JOINT TRANSMIT BEAMFORMING OPTIMIZATION AND UPLINK/DOWNLINK USER SELECTION IN A FULL-DUPLEX MULTI-USER MIMO SYSTEM

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ABSTRACT

This paper considers practical deployment issues of a multi-user MIMO system with full-duplex (FD) base station and half-duplex (HD) user equipment. The aim is to select a set of uplink (UL) and downlink (DL) users at any instant that will provide a satisfactory performance in system resource allocation. Furthermore, it is also necessary to deal with the interference created by the UL users to the DL users, which limits communication quality. In this work, we consider implementing a joint processing beamforming algorithm that can provide effective UL/DL selection and achieve system utility maximization. Our results show that with 20 dB self-interference cancellation, FD system significantly outperforms HD system under proportional fairness utility.

Index Terms— Full-duplex communication, joint processing, MIMO, beamforming.

1. INTRODUCTION

Full-duplex (FD) data transmission is an emerging technique in wireless communication that can potentially double the throughput compared with conventional half-duplex (HD) system by transmitting and receiving information simultaneously in the same time slot and frequency band. However, the most critical issue in FD system is its own transmission interfering with the device itself, thereby creating the so-called self-interference (SI). Due to hardware impairment such as amplifier nonlinearity, phase noise, and quantization error, SI cancellation (SIC) in practice is far from trivial. For single antenna devices, [1–6] showed that an SIC of 100 dB can be achieved with the use of additional hardware, such as FIR filters. Additionally, [7] proposed that SI can be avoided by transmitting signals in the null space of the SI channel in multiple antennas devices.

Recently, given the growth in the demand of both mobile uplink (UL) and downlink (DL) data traffic, increased attention has been paid to using signal processing techniques such as beamforming to unlock the capability of FD multi-user multi-input multi-output (MU-MIMO) systems, where multiple UL and DL user equipment (UEs) are served by an FD base station (BS) simultaneously [8–10]. In fact, FD systems are considered to be suitable for deployment in small cell systems which have low SIC requirement because of the small operating power and short transmission distance. However, the interference from the UL UE to the DL UE (UL-DL interference) within their transmission range is the most critical factor that limits system-level performance.

While some works have focused on sum rate maximization [8–10], it may not be adequate for practical systems as they usually have stringent requirements on the data rates, e.g., user fairness. Moreover, most of these studies assume the use of FD UE, which allows a UE to have a flexible choice in transmitting and receiving data. To avoid the use of expensive FD UE, some researchers have also considered the use of HD UE and have assumed a set of specific UL/DL users being selected a priori by the systems. However, UL/DL user selection is a nontrivial task, especially when resource allocation and UL-DL interference are taken into account.

Motivated by the above practical considerations, the presented work focuses on a joint processing beamforming algorithm which considers the problem of system utility maximization, UL-DL interference management and UL/DL user selection together. An iterative weighted minimum mean square error (WMMSE) algorithm [11, 12] has been used to handle the nonconvex utility maximization problem. Inspired by [13,14], we propose use the idea of user grouping to handle the UL-DL interference and integer programming to deal with the UL/DL user selection problem. The effectiveness of the proposed algorithm is verified by numerical simulations.

Notations: We use $\mathbf{X}^T$, $\mathbf{X}^H$ and $\|\mathbf{X}\|_F$ to denotes the transpose, Hermitian and Frobenius norm of matrix $\mathbf{X}$, and diag$(\mathbf{X})$ creates a diagonal matrix by setting the off-diagonal entries of matrix $\mathbf{X}$ to 0. $\perp\!\!\!\perp$ stand for statistical independence. $\mathcal{CN}(\mathbf{0}, \mathbf{X})$ represents the zero mean circularly symmetric complex Gaussian distribution with covariance $\mathbf{X}$ and Cov$\{\mathbf{x}\} = \mathbb{E}\{\mathbf{x}\mathbf{x}^H\}$ means the covariance of vector $\mathbf{x}$.

2. SYSTEM MODEL

Consider an MU-MIMO system where an FD BS communicates with K UEs. Users are divided into G groups, and different groups are served in different orthogonal time slots (i.e., TDMA) so that users in one group do not interfere with users in other groups. The proportion of time resources allocated to the users in group $g = 1, ..., G$ is denoted as τ^g, with $\sum_{g=1}^{G} \tau^g = 1$. We use index $i = 0$ to denote the BS and $i = 1, \ldots, K$ to denote the UEs. The channel from node i to i' is denoted as $\boldsymbol{H}_{i,i'} \in \mathbb{C}^{N_{r_{i'}} \times N_{t_i}}$, where N_{t_i} and N_{r_i} are the number of transmit and receive antennas at node i, respectively. Therefore, $\boldsymbol{H}_{i,i} \in \mathbb{C}^{N_{r_i} \times N_{t_i}}$ is the SI channel. We use similar notations to denote the beamformer and data streams, i.e., $\boldsymbol{V}^g_{i,i'} \in \mathbb{C}^{N_{t_i} \times d_{i,i'}}$ and $\boldsymbol{s}^g_{i,i'} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{I}) \in \mathbb{C}^{d_{i,i'} \times 1}$, which are the beamformer and data streams from node i to i' in group g, respectively, and $d_{i,i'}$ is the number of data streams intended to be transmitted from node i to node i'.

The intended DL and UL transmitting signals of the BS and UE $k = 1, ..., K$ in group g are given, respectively, as

$$\boldsymbol{x}^g_0 = \sum_{k'=1}^{K} \boldsymbol{V}^g_{0,k'} \boldsymbol{s}^g_{0,k'}, \qquad \boldsymbol{x}^g_k = \boldsymbol{V}^g_{k,0} \boldsymbol{s}^g_{k,0}. \tag{1}$$

In fact, there is a mismatch between the intended transmitting signal and the actual radiated signal due to hardware impairment. Similarly, the received signal is also distorted after it is digitized. According to [9,15,16], the mismatch and distortion can be modeled as an additive distortion noise at the transceiver. Let $\boldsymbol{u}_i^g$ be the undistorted received signal at node i in group g, $\boldsymbol{c}_i^g$ and $\boldsymbol{e}_i^g$ be the transmitter and receiver distortion noise with respect to $\boldsymbol{x}_i^g$ and $\boldsymbol{u}_i^g$, respectively. The signal received at the BS in group g is

$$\overline{\boldsymbol{y}}_0^g = \underbrace{\sum_{k'=1}^{K} \boldsymbol{H}_{k',0}(\boldsymbol{x}_{k'}^g + \boldsymbol{c}_{k'}^g) + \boldsymbol{H}_{0,0}(\boldsymbol{x}_0^g + \boldsymbol{c}_0^g) + \boldsymbol{\nu}_0}_{\boldsymbol{u}_0^g} + \boldsymbol{e}_0^g, \quad (2)$$

where $\boldsymbol{u}_0^g$ consists of the UL signal from the UE, SI signal, and additive white Gaussian noise (AWGN) vector ($\boldsymbol{\nu}_0$) with covariance $\sigma_0 \boldsymbol{I}$. Similarly, the signal received at UE k in group g is given by

$$\overline{\boldsymbol{y}}_k^g = \underbrace{\boldsymbol{H}_{0,k}(\boldsymbol{x}_0^g + \boldsymbol{c}_0^g) + \sum_{k'=1}^{K} \boldsymbol{H}_{k',k}(\boldsymbol{x}_{k'}^g + \boldsymbol{c}_{k'}^g) + \boldsymbol{\nu}_k}_{\boldsymbol{u}_k^g} + \boldsymbol{e}_k^g. \quad (3)$$

The first term in (3) is the DL signal from the BS, the second term contains the SI signal ($k' = k$) and UL-DL interference signal ($k' \neq k$), and the last term is the AWGN vector. For HD UE, $\boldsymbol{H}_{k,k}$ is zero (i.e., $\boldsymbol{H}_{k,k} = \boldsymbol{0}$) and either $\boldsymbol{V}_{0,k}^g$ or $\boldsymbol{V}_{k,0}^g$ is zero.

Assuming proper compensation and calibration at the transceiver, the residual transmitter and receiver hardware impairments can be approximated as an independent zero mean AWGN with covariance depending on the signal power [9, 15, 16]. i.e.,

$$\boldsymbol{c}_i^g \perp\!\!\!\perp \boldsymbol{x}_i^g, \quad \boldsymbol{c}_i^g \sim \mathcal{CN}(\boldsymbol{0}, \kappa \mathrm{diag}(\mathrm{Cov}\{\boldsymbol{x}_i^g\})), \quad (4)$$

$$\boldsymbol{e}_i^g \perp\!\!\!\perp \boldsymbol{u}_i^g, \quad \boldsymbol{e}_i^g \sim \mathcal{CN}(\boldsymbol{0}, \beta \mathrm{diag}(\mathrm{Cov}\{\boldsymbol{u}_i^g\})), \quad (5)$$

where $\kappa, \beta \ll 1$ is the transmit and receive *error vector magnitude* within the range [-22 dB, -15 dB] according to the 3GPP LTE standard [17].

Although perfect channel state information (CSI) is usually unavailable, we aim at finding an achievable upper bound of performance by assuming that perfect CSI is available throughout this paper. Given that node i knows the intended transmitting signals $\boldsymbol{x}_i^g$, the SI term $\boldsymbol{H}_{i,i}\boldsymbol{x}_i^g$ can be cancelled. Hence, the residual received signal of node i in group g is

$$\boldsymbol{y}_i^g = \overline{\boldsymbol{y}}_i^g - \boldsymbol{H}_{i,i}\boldsymbol{x}_i^g \quad (6)$$

Assuming that the decoder treats the interference as noise, the DL and UL rates of UE k in group g are

$$R_{0,k}^g = \log\det(\boldsymbol{Q}_{0,k}^g), \quad R_{k,0}^g = \log\det(\boldsymbol{Q}_{k,0}^g), \quad (7)$$

respectively, where

$$\boldsymbol{Q}_{i,i'}^g \triangleq \boldsymbol{I} + (\boldsymbol{V}_{i,i'}^g)^H \boldsymbol{H}_{i,i'}^H (\boldsymbol{J}_{i'}^g - \boldsymbol{H}_{i,i'}\boldsymbol{V}_{i,i'}^g (\boldsymbol{V}_{i,i'}^g)^H \boldsymbol{H}_{i,i'}^H)^{-1} \boldsymbol{H}_{i,i'}\boldsymbol{V}_{i,i'}^g, \quad (8)$$

with $\boldsymbol{J}_i^g$ being the covariance of $\boldsymbol{y}_i^g$

$$\boldsymbol{J}_i^g = \mathrm{Cov}\{\boldsymbol{u}_i^g\} + \beta \mathrm{diag}(\mathrm{Cov}\{\boldsymbol{u}_i^g\}) - \boldsymbol{H}_{i,i}\boldsymbol{\mathcal{V}}_i^g \boldsymbol{H}_{i,i}^H. \quad (9)$$

Here, $\boldsymbol{\mathcal{V}}_i^g = \sum_{k'=1}^{K} \boldsymbol{V}_{0,k'}^g (\boldsymbol{V}_{0,k'}^g)^H$ for $i = 0$, and $\boldsymbol{\mathcal{V}}_i^g = \boldsymbol{V}_{i,0}^g (\boldsymbol{V}_{i,0}^g)^H$ for $i = 1, ..., K$. $\mathrm{Cov}\{\boldsymbol{u}_i^g\}$ is the covariance of the undistorted received signal of $\boldsymbol{u}_i^g$, and is given by

$$\mathrm{Cov}\{\boldsymbol{u}_i^g\} = \sum_{i'=0}^{K} \boldsymbol{H}_{i',i}\Big(\boldsymbol{\mathcal{V}}_{i'}^g + \kappa \mathrm{diag}(\boldsymbol{\mathcal{V}}_{i'}^g)\Big)\boldsymbol{H}_{i',i}^H + \sigma_i^2 \boldsymbol{I}. \quad (10)$$

3. PROBLEM FORMULATION

Define $\mathcal{U}_{0,k}(\cdot)$ and $\mathcal{U}_{k,0}(\cdot)$ as a general utility function for DL and UL achievable rate of UE k, respectively, P_i be the power budget of node i, and $\alpha_{0,k}^g, \alpha_{k,0}^g \in \{0,1\}, \forall g, k$ as two binary assignment variables. If $\alpha_{0,k}^g = 1$, UE k in group g operates in downlink transmission mode. Similarly, if $\alpha_{k,0}^g = 1$, UE k in group g operates in uplink transmission mode. Therefore, we have $\alpha_{0,k}^g + \alpha_{k,0}^g \leq 1$ if UE k is an HD device. Since handling problems with discrete variables, the 0/1 restriction is relaxed to $\alpha_{0,k}^g \geq 0$ and $\alpha_{k,0}^g \geq 0$. The utility maximization problem is formulated as follows

$$\max_{\boldsymbol{V},\boldsymbol{\tau},\boldsymbol{\alpha}} \sum_{k=1}^{K} \mathcal{U}_{0,k}\Big(\sum_{g=1}^{G} \tau^g \alpha_{0,k}^g R_{0,k}^g\Big) + \mathcal{U}_{k,0}\Big(\sum_{g=1}^{G} \tau^g \alpha_{k,0}^g R_{k,0}^g\Big) \quad \text{(P)}$$

$$\text{subject to} \begin{cases} \sum_{k=1}^{K} \|\boldsymbol{V}_{0,k}^g\|_F^2 \leq P_0 \text{ and } \|\boldsymbol{V}_{k,0}^g\|_F^2 \leq P_k, \forall k, g, & \text{(C1)} \\ \sum_{g=1}^{G} \tau^g = 1 \text{ and } \tau^g \geq 0, \forall g, & \text{(C2)} \\ \alpha_{0,k}^g + \alpha_{k,0}^g \leq 1, \forall k, g, & \text{(C3)} \\ \alpha_{0,k} \geq 0 \text{ and } \alpha_{k,0} \geq 0, \forall k, g. & \text{(C4)} \end{cases}$$

Generally, utility maximization is known to be nonconvex. A stationary point can be computed by the WMMSE approach [11–14], which can decompose the problem into subproblems that can be easily optimized in an alternating optimization (AO) fashion.

Theorem 1 *Given τ^g and $\alpha_{i,i'}^g$, define*

$$\mathcal{C}_{i,i'}(\cdot) \triangleq -\mathcal{U}_{i,i'}\Big(-\sum_{g=1}^{G} \tau^g \alpha_{i,i'}^g \mathrm{logdet}(\cdot)\Big). \quad (11)$$

Also, assume that $\mathcal{U}_{i,i'}(\cdot)$ is a strictly increasing function. Thus, a well-defined inverse mapping of $\nabla \mathcal{C}_{i,i'}(\cdot)$, denoted as $\boldsymbol{\gamma}_{i,i'}(\cdot)$, exists, such that

$$\mathcal{U}_{i,i'}(R_{i,i'}) = -\min_{\substack{\boldsymbol{U}_{i,i'} \\ \boldsymbol{W}_{i,i'}}} \sum_{g=1}^{G} \mathrm{tr}(\boldsymbol{E}_{i,i'}^g \boldsymbol{W}_{i,i'}^g) + \mathcal{C}_{i,i'}(\boldsymbol{\gamma}_{i,i'}(\boldsymbol{W}_{i,i'}^g)) - \mathrm{tr}(\boldsymbol{W}_{i,i'}^g \boldsymbol{\gamma}_{i,i'}(\boldsymbol{W}_{i,i'}^g)), \quad (12)$$

where $\boldsymbol{E}_{i,i'}^g = \boldsymbol{I} - 2(\boldsymbol{U}_{i,i'}^g)^H \boldsymbol{H}_{i,i'} \boldsymbol{V}_{i,i'}^g + (\boldsymbol{U}_{i,i'}^g)^H \boldsymbol{J}_{i'}^g \boldsymbol{U}_{i,i'}^g$ and $\boldsymbol{W}_{i,i'}^g$ is positive semidefinite for all i, $i'$and g. The optimal $\boldsymbol{U}_{i,i'}^g$ and $\boldsymbol{W}_{i,i'}^g$ is given by

$$(\boldsymbol{U}_{i,i'}^g)^\star = (\boldsymbol{J}_{i'}^g)^{-1} \boldsymbol{H}_{i,i'} \boldsymbol{V}_{i,i'}^g, \quad (13a)$$

$$(\boldsymbol{W}_{i,i'}^g)^\star = \eta_{i,i'} \tau^g \alpha_{i,i'i}^g (\boldsymbol{I} - (\boldsymbol{U}_{i,i'}^g)^H \boldsymbol{H}_{i,i'} \boldsymbol{V}_{i,i'}^g)^{-1}, \quad (13b)$$

where $\eta_{i,i'} = \nabla \mathcal{U}_{i,i'}(\cdot)$.

The proof of Theorem 1 is similar to the proof in [12,14]. Hence, a WMMSE reformulation of problem (P) is

$$\min_{\substack{\boldsymbol{V},\boldsymbol{U},\boldsymbol{W} \\ \boldsymbol{\alpha},\boldsymbol{\tau}}} \sum_{k=1}^{K} \sum_{g=1}^{G} \left\{ \begin{array}{l} \mathrm{tr}(\boldsymbol{E}_{0,k}^g \boldsymbol{W}_{0,k}^g) + \mathrm{tr}(\boldsymbol{E}_{k,0}^g \boldsymbol{W}_{k,0}^g) \\ + \mathcal{C}_{0,k}(\boldsymbol{\gamma}_{0,k}(\boldsymbol{W}_{0,k}^g)) - \mathrm{tr}(\boldsymbol{W}_{0,k}^g \boldsymbol{\gamma}_{0,k}(\boldsymbol{W}_{0,k}^g)) \\ + \mathcal{C}_{k,0}(\boldsymbol{\gamma}_{k,0}(\boldsymbol{W}_{k,0}^g)) - \mathrm{tr}(\boldsymbol{W}_{k,0}^g \boldsymbol{\gamma}_{k,0}(\boldsymbol{W}_{k,0}^g)) \end{array} \right\} \quad \text{(P-WMMSE)}$$

subject to (C1)-(C4) being satisfied.

To apply AO, we first initialize $\alpha_{i,i'}^g = \frac{1}{2}, \tau^g = \frac{1}{G}$ and randomly generate $\boldsymbol{V}$ such that (C1) is satisfied. Then, $\{\boldsymbol{U}, \boldsymbol{W}\}$ is updated using (13). With $\{\boldsymbol{U}^\star, \boldsymbol{W}^\star\}$, and by the first order optimality condition, the closed-form update of $\boldsymbol{V}$ is

$$(\boldsymbol{V}_{i,i'}^g)^\star = (\boldsymbol{S}_i^g + \lambda_i^g \boldsymbol{I})^{-1} \boldsymbol{H}_{i,i'}^H \boldsymbol{U}_{i,i'}^g \boldsymbol{W}_{i,i'}^g, \tag{14}$$

where λ_i^g is the Lagrangian multiplier for node i obtained by bisection search on (14) such that (C1) is satisfied, and $\boldsymbol{S}_i^g$ is

$$\begin{aligned}\boldsymbol{S}_i^g \triangleq \sum_{k=1}^K \Big(& \overline{\boldsymbol{S}}_{i,k}\big(\boldsymbol{U}_{0,k}^g \boldsymbol{W}_{0,k}^g (\boldsymbol{U}_{0,k}^g)^H\big) \\ & + \overline{\boldsymbol{S}}_{i,0}\big(\boldsymbol{U}_{k,0}^g \boldsymbol{W}_{k,0}^g (\boldsymbol{U}_{k,0}^g)^H\big)\Big) - \boldsymbol{H}_{i,i}^H \boldsymbol{A}_i^g \boldsymbol{H}_{i,i},\end{aligned} \tag{15a}$$

where

$$\begin{aligned}\overline{\boldsymbol{S}}_{i,i'}(\boldsymbol{B}) \triangleq & \kappa \text{diag}(\boldsymbol{H}_{i,i'}^H \boldsymbol{B} \boldsymbol{H}_{i,i'}) + \kappa \text{diag}(\beta \boldsymbol{H}_{i,i'}^H \text{diag}(\boldsymbol{B}) \boldsymbol{H}_{i,i'}) \\ & + \boldsymbol{H}_{i,i'}^H \boldsymbol{B} \boldsymbol{H}_{i,i'} + \beta \boldsymbol{H}_{i,i'}^H \text{diag}(\boldsymbol{B}) \boldsymbol{H}_{i,i'}.\end{aligned} \tag{15b}$$

Here, $\boldsymbol{A}_i = \sum_{k=1}^K \boldsymbol{U}_{k,0}^g \boldsymbol{W}_{k,0}^g (\boldsymbol{U}_{k,0}^g)^H$ for $i = 0$, and $\boldsymbol{A}_i = \boldsymbol{U}_{0,i}^g \boldsymbol{W}_{0,i}^g (\boldsymbol{U}_{0,i}^g)^H$ for $i = 1, ..., K$.

Once $\boldsymbol{V}^\star$ is obtained, $R_{0,k}^g$ and $R_{k,0}^g$ can be found by (7). $\boldsymbol{\tau} = [\tau^1, ..., \tau^G]^T$ is updated by solving (P) with respect to $\boldsymbol{\tau}$ while fixing the other variables. Provided that when $\mathcal{U}_{i,i'}(\cdot)$ is concave, the solution of $\boldsymbol{\tau}$ can be computed by the gradient-projection method. Afterwards, define $\boldsymbol{\alpha}_{i,i'} = [\alpha_{i,i'}^1, ..., \alpha_{i,i'}^G]^T$, and $\{\boldsymbol{\alpha}_{0,k}, \boldsymbol{\alpha}_{k,0}\}_{k=1}^K$ is updated by solving (P) using $\{\boldsymbol{V}^\star, \boldsymbol{\tau}^\star\}$. Since finding the global optimal solution of $\{\boldsymbol{\alpha}_{0,k}, \boldsymbol{\alpha}_{k,0}\}_{k=1}^K$ in each step may fix $\{\boldsymbol{\alpha}_{0,k}, \boldsymbol{\alpha}_{k,0}\}_{k=1}^K$ at 0 or 1 at early iteration, one-step gradient projection is applied in each iteration. Therefore, the update of $\boldsymbol{\alpha}_{0,k}$ and $\boldsymbol{\alpha}_{k,0}$ is

$$\begin{bmatrix} \boldsymbol{\alpha}_{0,k}^\star \\ \boldsymbol{\alpha}_{k,0}^\star \end{bmatrix} \leftarrow P_\Omega\left(\begin{bmatrix} \boldsymbol{\alpha}_{0,k} + t\eta_{0,k} \boldsymbol{r}_{0,k} \\ \boldsymbol{\alpha}_{k,0} + t\eta_{k,0} \boldsymbol{r}_{k,0} \end{bmatrix} \right), \tag{16}$$

where $\boldsymbol{r}_{i,i'} = [\tau^1 R_{i,i'}^1, ..., \tau^G R_{i,i'}^G]^T$, t is the step size obtained by backtracking line search, and $P_\Omega(\cdot)$ is the projection onto the set $\Omega = \{\boldsymbol{\alpha}_{0,k}, \boldsymbol{\alpha}_{k,0} \in \mathbb{R}^G | \text{(C3), (C4)}\}$. The iterations continues until $\boldsymbol{V}$ converges. The process is summarized in Algorithm 1.

Algorithm 1: Proposed joint processing algorithm

1 Randomly initialize $\boldsymbol{V}$ such that (C1) is satisfied, and set $\tau^g = \frac{1}{G}$, $\forall g$ and $\alpha_{0,k}^g = \alpha_{k,0}^g = \frac{1}{2}, \forall k, g$

repeat for all k, g

2 $\boldsymbol{U}_{0,k}^g \leftarrow (\boldsymbol{J}_k^g)^{-1} \boldsymbol{H}_{0,k} \boldsymbol{V}_{0,k}^g$, $\boldsymbol{U}_{k,0}^g \leftarrow (\boldsymbol{J}_0^g)^{-1} \boldsymbol{H}_{k,0} \boldsymbol{V}_{k,0}^g$.

3 $\boldsymbol{W}_{0,k}^g \leftarrow \eta_{0,k} \tau^g \alpha_{0,k}^g \Big(\boldsymbol{I} - (\boldsymbol{U}_{0,k}^g)^H \boldsymbol{H}_{0,k} \boldsymbol{V}_{0,k}^g\Big)^{-1}$, $\boldsymbol{W}_{k,0}^g \leftarrow \eta_{k,0} \tau^g \alpha_{k,0}^g \Big(\boldsymbol{I} - (\boldsymbol{U}_{k,0}^g)^H \boldsymbol{H}_{k,0} \boldsymbol{V}_{k,0}^g\Big)^{-1}$.

4 $\boldsymbol{V}_{0,k}^g \leftarrow \Big(\boldsymbol{S}_0^g + \lambda_0^g \boldsymbol{I}\Big)^{-1} \boldsymbol{H}_{0,k}^H \boldsymbol{U}_{0,k}^g \boldsymbol{W}_{0,k}^g$. $\boldsymbol{V}_{k,0}^g \leftarrow \Big(\boldsymbol{S}_k^g + \lambda_k^g \boldsymbol{I}\Big)^{-1} \boldsymbol{H}_{k,0}^H \boldsymbol{U}_{k,0}^g \boldsymbol{W}_{k,0}^g$.

5 Update $\boldsymbol{\tau}$ by solving (P) w.r.t. $\boldsymbol{\tau}$ subject to (C2).

6 $\begin{bmatrix} \boldsymbol{\alpha}_{0,k}^\star \\ \boldsymbol{\alpha}_{k,0}^\star \end{bmatrix} \leftarrow P_\Omega\left(\begin{bmatrix} \boldsymbol{\alpha}_{0,k} + t\eta_{0,k} \boldsymbol{r}_{0,k} \\ \boldsymbol{\alpha}_{k,0} + t\eta_{k,0} \boldsymbol{r}_{k,0} \end{bmatrix} \right)$.

7 Round $\alpha_{0,k}^g$ and $\alpha_{k,0}^g$ to 0 or 1 after I iterations.

until $\boldsymbol{V}_{0,k}^g$ and $\boldsymbol{V}_{k,0}^g$ converge.

One can verify that the update of the variables at each step is minimizing a locally tight upper bound of the objective function in (P) by following the proof of Theorem 1 in [13] and Theorem 2 in [14]. By the convergence result of Block Successive Upper-bound Minimization (BSUM) (see Theorem 2 in [18]), the proposed algorithm is guaranteed to converge to a stationary point.

4. NUMERICAL RESULTS

Numerical results of the proposed algorithm are presented in this section. We randomly generate $K = 10$ users within a single hexagon picocell; we use the source code in [15] to do so. Proportional fairness utility is considered in our simulation, i.e., $\mathcal{U}_{0,k}(\cdot) = \mathcal{U}_{k,0}(\cdot) = \log(\cdot), \forall k$. The number of transmit and receive antennas at the BS is $N_{t_0} = N_{r_0} = 4$. Furthermore, we assume $N_{t_k} = N_{r_k} = d_{0,k} = d_{k,0} = 2, \forall k$. For picocell deployment, according to the 3GPP LTE (TR 36.828) evaluation methodology [19], the cell radius is 40 meters, the minimum distance between BS and UE is 10 meters, the noise power $\sigma_i^2 = -174$ dBm, and the power budget of BS and UE are $P_0 = 24$ dBm and $P_k = 23$ dBm, respectively. We model the channels as $\boldsymbol{H}_{i,i'} = \vartheta_{i,i'} \overline{\boldsymbol{H}}_{i,i'}$, where each entry of $\overline{\boldsymbol{H}}_{i,i'}$ are independently generated according to $\mathcal{CN}(0,1)$ and $\vartheta_{i,i'}$ is the path loss between node i and i'. Hence $\vartheta_{i,i}$ is the path loss of the SI channel, or the SIC in analog domain, of node i. We set $\vartheta_{0,0} = -20$ dB. The path loss (dB) between BS and UE k is

$$\begin{matrix}\vartheta_{0,k} \\ (\text{or } \vartheta_{k,0})\end{matrix} = \begin{cases} 103.8 + 20.9 \log_{10} d, & \text{for LOS}, \\ 145.4 + 37.5 \log_{10} d, & \text{for NLOS}, \end{cases} \tag{17}$$

and the path loss (dB) between UE k and k' is

$$\begin{matrix}\vartheta_{k,k'} \\ (k' \neq 0, k)\end{matrix} = \begin{cases} 98.45 + 20 \log_{10} d, & d \leq 50 \text{ meters}, \\ 175.78 + 40 \log_{10} d, & d > 50 \text{ meters}. \end{cases} \tag{18}$$

Here, d is the distance between two nodes in kilometers. LOS and NLOS stand for line-of-sight and non-line-of-sight communication, respectively. The probability for LOS communication is given by

$$\begin{aligned}\text{Prob}_{\text{LOS}} = 0.5 - & \min(0.5, 5\exp(-0.156/d)) \\ & + \min(0.5, 5\exp(-d/0.03)).\end{aligned} \tag{19}$$

We choose $\kappa = \beta = -20$ dB. The performance is evaluated by measuring the cumulative distribution function (CDF) of the proportional fairness utility. The results are based on 300 channel realizations. We compare the proposed algorithm with the following baseline systems:

FD-Z: Both BS and UE can operate in FD mode. This system assumes the ideal case in which UE have no SI, i.e., $\boldsymbol{H}_{k,k} = \boldsymbol{0}$ for all k. This system serves as the upper bound of performance evaluation.

FD-83: This is the traditional FD system that considers the use of FD BS and FD UE, and we assume the SIC of UE in analog domain is 83 dB, i.e. $\vartheta_{k,k} = -83$ dB $\forall k$, which is a very high value.

HD-R: System with FD BS and HD UE. A set of UL/DL users are randomly selected with equal probability.

Half-duplex system: Both BS and UE are HD devices. The optimal UL and DL beamformer is obtained by solving the following optimization problem

$$\max_{\boldsymbol{V}, \boldsymbol{\tau}} \quad \sum_{k=1}^K \log\Big(\sum_{g_1=1}^{G/2} \tau^{g_1} R_{0,k}^{g_1}\Big) + \log\Big(\sum_{g_2=\frac{G}{2}+1}^{G} \tau^{g_2} R_{k,0}^{g_2}\Big) \tag{20}$$

subject to both (C1) and (C2) being satisfied.

Note that we initialize $\boldsymbol{V}_{k,0}^{g_1} = \mathbf{0}$ and $\boldsymbol{V}_{0,k}^{g_2} = \mathbf{0}, \forall k$ when solving (20).

We first compare the proportional fairness CDF with $G = 2, 10$ and 20 in Fig. 1. To avoid most of the $\boldsymbol{\tau}$ from becoming zero at early iterations, the update of $\boldsymbol{\tau}$ starts after 100 iterations and the total number of iterations is 200, where one iteration means all variables are being updated once. The curve "—○—" represents the proposed algorithm considering FD BS and HD UE. Here, $\boldsymbol{\alpha}$ will be rounded to 0 or 1 at the final iteration step. From Fig. 1, we see that the system performance can be significantly improved by increasing the number of groups. When $G \geq 10$, systems with the use of FD BS outperform the half-duplex system even when SIC is only 20 dB. In addition, the performance of the proposed approach is the closest to the upper bound (i.e., FD-Z).

Table 1 shows the average computation time of the proposed algorithm and the baseline systems (implemented in Mathworks Matlab 8.5) running in a desktop computer (equipped with Intel Core i7-5820 CPU 3.30 GHz, 32 GB memory). We observed that the average computation time of the proposed algorithm is similar with that of the other baseline systems, because it only has an additional one-step gradient projection in each iteration. Also, the running time increases with G. We see that there is a tradeoff between computational complexity and system performance, but the improvement of system performance shows in Fig. 1 are well worth the trade-off.

Table 1: Average computation time of various systems (in seconds)

	Proposed	FD-Z	FD-83	HD-R	Half-duplex system
$G = 2$	1.24	1.16	1.21	1.66	1.00
$G = 10$	3.79	3.98	3.01	3.63	3.58
$G = 20$	8.80	9.52	9.40	8.47	10.53

We next compare the convergence speed of the proposed algorithm and the baseline systems in Fig. 2. We plot the average value of the system utility versus iteration number with $G = 20$. Updating of $\boldsymbol{\tau}$ after the 100th iteration leads to a sudden increase of the system utility. As can be seen, the convergence speed of the proposed algorithm and the baseline systems are very similar.

Finally, Table 2 shows the percentage of active UEs operating in FD mode. For active UEs, we mean the UEs that are transmitting and/or receiving data. For FD-83, we observed that less than 1.5% of the active UE are operating in FD mode. In addition, Fig. 1 shows that the proposed algorithm outperforms FD-83. It is worth mentioning that 83 dB SIC in analog domain is already very high and very difficult to achieve. These results imply that the use of FD UE may be unnecessary.

Table 2: Percentage of active UE operate in FD mode

G = 1		G = 10		G = 20	
FD-Z	FD-83	FD-Z	FD-83	FD-Z	FD-83
1.67%	1.27%	8.17%	0.55%	9.53%	0.17%

5. CONCLUSION

In this paper, we proposed a joint processing beamforming algorithm that takes practical factors into consideration: system utility maximization, UL/DL user selection and UL-DL interference management. We formulated the problem as an integer programming based WMMSE algorithm . The proposed algorithm also employed a user grouping technique to deal with the UL-DL interference. Numerical results demonstrated the superior performance of the proposed algorithm with FD BS and HD UE by comparing the CDF of the proportional fairness utility with that of half-duplex systems. Lastly, we observed that even though high SIC FD UEs are used, UEs tend to operate in HD mode. Therefore, high cost FD UEs are not required when carrying out FD data transmission in MU-MIMO systems.

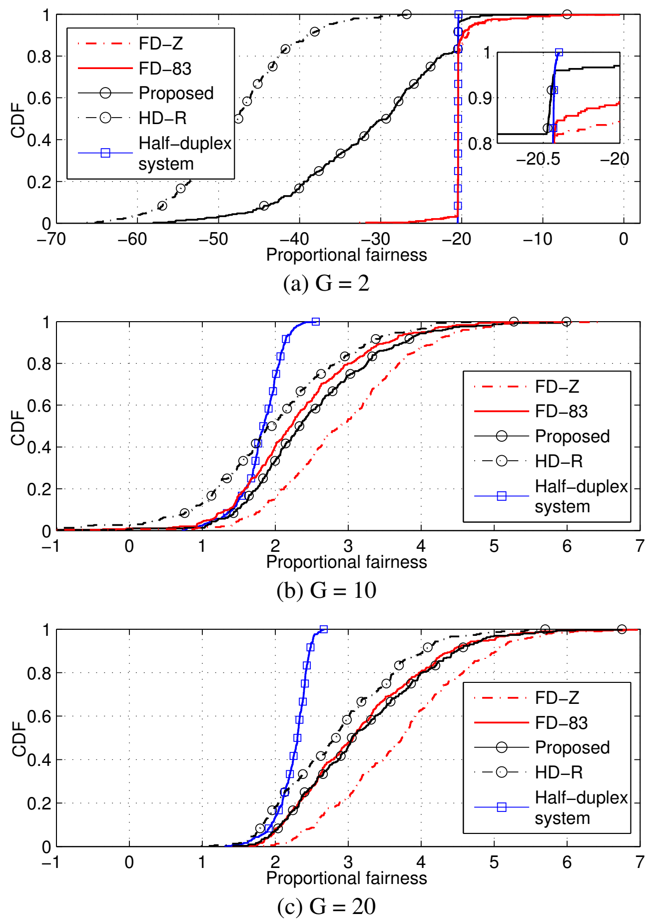

Fig. 1: Proportional fairness CDF with different number of groups of various systems.

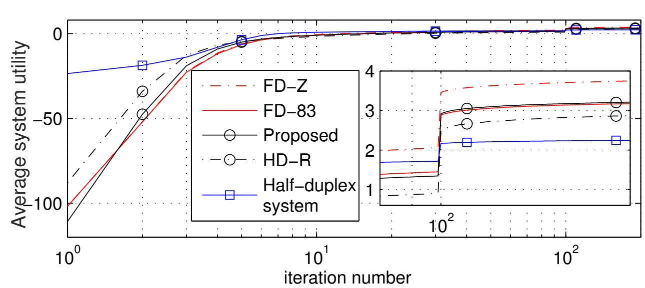

Fig. 2: Average system utility vs iteration number with $G = 20$ of various systems

6. REFERENCES

[1] J. I. Choi, M. Jain, K. Srinivasan, P. Levis, and S. Katti, "Achieving Single Channel, Full Duplex Wireless Communication," in *Proceedings of the Sixteenth Annual International Conference on Mobile Computing and Networking*, New York, NY, USA, 2010, MobiCom '10, pp. 1–12, ACM.

[2] M. Duarte and A. Sabharwal, "Full-duplex wireless communications using off-the-shelf radios: Feasibility and first results," in *2010 Conference Record of the Forty Fourth Asilomar Conference on Signals, Systems and Computers*, Nov 2010, pp. 1558–1562.

[3] M. Jain, J. I. Choi, T. M. Kim, D. Bharadia, S. Seth, K. Srinivasan, S. Katti P. Levis, and P. Sinha, "Practical, Real-time, Full Duplex Wireless," in *Proceedings of the 17th Annual International Conference on Mobile Computing and Networking*, New York, NY, USA, 2011, MobiCom '11, pp. 301–312, ACM.

[4] E. Aryafar, M. A. Khojastepour, K. Sundaresan, S. Rangarajan, and M. Chiang, "MIDU: Enabling MIMO Full Duplex," in *Proceedings of the 18th Annual International Conference on Mobile Computing and Networking*, New York, NY, USA, 2012, Mobicom '12, pp. 257–268, ACM.

[5] Bharadia, McMilin D., E., and S Katti, "Full Duplex Radios," *SIGCOMM Comput. Commun. Rev.*, vol. 43, no. 4, pp. 375–386, Aug. 2013.

[6] E. Everett, A. Sahai, and A. Sabharwal, "Passive Self-Interference Suppression for Full-Duplex Infrastructure Nodes," *IEEE Transactions on Wireless Communications*, vol. 13, no. 2, pp. 680–694, February 2014.

[7] T. Riihonen, S. Werner, and R. Wichman, "Mitigation of Loopback Self-Interference in Full-Duplex MIMO Relays," *IEEE Transactions on Signal Processing*, vol. 59, no. 12, pp. 5983–5993, Dec 2011.

[8] S. Li, R. D. Murch, and V. K. N. Lau, "Linear transceiver design for full-duplex multi-user MIMO system," in *2014 IEEE International Conference on Communications (ICC)*, June 2014, pp. 4921–4926.

[9] A. C. Cirik, R. Wang, Y. Hua, and M. Latva-aho, "Weighted Sum-Rate Maximization for Full-Duplex MIMO Interference Channels," *IEEE Transactions on Communications*, vol. 63, no. 3, pp. 801–815, March 2015.

[10] S. Huberman and T. Le-Ngoc, "Full-Duplex MIMO Precoding for Sum-Rate Maximization With Sequential Convex Programming," *IEEE Transactions on Vehicular Technology*, vol. 64, no. 11, pp. 5103–5112, Nov 2015.

[11] S. S. Christensen, R. Agarwal, E. D. Carvalho, and J. M. Cioffi, "Weighted sum-rate maximization using weighted MMSE for MIMO-BC beamforming design," *IEEE Transactions on Wireless Communications*, vol. 7, no. 12, pp. 4792–4799, December 2008.

[12] Q. Shi, M. Razaviyayn, Z. Q. Luo, and C. He, "An Iteratively Weighted MMSE Approach to Distributed Sum-Utility Maximization for a MIMO Interfering Broadcast Channel," *IEEE Transactions on Signal Processing*, vol. 59, no. 9, pp. 4331–4340, Sept 2011.

[13] M. Razaviyayn, M. Baligh, A. Callard, and Z. Q. Luo, "Joint User Grouping and Transceiver Design in a MIMO Interfering Broadcast Channel," *IEEE Transactions on Signal Processing*, vol. 62, no. 1, pp. 85–94, Jan 2014.

[14] M. Sanjabi, M. Razaviyayn, and Z. Q. Luo, "Optimal Joint Base Station Assignment and Beamforming for Heterogeneous Networks," *IEEE Transactions on Signal Processing*, vol. 62, no. 8, pp. 1950–1961, April 2014.

[15] R. Brandt, E. Björnson, and M. Bengtsson, "Weighted sum rate optimization for multicell MIMO systems with hardware-impaired transceivers," in *2014 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, May 2014, pp. 479–483.

[16] B. P. Day, A. R. Margetts, D. W. Bliss, and P. Schniter, "Full-Duplex Bidirectional MIMO: Achievable Rates Under Limited Dynamic Range," *IEEE Transactions on Signal Processing*, vol. 60, no. 7, pp. 3702–3713, July 2012.

[17] H. Holma and A. Toskala, *LTE for UMTS: Evolution to LTE-Advanced*, John Wiley & Sons, 2011.

[18] Meisam Razaviyayn, Mingyi Hong, and Zhi-Quan Luo, "A unified convergence analysis of block successive minimization methods for nonsmooth optimization," *SIAM Journal on Optimization*, vol. 23, no. 2, pp. 1126–1153, 2013.

[19] "3GPP TR 36.828, Evolved Universal Terrestrial Radio Access (E-UTRA); Further enhancements to LTE Time Division Duplex (TDD) for Downlink-Uplink (DL-UL) interference management and traffic adaptation," 2012.