NON-PARAMETRIC ANALOG JOINT SOURCE CHANNEL CODING FOR AMPLIFY-AND-FORWARD TWO-HOP NETWORKS

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ABSTRACT

We investigate the use of discrete-time analog joint source channel coding (JSCC) in an amplify-and-forward two-hop wireless network subject to Rayleigh fading channels. The discrete-time analog source is compressed using either 2:1, 3:1 or 4:1 low delay dimension-compression parametric and non-parametric analog JSCC. Our results show that both, non-parametric and parametric, analog schemes outperform an ideal fully digital system based on scalar quantization, while the performance obtained with non parametric mappings is superior to that achieved with parametric systems.

Index Terms**—** Joint Source Channel Coding, Analog Mappings, Bandwidth Compression

1. INTRODUCTION

Discrete-time analog Joint Source Channel Coding (JSCC) is a robust, low complexity and low delay alternative to traditional digital systems based on the use of separated source and channel encoders for the transmission of analog sources [1–11]. Most of the work on analog JSCC has focused on point-to-point networks [1–8], while only a few studies, such as [9–11], have proposed the application of analog schemes for relay networks. The authors in [9] analyze the use of parametric analog JSCC [6] for a two-hop network [12], where the relay compresses its own information together with the received data from another source node and then forwards it to a sink node. In [10], a non-parametric analog coding [2] scheme for the relay channel [13] is proposed. In this case, the destination can also observe the source. However, both [9, 10] consider non-fading channels. Different from [9, 10], Rayleigh fading channels are considered in [11], which proposes the use of parametric analog JSCC over a relay channel with the Amplify-and-Forward (AF) protocol.

In this paper, we propose a dimension-compression non-parametric analog JSCC scheme in a two-hop network with Rayleigh fading channels, where a half-duplex relay assists the source node. We consider the AF protocol as we propose an all-analog-processing communication system. A power constrained channel optimized vector quantization (PCCOVQ) [2, 4] is considered as our non-parametric analog coding, using low delay 2:1, 3:1, or 4:1 dimension compression. The mappings are optimized for different instantaneous CSNR at the destination. Finally, we compare non-parametric and parametric analog JSCC schemes with an ideal digital system based on scalar quantization. The low delay analog coding schemes, based on either parametric or non parametric mappings, outperform the digital system, while non-parametric mappings achieve the best performance. The advantage of the novel non-parametric schemes over parametric mappings increases with the compression ratio.

The remainder of this paper is organized as follows. Section 2 presents the system model, while Section 3 discusses the non-parametric mapping optimization. Numerical results are discussed in Section 4 and some final comments are given in Section 5.

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2. SYSTEM MODEL

The network topology consists of three nodes: source ($\mathbb{S}$), relay ($\mathbb{R}$) and destination ($\mathbb{D}$). We consider that the direct link between $\mathbb{S}$ and $\mathbb{D}$ is blocked due to the effects of strong shadowing. The source generates discrete-time memoryless Gaussian samples $\mathbf{x}$ with zero mean and variance $\sigma_x^2 = 1$. Then, a group of N source samples are encoded into a single channel symbol s through analog JSCC as detailed in Section 3. We also consider that $\mathbb{E}[|s|^2] = 1$, where $\mathbb{E}[\cdot]$ is the expectation operator. Moreover, we denote the channel fading envelopes by h (from $\mathbb{S}$ to $\mathbb{R}$) and g (from $\mathbb{R}$ to $\mathbb{D}$), which are identically and independent Rayleigh distributed with variance $\sigma_i^2 = 1$, $i \in \{h, g\}$, and are constant during the transmission of s, changing independently from one symbol to another, characteristic thus of a quasi-static fading channel. In addition, similar to previous work [11], it is assumed that the channel state information is available only at $\mathbb{D}$.

In the first hop, the channel symbol s is transmitted through the wireless channel h so that the relay receives

$$y_r = \sqrt{P_s\, d_1^{-\nu}}\, h\, s + w_r, \tag{1}$$

where P_s is the source power, d_1 is the distance between $\mathbb{S}$ and $\mathbb{R}$, ν is the path loss exponent and w_r is the additive Gaussian noise at the relay with zero mean and variance $\sigma_{w_r}^2$. Then, the relay compensates the loss in the $\mathbb{S}$-$\mathbb{R}$ link by amplifying the received signal with a gain

$$\beta = \sqrt{\frac{1}{\mathbb{E}[|y_r|^2]}} = \frac{1}{\sqrt{P_s\, h^2\, d_1^{-\nu} + \sigma_{w_r}^2}}, \tag{2}$$

so that the relay transmitted channel symbol becomes

$$x_r = \sqrt{P_r}\, \beta\, y_r = \frac{\sqrt{P_r P_s\, d_1^{-\nu}}\, h\, s + w_r}{\sqrt{P_s\, h^2\, d_1^{-\nu} + \sigma_{w_r}^2}} \tag{3}$$

where P_r is the relay power. Then, the received signal at the destination can be expressed as

$$y_d = \underbrace{\frac{\sqrt{P_r P_s}\, h\, g\, s}{\sqrt{d_2^\nu (P_s h^2 + d_1^\nu \sigma_{w_r}^2)}}}_{\text{information}} + \underbrace{\frac{\sqrt{P_r d_1^\nu}\, g\, w_r}{\sqrt{d_2^\nu (P_s h^2 + d_1^\nu \sigma_{w_r}^2)}} + w_d}_{\text{overall noise}}, \quad (4)$$

where d_2 is the distance between $\mathbb{R}$ and $\mathbb{D}$, and w_d is the additive Gaussian noise at the destination with zero mean and variance $\sigma_{w_d}^2$. Using (4), the instantaneous CSNR at the destination is given by $\gamma_d = \frac{\mathbb{E}[|\text{information in (4)}|^2]}{\mathbb{E}[|\text{overall noise in (4)}|^2]}$, yielding

$$\gamma_d = \frac{P_r P_s h^2 g^2}{P_r d_1^\nu g^2 \sigma_{w_r}^2 + d_2^\nu (P_s h^2 + d_1^\nu \sigma_{w_r}^2)\sigma_{w_d}^2}. \quad (5)$$

3. ANALOG JOINT SOURCE CHANNEL CODING SCHEMES

We consider N:1 dimension compression analog JSCC[1] where a source vector $\mathbf{x}$ with N source symbols is compressed into a single channel symbol s. Then, an estimate $\hat{\mathbf{x}}$ is obtained at the destination based on the received observation y_d, so that the system performance can be measured in terms of the signal-to-distortion ratio, defined as $\mathrm{SDR} = \frac{\sigma_x^2}{D}$, where $D = \mathbb{E}[\|\mathbf{x} - \hat{\mathbf{x}}\|^2]$ is the distortion measured as the mean square error (MSE) between $\mathbf{x}$ and its estimate $\hat{\mathbf{x}}$.

We analyze the performance of both parametric and non-parametric analog JSCC. Parametric schemes can be represented by simple non-linear functions, while non-parametric mappings are obtained through more complex optimization algorithms in order to reduce the gap to the theoretic limits.

3.1. Parametric Dimension Compression Mappings

3.1.1. 2:1 *mapping*

The 2:1 encoder consists of a mapping function $M_\delta(\cdot)$, a non-linear transform function $T_\varphi(\cdot)$ and a normalization factor $\sqrt{\tau}$. We initially encode $N = 2$ source samples, $\mathbf{x} = (x_1, x_2)$, into a single sample θ by

$$\theta = M_\delta(\mathbf{x}) = \arg\min_\theta ||\mathbf{x} - \mathbf{x}_\theta(\theta)||^2, \quad (6)$$

where $\mathbf{x}_\theta(\cdot)$ is a spiral-like function that maps θ into $\mathbf{x}_\theta = (x_{\theta 1}, x_{\theta 2})$, given by [6]

$$\mathbf{x}_\theta(\theta) = \begin{bmatrix} \mathrm{sign}(\theta)\frac{\delta}{\pi}\theta \sin\theta \\ \frac{\delta}{\pi}\theta\cos\theta \end{bmatrix} \text{ for } \theta \in \mathbb{R}, \quad (7)$$

and δ represents the distance between the two neighboring arms of the spiral.

We also employ a matching function $T_\varphi(\theta) = \mathrm{sign}(\theta)\,|\theta|^\varphi$ in order to improve the SDR performance by numerically optimizing the shape parameter φ along with δ [7]. Moreover, the normalization factor $\sqrt{\tau}$ is used to ensure that $\mathbb{E}[|s|^2] = \mathbb{E}[|\frac{T_\varphi(M_\delta(\mathbf{x}))}{\sqrt{\tau}}|^2] = 1$ before transmission.

At the decoder, we perform maximum likelihood (ML) demapping, preeceded by linear minimum MSE (MMSE) estimation of the channel input, a two-stage decoding scheme that in many environments achieves near MMSE end-to-end performance with very low complexity [14]. Therefore, the proposed decoder consists of the de-normalization of $\sqrt{\tau}$, a linear MMSE estimator, the inverse transform function $T_\varphi^{-1}(\cdot)$ and the inverse mapping function $M_\delta^{-1}(\cdot)$. The MMSE estimate of s is obtained as $\hat{s} = \mathbb{E}[s|y_d] = R_{sy_d} R_{y_d y_d}^{-1} y_d$ [7], where R_{sy_d} is the cross-correlation between s and y_d and $R_{y_d y_d}$ is the auto-correlation of y_d, which results in

$$\hat{s} = \frac{\sqrt{P_r P_s} h g d_2^\nu (P_s h^2 + d_1^\nu \sigma_{w_r}^2)}{P_r P_s h^2 g^2 + P_r d_1^\nu g^2 \sigma_{w_r}^4 + \sigma_{w_d}^2 d_2^\nu (P_s h^2 + d_1^\nu \sigma_{w_r}^2)} y_d. \quad (8)$$

Then, the decoded symbol is given by $\hat{\theta} = T_\varphi^{-1}(\hat{s}) = \mathrm{sign}(\hat{s})\,|\hat{s}|^{\frac{1}{\varphi}}$ and $\hat{\mathbf{x}}$ is finally obtained by simply applying $\hat{\theta}$ in (7), so that $\hat{\mathbf{x}} = M_\delta^{-1}(\hat{\theta}) = \mathbf{x}_\theta(\hat{\theta})$.

3.1.2. 3:1 *and* 4:1 *mappings*

These schemes were initially proposed in [6] and have a structure similar to the 2:1 mapping, but using other particular mapping functions. For the sake of brevity, we skip the details and we refer the reader to [6] for detailed information.

3.2. Non-Parametric Dimension Compression Mappings

In order to improve the SDR performance of the analog JSCC mapping schemes, we employ a PCCOVQ method, initially proposed in [2] for the AWGN channel and extended in [4] to Rayleigh fading channels. Here, we further extend the technique to a two-hop network scenario

In the PCCOVQ encoder, the N-dimensional source space is split into Q partitions, $\mathbf{p} = \{\Omega_0, \Omega_1, \ldots, \Omega_{Q-1}\}$. Therefore, if $\mathbf{x} \in \Omega_i$, the encoder assigns it to the discrete index i. Then, each index i corresponds to a specific pulse amplitude modulation (PAM) channel symbol, given by $s = \Delta u_i$, where u_i is a unit distance PAM signal in the one-dimensional channel space, and Δ is the constant distance between two neighboring channel symbols. The Q-PAM symbol s is transmitted through the two-hop network according to (4). At the receiver, the ML estimate of s is $\hat{s} = s + \frac{\sqrt{d_1^\nu}\, w_r}{\sqrt{P_s}\, h} + \frac{\sqrt{d_2^\nu (P_s h^2 + d_1^\nu \sigma_{w_r}^2)}\, w_d}{\sqrt{P_r P_s}\, hg}$.

Instead of employing only one reconstruction codebook optimized for all instantaneous CSNR γ_d at the destination, we follow [4, 8] and discretize γ_d into H uniform discrete values, given by $\gamma_{d,k}$. Then, we optimize a different codebook $\mathbf{b}_k \in \mathbf{a} = \{\mathbf{b}_0, \mathbf{b}_1, \ldots, \mathbf{b}_{H-1}\}$ for each $\gamma_{d,k}$. Thus, the number of optimized mappings is also H. Given a particular instantaneous CSNR, the decoder searches for the closest discrete CSNR $\gamma_{d,k}$ and chooses its respective optimized reconstruction codebook $\mathbf{b}_k$. Then, the decoder chooses the index j which minimizes $\|\hat{s} - \Delta u_j\|^2$. Finally, with k and j we can obtain $\hat{\mathbf{x}} = \mathbf{c}_{k,j}$, where $\mathbf{c}_{k,j}$ is the reconstruction vector from $\mathbf{b}_k = \{\mathbf{c}_{k,0}, \mathbf{c}_{k,1}, \ldots, \mathbf{c}_{k,Q-1}\}$.

3.2.1. *Simplified two-hop CSNR pdf*

As discussed in [2], the aim of the PCCOVQ algorithm is to find a vector quantizer that minimizes the distortion with a power constraint using the Lagrange multiplier λ, which is used to control the average source power P_s. However, we notice that in order to apply the PCCOVQ algorithm, we need to consider discretized Rayleigh fadings for each hop (with respect to h and g) as well as their respective pdfs. Then, in order to reduce the algorithm computations, we follow [12] and resort to a high CSNR approximation of the AF gain

[1]Dimension expansion analog JSCC is not considered here as its performance over Rayleigh fading channels may be inferor [5].

by neglecting the noise variance $\sigma_{w_r}^2$ in (2), so that $\tilde{\beta} = \frac{1}{\sqrt{P_s h^2 d_1^{-\nu}}}$. With this simplification, the pdf of the CSNR in (5) can be expressed as [12]

$$f_\gamma(\gamma) = \frac{2\gamma\, e^{-\gamma\left(\frac{1}{\overline{\gamma}_1}+\frac{1}{\overline{\gamma}_2}\right)}}{\overline{\gamma}_1\overline{\gamma}_2}\left[\left(\frac{\overline{\gamma}_1+\overline{\gamma}_2}{\sqrt{\overline{\gamma}_1\overline{\gamma}_2}}\right)K_1\left(\frac{2\gamma}{\sqrt{\overline{\gamma}_1\overline{\gamma}_2}}\right)\right.$$
$$\left. +2K_0\left(\frac{2\gamma}{\sqrt{\overline{\gamma}_1\overline{\gamma}_2}}\right)\right], \quad (9)$$

where $\overline{\gamma}_1 = \frac{P_s\sigma_h^2}{\sigma_{w_r}^2 d_1^\nu}$ and $\overline{\gamma}_2 = \frac{P_r\sigma_g^2}{\sigma_{w_d}^2 d_2^\nu}$ are the per hop average CSNR, and $K_0(\cdot)$ and $K_1(\cdot)$ are the zero-order and first order modified Bessel function of the second kind [15], respectively.

Then, we consider that both source and relay use the same power ($P_r = P_s$) and, similar to [4], we employ a uniform quantizer to obtain the discrete CSNR values $\gamma_{d,k}$ and their respective probabilities $p_{\gamma_d}(k)$ with the simplified pdf in (9). Thus, the optimization problem can be expressed as

$$\min_{\{\mathbf{p},\mathbf{a},\Delta\}} = [D(\mathbf{p},\mathbf{a},\Delta) + \lambda P_s], \quad (10)$$

where

$$D(\mathbf{p},\mathbf{a},\Delta) = \frac{\mathbb{E}[\|\mathbf{x}-\hat{\mathbf{x}}\|^2]}{N} = \sum_{i=0}^{Q-1}\int_{\Omega_i}\mathcal{D}_i(\mathbf{x})f_x(\mathbf{x})\mathrm{d}\mathbf{x} \quad (11)$$

is the average total distortion per source vector $\mathbf{x}$, where $f_x(\mathbf{x})$ is the pdf of $\mathbf{x}$ and

$$\mathcal{D}_i(\mathbf{x}) = \frac{1}{N}\sum_{j=0}^{Q-1}\sum_{k=0}^{H-1}p_{\gamma_d}(k)p(j|i,k)\|\mathbf{x}-\mathbf{c}_{k,j}\|^2 \quad (12)$$

is the distortion of $\mathbf{x}$ associated with the partition i, where $p_{\gamma_d}(k)$ is the probability of the discretized CSNR $\gamma_{d,k}$, and $p(j|i,k)$ is the probability of receiving index j given that i was transmitted with a CSNR $\gamma_{d,k}$. Finally, P_s in (10) is the power per channel symbol, with $P_s = \Delta^2\sum_{i=0}^{Q-1}\|u_i\|^2\int_{\Omega_i}f_x(\mathbf{x})\mathrm{d}\mathbf{x}$.

3.2.2. *Optimization algorithm*

The minimization of (10) can be achieved by a modified generalized Lloyd algorithm [16], which consists of four steps.

i.) Optimize the partitioning $\mathbf{p}$: Considering an initial λ and an initial codebook $\mathbf{a}$ (details in Section 3.2.3), the optimal partition that minimizes $\mathbf{p}$ in (10) is

$$\Omega_i = \{\mathbf{x}|\mathcal{G}_i(\mathbf{x}) \le \mathcal{G}_j(\mathbf{x}), \forall j\in\mathbf{i}\}, \ \ i\in\mathbf{i}, \quad (13)$$

where $\mathbf{i} = \{0,1,\ldots,Q-1\}$ is the index set and $\mathcal{G}_i(\mathbf{x}) = \mathcal{D}_i(\mathbf{x}) + \lambda\Delta^2\|u_i\|^2$ is the distortion cost function.

ii.) Optimize the codebook set $\mathbf{a}$: Given Δ and the updated partition $\mathbf{p}$, the optimal codebook set $\mathbf{a}$ is formed by elements given by [4]

$$\mathbf{c}_{k,j} = \frac{\sum_{i=0}^{Q-1}p(j|i,k)\int_{\Omega_i}\mathbf{x}f_x(\mathbf{x})\mathrm{d}\mathbf{x}}{\sum_{i=0}^{Q-1}p(j|i,k)\int_{\Omega_i}f_x(\mathbf{x})\mathrm{d}\mathbf{x}}, \ \ j\in\mathbf{i}. \quad (14)$$

iii.) Optimize Δ: The parameter Δ is numerically optimized through an unconstrained nonlinear iterative method[2] which minimizes (10).

iv.) Repeat or stop: If the difference between the cost function in (10) at the current and at the previous iteration is smaller than a convergence criterion ϵ, then stop. Otherwise, repeat all four steps again.

3.2.3. *Implementation considerations*

In order to efficiently implement the algorithm, we follow a procedure known as noisy channel relaxation [2], so that the system is initially optimized for a very low $\overline{\gamma}_d$ by setting a high value for λ and an initial set of ramp mappings. After the optimization of the first codebooks for the initial value of λ, we reduce λ in order to obtain new mappings optimized for a higher $\overline{\gamma}_d$. As P_s and P_r increase when λ is reduced, both source and relay noise variances are fixed at $\sigma_i^2 = 1$, $i\in\{w_r, w_d\}$.

3.2.4. $H=1$ *optimized mapping*

In order to have a simplified version of the non-parametric mapping scheme, we consider the case with a single codebook $\mathbf{b} = \{\mathbf{c}_0,\mathbf{c}_1,\ldots,\mathbf{c}_{Q-1}\}$ taking into account all the G elements in a set of instantaneous CSNR $\gamma_{d,k}$ for each $\overline{\gamma}_d$. The same optimization algorithm can be used by just replacing (12) by

$$\mathcal{D}_i(\mathbf{x}) = \frac{1}{N}\sum_{j=0}^{Q-1}\sum_{k=0}^{G-1}p_{\gamma_d}(k)p(j|i,k)\|\mathbf{x}-\mathbf{c}_j\|^2, \quad (15)$$

while the optimal codebook in (14) simplifies to

$$\mathbf{c}_j = \sum_{k=0}^{G-1}p_{\gamma_d}(k)\frac{\sum_{i=0}^{Q-1}p(j|i,k)\int_{\Omega_i}\mathbf{x}f_x(\mathbf{x})\mathrm{d}\mathbf{x}}{\sum_{i=0}^{Q-1}p(j|i,k)\int_{\Omega_i}f_x(\mathbf{x})\mathrm{d}\mathbf{x}}, \ \ j\in\mathbf{i}. \quad (16)$$

3.2.5. *Optimized mappings without the simplified CSNR pdf*

If the simplified two-hop CSNR pdf in (9) is not considered, then in the mapping optimization we have to consider H discretized Rayleigh fadings for each hop, as well as their respective probabilities. As a result, the algorithm needs to handle H^2 discrete instantaneous CSNR values. Many of these H^2 discrete CSNR values are very similar, which is highly inefficient when compared to the proposed scheme with uniformly distributed discrete CSNR values. The distortion of $\mathbf{x}$ associated with partition i would be

$$\mathcal{D}_i(\mathbf{x}) = \frac{1}{N}\sum_{j=0}^{Q-1}\sum_{l=0}^{H-1}\sum_{m=0}^{H-1}p_h(l)p_g(m)p(j|i,l,m)\|\mathbf{x}-\mathbf{c}_{l,m,j}\|^2 \quad (17)$$

and the optimal codebook is given by

$$\mathbf{c}_{l,m,j} = \frac{\sum_{i=0}^{Q-1}p(j|i,l,m)\int_{\Omega_i}\mathbf{x}f_x(\mathbf{x})\mathrm{d}\mathbf{x}}{\sum_{i=0}^{Q-1}p(j|i,l,m)\int_{\Omega_i}f_x(\mathbf{x})\mathrm{d}\mathbf{x}}, \ \ j\in\mathbf{i}. \quad (18)$$

3.3. Optimal Performance Theoretically Attainable (OPTA)

The OPTA equates the rate distortion function to the channel capacity [17], which in the case of the two-hop channel with AF relaying must take into account all possible realizations for both h and g. Thus,

$$N\log_{10}\left(\frac{\sigma_x^2}{D}\right) = \int_{\boldsymbol{h}}\int_{\boldsymbol{g}}\log_{10}(1+\gamma_d)\,f_{h^2}(\boldsymbol{h})f_{g^2}(\boldsymbol{g})\mathrm{d}\boldsymbol{g}\mathrm{d}\boldsymbol{h}, \quad (19)$$

where $f_{h^2}(\boldsymbol{h})$ and $f_{g^2}(\boldsymbol{g})$ are exponentially distributed.

[2] A readily implemented option the for unconstrained nonlinear optimization, used throughout the numerical results of this paper, is the `fminsearch` function in Matlab®.

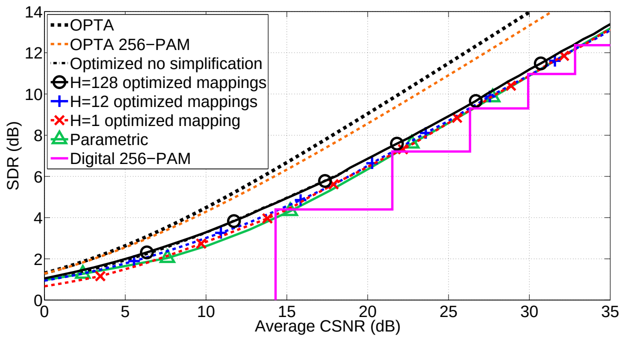

Fig. 1. SDR performance *vs.* average CSNR at destination ($\overline{\gamma}_d$) of the 2:1 analog coding schemes over AF two-hop relaying.

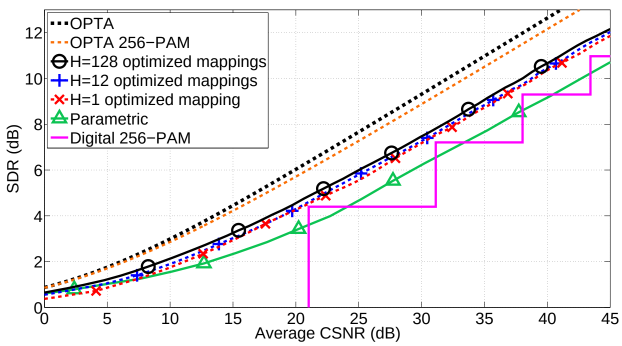

Fig. 2. SDR performance *vs.* average CSNR at destination ($\overline{\gamma}_d$) of the 3:1 analog coding schemes over AF two-hop relaying.

3.4. Analog versus Digital Coding

Similar to [1, 10, 11], we compare the considered analog JSCC schemes with an ideal digital M-PAM system which includes an optimum q-level non-uniform scalar quantizer [18] that maps the discrete-time memoryless Gaussian analog source vector $\mathbf{x}$ into a discrete set of values. After quantization, we assume an ideal digital source encoder where the average codeword length is equal to the source entropy. Finally, after ideal (capacity achieving) channel coding, the encoded bits are modulated in a one-dimensional 256-PAM constellation with Gray mapping. Please see [1, 8, 11] for additional details to calculate the OPTA of the ideal digital system, which is an optimistic bound on the performance of digital systems based on scalar quantization.

4. SIMULATION RESULTS

In this section we numerically evaluate the SDR performance of the analog JSCC scheme as a function of the average CSNR at the destination, $\overline{\gamma}_d$, comparing it with the OPTA and with the ideal digital communication system defined before. In the case of the non-parametric schemes, we consider 10^4 Gaussian source samples, $Q = 256$ mapping samples and $\epsilon = 10^{-4}$. We also assume that $P_r = P_s$, $\nu = 2$, $\sigma^2_{w_r} = \sigma^2_{w_d} = 1$, and $d_1 = d_2 = 1$.

In Figure 1, we can observe that the gaps to the OPTA at $\overline{\gamma}_d = 10$ dB for the 2:1 non-parametric mapping with $H = \{128, 12, 1\}$ and for the 2:1 parametric scheme are, respectively, 1.2 dB, 1.5 dB, 1.7 dB and 1.9 dB. For high $\overline{\gamma}_d$, the performance of all analog schemes are similar, with the non-parametric scheme with $H = 128$ improving only 0.3 dB in terms of SDR with respect to the case of $H = 1$. The 3:1 and 4:1 systems based on non-parametric mappings with $H = 128$ also present a gain of around 0.3 dB with respect to the $H = 1$ case, as shown in Figures 2 and 3. It is relevant, however, that for these compression ratios the improvement obtained by using non-parametric mappings rather that parametric ones is quite substantial, much more significant that for the 2:1 case.

We also compare the non-parametric schemes with and without the CSNR pdf simplification, considering $H = 128$ for the case with the simplified pdf and $H^2 = 400$ for the case without the simplification. As show in Figure 1, the SDR performance in both cases is almost the same, but it is important to remark that with the proposed simplification the computational complexity for mapping optimization is greatly reduced.

Notice that, as shown in Figures 1 to 3, for all the compression ratios the non-parametric mappings outperform the ideal digital system. This occurs even though the analog coding scheme has nearly zero delay, while the ideal digital system has, in theory, an infinite delay. A practical digital system, with a finite delay, would achieve even worse performance.

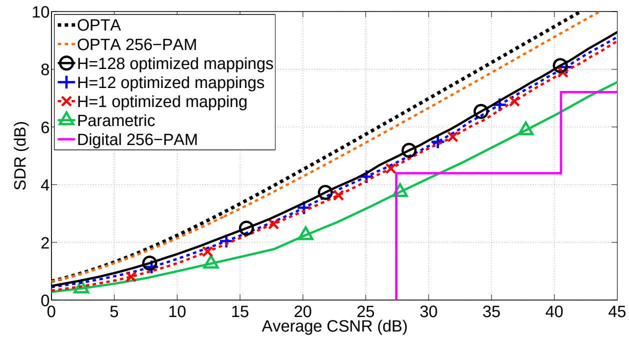

Fig. 3. SDR performance *vs.* average CSNR at destination ($\overline{\gamma}_d$) of the 4:1 analog coding schemes over AF two-hop relaying.

5. CONCLUSION

We have presented a low delay discrete-time non-parametric analog JSCC scheme for the AF two-hop relaying network with Rayleigh fading channels. Simulation results have shown that the novel non-parametric schemes outperform the existing parametric ones, with the performance advantage increasing with the compression ratio. Moreover, the systems based on low delay non-parametric mappings easily outperform a fully ideal digital system based on scalar quantization and infinite block length. Practical source and channel digital encoders would perform even worse than the considered ideal scheme, and they would still experience relatively long delays, while the low delay analog schemes operate on a sample-by-sample basis.

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