}\left\{(V_{II}(\mathbf{p},t))^2 \middle| \mathscr{C}(\mathcal{T}_{t-1})\right\}}{(\mathbb{E}\{V_{II}(\mathbf{p},t) | \mathscr{C}(\mathcal{T}_{t-1})\})^3}, \quad (18)$$

where the square on the numerator can be expanded into a sum of terms of the form $C(m,n) \times |f(\mathbf{p},t)|^m |g(\mathbf{p},t)|^n$, for some $(m,n) \in \mathbb{Z} \times \mathbb{Z}$ and some constant $C(m,n)$. Now we invoke ([17], Lemma 2), a result very similar to ([14], Theorem 1), which exploits the Gaussian spatiotemporal structure of the channel, as presented in Section 2.2 (see (3) to (6)), along with the representation trick (1) and the definition of the moment generating function of the Gaussian distribution. This provides a closed form expression for $\mathbb{E}\{|f(\mathbf{p},t)|^m |g(\mathbf{p},t)|^n | \mathscr{C}(\mathcal{T}_{t-1})\}$, for any $(m,n) \in \mathbb{Z} \times \mathbb{Z}$. Thus, the conditional expectations $\mathbb{E}\{V_{II}(\mathbf{p},t) | \mathscr{C}(\mathcal{T}_{t-1})\}$, $\mathbb{E}\left\{(V_{II}(\mathbf{p},t))^2 \middle| \mathscr{C}(\mathcal{T}_{t-1})\right\}$ and, in turn, (18), may be evaluated at any point $\mathbf{p} \in \mathcal{S}$. Then, we propose the replacement of the initial, pointwise problem (16), with

$$\begin{array}{ll} \underset{\mathbf{p}}{\text{maximize}} & \dfrac{\mathbb{E}\left\{(V_{II}(\mathbf{p},t))^2 \middle| \mathscr{C}(\mathcal{T}_{t-1})\right\}}{\left(\mathbb{E}\{V_{II}(\mathbf{p},t) | \mathscr{C}(\mathcal{T}_{t-1})\}\right)^3} \\ \text{subject to} & \mathbf{p} \in \mathcal{C}_i(\mathbf{p}^o(t-1)) \end{array}, \qquad (19)$$

to be solved at relay $i \in \mathbb{N}_R^+$, at each time $t-1 \in \mathbb{N}_{N_T-1}^+$, therefore determining the (approximately/locally) optimal decisions on the position of each relay at the next time slot, t. Various methods may be employed for the solution of (19). In the typical case where $\mathcal{C}_i$ is finite, for $i \in \mathbb{N}_R^+$, corresponding to the situation where the relays move on a grid, (19) may be easily solved via exhaustive search.

4. NUMERICAL SIMULATIONS

In this section, we present synthetic numerical simulations, which essentially confirm that the proposed approach, presented in Sections 3.1 and 3.2 above, actually works, and results in relay motion control policies, which yield improved beamforming performance. All synthetic experiments were conducted on an imaginary square terrain of dimensions 30×30 squared units of length, with $\mathcal{S} \equiv [0,30]^2$, uniformly divided into $26 \times 26 \equiv 676$ square regions. The locations of the source and destination are fixed as $\mathbf{p}_S \equiv [15\,0]^{\boldsymbol{T}}$ and $\mathbf{p}_D \equiv [15\,30]^{\boldsymbol{T}}$. The beamforming temporal horizon is chosen as $T \equiv 6$ and the number of relays is fixed at $R \equiv 8$. The wavelength is chosen as $\lambda \equiv 0.125$, corresponding to a carrier frequency of $2.4\,GHz$. The various parameters of the assumed channel model are set as $\ell \equiv 3$, $\rho \equiv 20$, $\sigma_\xi^2 \equiv 20$, $\eta^2 \equiv 50$, $\beta \equiv 10$, $\gamma \equiv 5$ and $\delta \equiv 1$. The variances of the reception noises at the relays and the destination are fixed as $\sigma^2 \equiv \sigma_D^2 \equiv 1$. Lastly, both the transmission power of the source and the *total* transmission power budget of the relays are chosen as $P \equiv P_c \equiv 25$ ($\approx 14dB$) units of power.

Regarding implementation of the proposed approach, the relays are allowed to be located in the rectangular region $[0,30] \times [12,18]$, that is, inside a narrow strip in the middle of the terrain, with respect to the y-axis. Further, at each time instant, each of the relays is allowed to move inside a 9-region area, centered at each current position, thus defining its closed set of feasible directions $\mathcal{C}_i$, for each relay $i \in \mathbb{N}_R^+$. Basic collision and out-of-bounds control was also considered and implemented.

In order to assess the effectiveness of our proposed approach for strategic relay motion control, we compare it against the case where an *agnostic*, *purely randomized* relay control policy is adopted; in this case, at each time slot, each relay moves randomly to a new available position, without taking previously observed CSI into consideration. Of course, the comparison of the two controlled systems is made under exactly the same communication environment. The

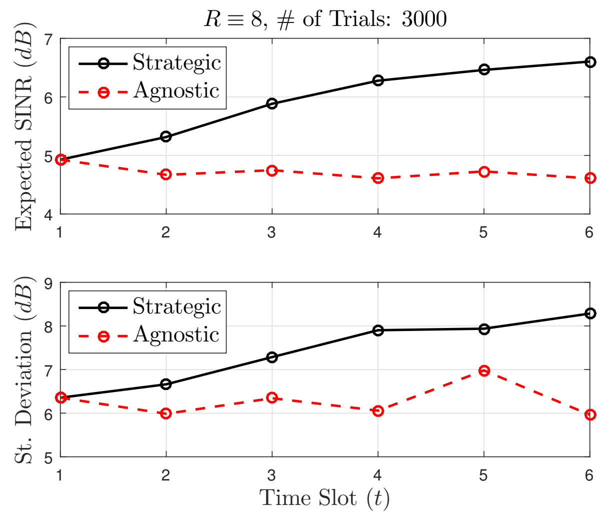

Fig. 3. Experimental comparison of our proposed strategic relay planning, versus an agnostic, randomized motion policy at the relays.

expected QoS achieved at each time instant was approximated by executing 3000 trials of the whole experiment, for both controlled systems under test. Fig. 3 shows the approximations of both the expectation and standard deviation of the QoS achieved by the two systems. As seen by the figure, there is a clear advantage in exploiting strategically designed relay motion control. Whereas the agnostic system maintains an average SINR of about $4.7\,dB$ at all times, the system based on the proposed approach is clearly superior, exhibiting an increasing trend in the achieved SINR, with a gap starting from about $0.65\,dB$, up to $2\,dB$. This increasing trend might reveal further useful properties of our stochastic programming formulation; this is a subject of current research. Finally, we should comment on the standard deviation of both systems, which, from Fig. 3, seems somewhat high, relative to the range of the respective average SINR. This behavior is *exclusively* due to the wild variations of the channel, which, in turn, are due to the effects of shadowing and multipath fading; it is *not* due to the adopted beamforming technique. Further, we readily observe that, although the standard deviation of the SINR (the objective) is uncontrolled in (14), the range of its values is only slightly (in fact, proportionally) higher, relative to the agnostic case. This is reasonable, since, when the channel is not *actually* in deep fade at time t, the relays, at time $t-1$, are predictively steered to locations, which, most probably, incur higher network QoS.

5. CONCLUSION

We have considered the problem of enhancing QoS in spatially controlled relay beamforming networks with one source/destination, via stochastic relay motion control. Modeling the wireless channel as a spatiotemporal stochastic field, we proposed a novel 2-stage stochastic programming formulation for predictively specifying relay positions, such that the future expected network QoS is maximized, based on causal CSI and under a total relay power constraint. We have shown that this problem can be meaningfully approximated by a set of simple, two dimensional subproblems, which can be distributively solved, one at each relay. Our simulations confirmed the success of the proposed approach, which results in relay motion control policies that yield significant improvement in the average achieved QoS, when compared to agnostic, randomized relay motion.

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