ADMM FOR HARMONIC RETRIEVAL FROM ONE-BIT SAMPLING WITH TIME-VARYING THRESHOLDS

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ABSTRACT

Parameter estimation of sinusoids is a problem of significance in many practical applications. This problem is revisited through a new alternating direction method of multipliers (ADMM) based approach, where the unknown parameters are estimated from one-bit quantized noisy measurements with time varying thresholds. The proposed method is computationally efficient and easy to implement, since each ADMM update has simple closed-form formula. Moreover, it provides accurate estimates by exploiting group sparsity hidden in the signal model. Simulations demonstrate the effectiveness of our algorithm.

***Index Terms*—** One-bit quantization, alternating direction method of multipliers (ADMM), group sparsity, parameter estimation.

1. INTRODUCTION

In digital signal processing, after sampling, the signal is rounded to one of a number of predefined levels. This procedure is called quantization, which is an essential component of the analog-to-digital converter (ADC). It is well known that if the quantization is fine enough, analog signals can be accurately represented in the digital domain. However, in some applications such as spectrum sensing for cognitive radio [1] and cognitive radar [2], the signal bandwidth is usually in the gigahertz range, which makes high bit quantization impractical. This is because both energy consumption and product cost of the ADC device scales exponentially as the number of bits increases. Therefore, using low bit quantizers become meaningful in practice, especially in large-bandwidth systems. Particularly, one-bit quantization is a competitive choice since it allows for an extremely high sampling rate at a low cost.

The effects of one-bit quantization on parameter estimation have been studied for many years, through reformulating classical signal processing problems as one-bit quantization based models [3], including radar processing [4–6], statistical signal processing [7–9], sampling [10] and compressive sensing [11–15]. Earlier studies such as [8] and [9] considered the one-bit harmonic retrieval, where [8] studied the influence of frequency estimation of a single-tone and [9] revisited the direction-of-arrival problem. Both of them derived the Cramér-Rao bound for their respective scenarios, and showed that one-bit quantization gives slightly worse performance. The one-bit compressive sensing was introduced in [11], and studied further in [12] and [13]. Until very recently, the one-bit sampling study is mainly focused on comparing with the zero threshold, and signal norm information cannot be recovered. With properly chosen thresholds, however, it is shown that the signal norm can be estimated from one-bit measurements [15].

Our work in [16] and [19] considered the frequency and phase estimation of multiple sinusoids from one-bit samples with time-varying thresholds. It was shown that accurately estimating the parameters including the amplitudes of sinusoids is possible. Moreover, two algorithms which are based on ℓ_1-norm and logarithm approximation, respectively, were developed in [16] to estimate the unknown parameters of sinusoids via introducing a frequency dictionary and then modeling the original estimation problem as a sparse signal recovery problem. Note that although these two algorithms utilize convex programming forms which can be solved by using interior-point methods [17, 18], their complexities are too high to allow them to deal with large sized signals. To circumvent this problem, in this paper, we aim at developing a computationally efficient method to accurately estimate the unknown parameters of sinusoids from one-bit samples with time-varying thresholds.

The organization of this paper is as follows. The signal model under considering is introduced in Section 2, and an alternating direction method of multiplers (ADMM) based approach is devised in Section 3, to solve the parameter estimation problem. Simulations are included in Section 4 to examine the performance of our method by comparing it with the state-of-the-art methods. Finally, conclusion is drawn in Section 5.

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2. PROBLEM STATEMENT

The 1-D harmonic retrieval problem is encountered in many signal processing applications such as radar and wireless communication systems, and the signal can be modeled as a sum of several harmonics:

$$s(t) = \sum_{i=k}^{K} a_k \cos(\omega_k t + \phi_k) \tag{1}$$

where a_k, ω_k and ϕ_k represent the amplitude, frequency and phase of the kth sinusoid, respectively.

One essential process to implement modern signal processing techniques is to represent the continuous signal $s(t)$ in digital form, which involves the so-called quantization. Of course, fine quantization reduces the measurement error caused by the quantizer, but significantly burdens the ADC, resulting in a lower sample rate and higher power consumption. In practical systems, e.g., in cognitive radar, where the bandwidth is very wide, fine quantization is unrealistic (or at least prohibitively expensive). In such cases, low-bit quantizer is preferable.

In the following, we consider harmonic retrieval from one-bit quantized measurements of $s(t)$, referred to as one-bit harmonic retrieval, that has the form of

$$y(t) = \mathrm{sign}(s(t) - h(t)) \tag{2}$$

where $h(t)$ is a time-varying threshold and $\mathrm{sign}(\cdot)$ is a sign operator, which is characterized by

$$\mathrm{sign}(x) = \begin{cases} 1, x \geq 0 \\ -1, x < 0. \end{cases} \tag{3}$$

Here, the problem is to estimate the sinusoidal parameters from its sign measurements $y(t)$.

Define two over-complete dictionaries

$$\mathbf{A}_c = [\, \mathbf{a}_c(t_0) \ \cdots \ \mathbf{a}_c(t_{T-1}) \,]^T \tag{4}$$

$$\mathbf{A}_s = [\, \mathbf{a}_s(t_0) \ \cdots \ \mathbf{a}_s(t_{T-1}) \,]^T \tag{5}$$

where $(\cdot)^T$ is the transpose and

$$\mathbf{a}_c(t_i) = [\, \cos(\omega_0 t_i) \ \cdots \ \cos(\omega_{N-1} t_i) \,]^T \tag{6}$$

$$\mathbf{a}_s(t_i) = [\, \sin(\omega_0 t_i) \ \cdots \ \sin(\omega_{N-1} t_i) \,]^T \tag{7}$$

$$\text{for } \ i = 0, \cdots, T-1.$$

Note that the frequencies in the dictionaries are normalized as $\omega_n = \pi n/N, n = 0, \cdots, N-1$. Then, (1) can be rewritten as

$$\mathbf{s} = [s(t_0) \ \cdots \ s(t_{T-1})]^T = \mathbf{A}_c\mathbf{x}_c - \mathbf{A}_s\mathbf{x}_s \tag{8}$$

where $\mathbf{x}_c$ and $\mathbf{x}_s$ are K-sparse real-valued vectors corresponding to the cosine and sine amplitudes, respectively. Thus, (2) becomes

$$\mathbf{y} = \mathrm{sign}(\mathbf{A}_c\mathbf{x}_c - \mathbf{A}_s\mathbf{x}_s - \mathbf{h}) \tag{9}$$

where

$$\mathbf{y} = [\, y(t_0) \ \cdots \ y(t_{T-1}) \,]^T$$

$$\mathbf{h} = [h(t_0) \ \cdots \ h(t_{T-1})]^T .$$

Now the sinusoidal parameter estimation problem can be solved as the following one-bit sampling problem with time-varying thresholds [16], i.e.,

$$\begin{aligned} &\min_{\mathbf{x}_c, \mathbf{x}_s} \|\mathbf{x}_c\|_0 + \|\mathbf{x}_s\|_0 \\ &\text{s. t. } \ \mathbf{y} = \mathrm{sign}(\mathbf{A}_c\mathbf{x}_c - \mathbf{A}_s\mathbf{x}_s - \mathbf{h}) \end{aligned} \tag{10}$$

where $\|\cdot\|_0$ is the zero norm.

3. PROPOSED ALGORITHM

Problem (10) is non-convex. A simple way to avoid its complexities is to replace the zero norm with ℓ_1-norm, then formulate it as a second-order cone programming [16], which is usually time consuming. In this section, we aim at developing a low-complexity technique to estimate the sinusoidal parameters.

To start, let us define

$$\mathbf{A} = [\mathbf{A}_c \ \ -\mathbf{A}_s] \tag{11}$$

$$\mathbf{x} = \left[\mathbf{x}_c^T \ \ \mathbf{x}_s^T\right]^T \tag{12}$$

which leads to

$$\mathbf{y} = \mathrm{sign}(\mathbf{A}\mathbf{x} - \mathbf{h}). \tag{13}$$

Let $\mathbf{b}$ be the amplitude of $(\mathbf{A}\mathbf{x} - \mathbf{h})$. Thus,

$$\mathbf{y} \odot \mathbf{b} = \mathbf{A}\mathbf{x} - \mathbf{h} \tag{14}$$

where $\odot$ is the element-wise product of vectors or matrices. In the noisy case, since (14) approximately holds true, we can modify the harmonic retrieval problem as follows:

$$\begin{aligned} &\min_{\mathbf{x}, \mathbf{b}} \|\mathbf{A}\mathbf{x} - \mathbf{y} \odot \mathbf{b} - \mathbf{h}\|_2^2 \\ &\text{s. t. } \|\mathbf{x}\|_0 \leq 2K \end{aligned} \tag{15}$$

where $\|\cdot\|_2$ is the ℓ_2-norm. Note that due to the fact that $\mathbf{x}_c$ and $\mathbf{x}_s$ are both K-sparse vectors, $\mathbf{x}$ is a $2K$-sparse vector. Moreover, $\mathbf{x}_c$ and $\mathbf{x}_s$ have the same sparsity, which means that $\mathbf{x}$ is group sparse. This motivates us to propose

$$\min_{\mathbf{x}, \mathbf{b}} \ \|\mathbf{A}\mathbf{x} - \mathbf{y} \odot \mathbf{b} - \mathbf{h}\|_2^2 + \lambda\|\mathbf{x}\|_{2,1} \tag{16}$$

where $\|\cdot\|_{2,1}$ denotes the $\ell_{2,1}$-norm and is defined as

$$\|\mathbf{x}\|_{2,1} = \sum_{i=1}^{N} \left(x_i^2 + x_{i+N}^2\right)^{1/2} := \sum_{i=1}^{N} \|\mathbf{x}_{g_i}\|_2 \tag{17}$$

with

$$\mathbf{x}_{g_i} = [x_i, x_{i+N}]^T \tag{18}$$

being the ith sub-vector of $\mathbf{x}$ indexed by g_i.

We follow the rationale of ADMM to deal with Problem (16). To this end, define an auxiliary variable $\mathbf{z} = \mathbf{x}$, and write (16) as

$$\begin{aligned}
&\min_{\mathbf{x},\mathbf{b},\mathbf{z}} \|\mathbf{A}\mathbf{x} - \mathbf{y} \odot \mathbf{b} - \mathbf{h}\|_2^2 + \lambda \|\mathbf{z}\|_{2,1} \\
&\text{ s. t. } \mathbf{z} = \mathbf{x}.
\end{aligned} \tag{19}$$

The corresponding augmented Lagrangian is

$$L = \|\mathbf{A}\mathbf{x} - \mathbf{y} \odot \mathbf{b} - \mathbf{h}\|_2^2 + \lambda \|\mathbf{z}\|_{2,1} + \rho \|\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2 \tag{20}$$

where $\mathbf{u}$ is a scaled dual variable. Then the ADMM update takes the form of

$$\mathbf{x} \leftarrow \arg\min_{\mathbf{x}} \ \|\mathbf{A}\mathbf{x} - \mathbf{y} \odot \mathbf{b} - \mathbf{h}\|_2^2 + \rho \|\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2 \tag{21}$$

$$\mathbf{b} \leftarrow |\mathbf{A}\mathbf{x} - \mathbf{h}| \tag{22}$$

$$\mathbf{z} \leftarrow \arg\min_{\mathbf{z}} \ \lambda \|\mathbf{z}\|_{2,1} + \rho \|\mathbf{x} - \mathbf{z} + \mathbf{u}\|_2^2 \tag{23}$$

$$\mathbf{u} \leftarrow \mathbf{u} + \mathbf{x} - \mathbf{z} \tag{24}$$

where $|\cdot|$ denotes the absolute value. Problem (21) is equivalent to

$$(\mathbf{A}^H \mathbf{A} + \rho \mathbf{I}_N)\mathbf{x} = \mathbf{A}^H (\mathbf{y} \odot \mathbf{b} + \mathbf{h}) + \rho(\mathbf{z} - \mathbf{u}) \tag{25}$$

where $\mathbf{I}_m$ is the $m \times m$ identity matrix and $(\cdot)^H$ is the conjugate transpose. Using the matrix inversion lemma, the solution of (25) can be expressed as

$$\begin{aligned}
\mathbf{x} \ \leftarrow & \frac{1}{\rho} \left(\mathbf{I}_N - \mathbf{A}^H \left(\mathbf{A}\mathbf{A}^H + \rho \mathbf{I}_T \right)^{-1} \mathbf{A} \right) \\
& \times \left(\mathbf{A}^H (\mathbf{y} \odot \mathbf{b} + \mathbf{h}) + \rho(\mathbf{z} - \mathbf{u}) \right).
\end{aligned} \tag{26}$$

where $(\cdot)^{-1}$ represents matrix inverse.

In order to handle Problem (23), let us first split $\mathbf{z}$ and $\mathbf{u}$ into N subvectors as in (18). Then we have

$$\mathbf{z}_{g_i} \leftarrow \arg\min_{\mathbf{z}_{g_i}} \ \lambda \|\mathbf{z}_{g_i}\|_2 + \rho \|\mathbf{x}_{g_i} - \mathbf{z}_{g_i} + \mathbf{u}_{g_i}\|_2^2, \ \forall i \tag{27}$$

which is a block soft-thresholding problem [20,21], and therefore has a closed-form solution as

$$\mathbf{z}_{g_i} \leftarrow S_{\lambda/\rho}(\mathbf{x}_{g_i} + \mathbf{u}_{g_i}) \tag{28}$$

where $S_t(\mathbf{x}) = \max\left\{1 - \frac{t}{\|\mathbf{x}\|_2}, 0\right\} \mathbf{x}$.

When the algorithm converges, we obtain an estimate of $\mathbf{x}$, denoted by $\hat{\mathbf{x}}$. Then the frequency estimates are selected as the one corresponding to the K largest values of $\sqrt{\hat{\mathbf{x}}_c^2 + \hat{\mathbf{x}}_s^2}$. Since our method is an ADMM based spectrum estimation approach (SEA), we call it "SEA-ADMM".

The procedures for the SEA-ADMM algorithm are summarized in **Algorithm 1.**

Algorithm 1 SEA-ADMM for one-bit spectrum estimation

function $\hat{\mathbf{x}}$ =SEA-ADMM$(\mathbf{y}, \mathbf{A})$
    Initialize $\mathbf{b} = \mathbf{1}_T$, $\mathbf{z}^{(0)}$,$r = 1$ $\mathbf{u}^{(0)} = \mathbf{0}_{2N}$
        % $\mathbf{1}_T$ is a $T \times 1$ one vector
        % $\mathbf{0}_{2N}$ is a $2N \times 1$ zero vector
        % $\mathbf{z}^{(0)}$ is randomly generated.
    while Until some stopping criterion is reached **do**
        $\mathbf{x}^{(r)} =$ (26), where $\mathbf{z}, \mathbf{b}, \mathbf{u}$ in (26) are replaced by $\mathbf{z}^{(r-1)}$, $\mathbf{b}^{(r-1)}$, and $\mathbf{u}^{(r-1)}$, respectively.
        $\mathbf{b}^{(r)} = |\mathbf{A}\mathbf{x}^{(r)} - \mathbf{h}|$
        $\mathbf{z}_{g_i}^{(r)} = S_{\lambda/\rho}\left(\mathbf{x}_{g_i}^{(r)} + \mathbf{u}_{g_i}^{(r-1)}\right)$
        $\mathbf{u}^{(r)} = \mathbf{u}^{(r-1)} + \mathbf{x}^{(r)} - \mathbf{z}^{(r)}$
        $r = r + 1$
    end while
end function

4. SIMULATION RESULTS

We present the simulations of the proposed method and compare it with two methods in [16] that are ℓ_1- and log-norm based spectrum estimation approaches (SEA). Hence, we call them ℓ_1-SEA and log-SEA. The ℓ_1-SEA has the following formulation

$$\ell_1\text{−SEA} \begin{cases} \min\limits_{\mathbf{x}_c, \mathbf{x}_s, \tilde{\mathbf{x}}, \mathbf{e}, q} & \left[q, \tilde{\mathbf{x}}^T\right] \mathbf{c} \\ \quad \text{s. t.} & \mathbf{y} \odot (\mathbf{A}_c \mathbf{x}_c - \mathbf{A}_s \mathbf{x}_s + \mathbf{e} - \mathbf{h}) \succeq 0 \\ & q \geq \|\mathbf{e}\|_2 \\ & \tilde{x}_n \geq \sqrt{x_{c,n}^2 + x_{s,n}^2}, \ n = 1, \cdots, N \end{cases}$$

where $\mathbf{c} = [1, \lambda, \cdots, \lambda]^T \in \mathbb{R}^{N+1}$. The log-SEA is an iterative method which at the rth iteration solves the following problem

$$\text{log–SEA} \begin{cases} \min\limits_{\mathbf{x}_c, \mathbf{x}_s, \tilde{\mathbf{x}}, \mathbf{e}, q} & \left[q, \tilde{\mathbf{x}}^T\right] \mathbf{w}^{(r-1)} \\ \quad \text{s. t.} & \mathbf{y} \odot (\mathbf{A}_c \mathbf{x}_c - \mathbf{A}_s \mathbf{x}_s + \mathbf{e} - \mathbf{h}) \geq 0 \\ & q \geq \|\mathbf{e}\|_2 \\ & \tilde{x}_n \geq \sqrt{x_{c,n}^2 + x_{s,n}^2} \end{cases}$$

where $\mathbf{w}^{(r-1)} = \left[1, \frac{\lambda}{\tilde{x}_1^{(r-1)}}, \cdots, \frac{\lambda}{\tilde{x}_N^{(r-1)}}\right]^T$.

In the simulations, we assume that there are four sinusoids with frequencies being $\omega_1 = 1.0186$, $\omega_2 = 1.4972$, $\omega_3 = 1.9083$ and $\omega_4 = 2.1721$, amplitudes being $a_1 = 9$, $a_2 = 17$, $a_3 = 13$ and $a_4 = 15$, and phases being $\phi_1 = \frac{\pi}{3}$, $\phi_2 = \frac{\pi}{7}$, $\phi_3 = \frac{7\pi}{3}$ and $\phi_4 = \pi$, respectively. The number of measurements is $T = 512$ and the signal-to-noise ratio (SNR) is 15 dB. Moreover, the frequency dictionary is constructed via uniformly spacing $[0, \pi]$ by $N = 1024$ points. Thirty one threshold values are used, and each of which is chosen randomly at each sample. All results are obtained using 100

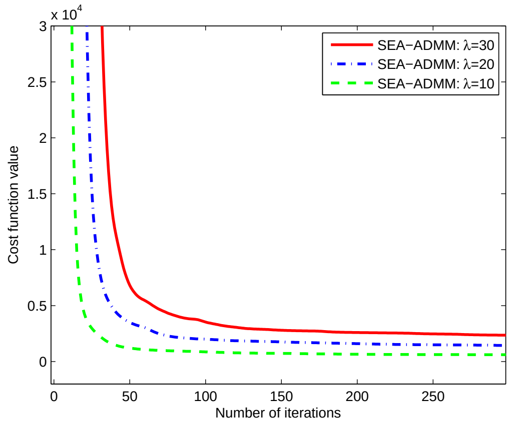

Fig. 1. Cost function value versus number of iterations.

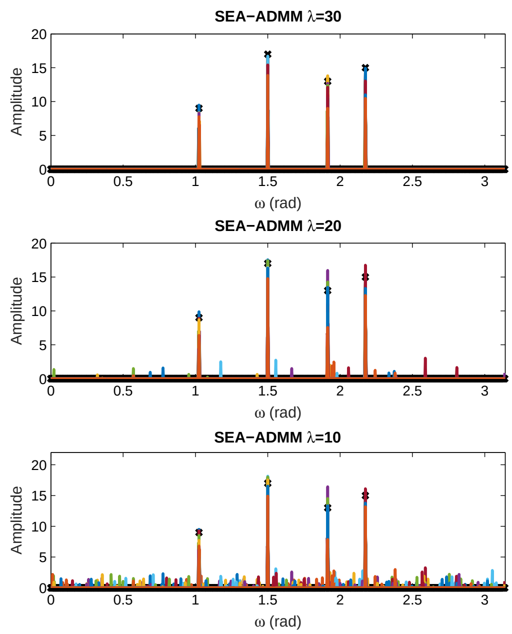

Fig. 2. Sparse spectra recovery of SEA-ADMM from 100 independent tests, where vertical lines stand for the spectrum estimates and $\times$ stands for the true spectral localization.

Monte-Carlo trials on a computer with 3.6 GHz i7-4790 CPU and 8 GB RAM.

In the first example, we study how the shrinkage parameter λ in (19) impacts the performance of SEA-ADMM in terms of convergence speed and estimation accuracy. We keep $\rho = 1$ and vary λ from 10 to 30. Fig. 1 shows the cost function value in (16) versus the number of iterations. As we can see, our scheme converges after 200 iterations and a smaller λ provides faster convergence speed. However, SEA-ADMM with a smaller λ produces less sparse spectrum estimates which is shown in Fig. 2.

We now compare the spectrum estimation performance. For the proposed method, we set $\lambda = 30$ and $\rho = 1$, and design stopping criterion as the relative cost function value error below 10^{-6}, i.e., $\frac{|f^{(r)} - f^{(r-1)}|}{f^{(r-1)}} \leq 10^{-6}$, where the cost function is defined in (16). For the ℓ_1-SEA and $\log$-SEA, the user defined parameter is set to be 2. It is seen from Fig. 3 that all the algorithms have four distinct peaks. Among the tested algorithm, the SEA-ADMM has the best performance while the ℓ_1-SEA is the worst since it has lots of small amplitudes associated with the incorrect indices. The $\log$-SEA scheme performs much better than the ℓ_1-SEA, and this performance improvement is mainly due to the weights that make a compromise between sparse promotion and noise suppression, but at the expense of increased complexity. It should be noted that the averaged CPU time for SEA-ADMM, ℓ_1-SEA and $\log$-SEA are 1.1507 s, 54.6100 s and 184.2318 s, respectively. It is obvious that our method is computationally the most efficient.

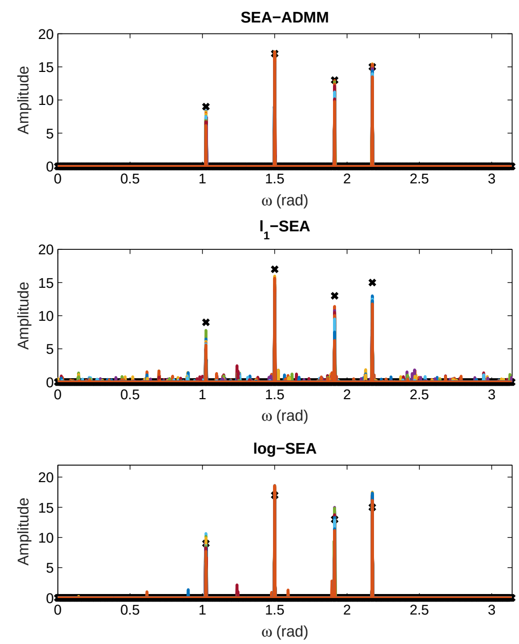

Fig. 3. Sparse spectra recovery performance comparison from 100 independent tests, where vertical lines stand for the spectrum estimates and $\times$ stands for the true line spectrum.

5. CONCLUSION

In this paper, we have considered the problem of estimating unknown parameters of sinusoids from its noisy one-bit thresholded measurements. We have provided an algorithmic framework that is based on ADMM, namely, SEA-ADMM, which is computationally efficient and easy to implement. Simulation results have been provided to show that the proposed scheme is faster and offers better spectral estimates than the existing algorithms.

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