# UPLINK AND DOWNLINK USER PAIRING IN FULL-DUPLEX MULTI-USER SYSTEMS: COMPLEXITY AND ALGORITHMS

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# ABSTRACT

In this paper, we consider a wireless network with one full-duplex (FD) base station (BS) and a set of half-duplex (HD) user equipments (UEs). In such scenario, in addition to the self-interference, the co-channel interference from uplink UEs to downlink UEs is the main bottleneck for the network performance. To overcome this, we consider the problem of maximizing the minimum fairness rate among all UEs by jointly determining the UE uplink/downlink directions and pairing the UEs over different resource blocks. We first show that the UE pairing problem is NP-hard in general. To develop efficient suboptimal algorithms, we formulate the considered problem as a mixed integer linear program and handle it by the iterative reweighted  $\ell_q$ -norm minimization (IRM) method. In particular, we propose a two-stage IRM algorithm that determines the UE transmission directions in the first stage followed by optimizing the UE pairs in the second stage. Simulation results are presented to show the efficacy of the proposed algorithm over some heuristic methods.

**Index Terms**— Full duplex, user pairing, max-min fairness,  $\ell_q$ -norm approximation, mixed integer linear programming.

# 1. INTRODUCTION

Full-duplex (FD) technology, which enables a transceiver to receive and transmit signals at the same time and over the same frequency, has drawn considerable attention recently. Conventionally, the selfinterference (SI) has limited the deployment of FD systems. However, recent advances in analog and digital interference cancellation schemes have shown that the SI can be effectively mitigated [1, 2]. Therefore, FD systems have been considered in a variety of scenarios including relay networks [3, 4], cellular networks [5, 6] and heterogeneous networks (HetNets) [7–9].

This paper considers a wireless system consisting of one FD base station (BS) and a set of half-duplex (HD) user equipments (UEs). Different from the traditional HD BS, the FD BS can communicate with the UEs for downlink and uplink transmissions simultaneously. In such FD system, however, the UEs performing uplink transmission (i.e., uplink UEs) can severely interfere with the UEs that are receiving data from the BS (i.e., downlink UEs). This new form of co-channel interference (CCI) significantly constrains the network performance and is a major bottleneck in FD multi-user systems [5, 10]. In view of this, there have been several researches

aiming at mitigating the CCI through judicious resource allocation and transmission scheduling. For example, reference [6] considered the problem of pairing an uplink UE and a downlink UE for transmission in a resource block (RB), and proposed heuristic algorithms for network throughput maximization and UE outage probability minimization. Reference [11] proposed joint UE pairing and power control in a time-sharing system, for maximizing the sum rate of the network. In both [6] and [11], whether a UE works as an uplink UE or a downlink UE is predetermined in advance of UE pairing. Different from [6] and [11], reference [12] considered that the UEs can either be an uplink UE or a downlink UE, and proposed to jointly determine the transmission directions of UEs (i.e., uplink/downlink UE assignment) and perform UE pairing for sum rate maximization in an orthogonal frequency division multiple access (OFDMA) system. Reference [7] considered a multi-cellular scenario with mixed FD/HD BSs and studied the joint FD/HD mode selection, user pairing and subcarrier allocation problem.

In this paper, we consider a multi-user OFDMA system with one FD BS and a set of HD UEs. Similar to [12], we consider the problem of joint uplink/downlink UE assignment and UE pairing. However, unlike [12], we consider the design problem of optimizing the max-min fairness (MMF) (uplink/downlink) rate of all UEs. The MMF criterion guarantees that each UE can be assigned to at least one RB as long as the number of RBs is sufficiently large. We focus on two aspects of such design problem, namely, complexity and algorithm. Firstly, we show that solving the UE assignment and pairing problem is at least as difficult as solving a 3-dimensional matching problem which is known to be NP-complete [13]. Secondly, we formulate the UE assignment and pairing problem as a mixed integer linear program (MILP), and propose to handle it by the  $\ell_a$ norm approximation and iterative reweighted minimization (IRM) method [14]. More specifically, we propose a two-stage IRM algorithm that assigns the uplink/downlink transmission directions of the UEs in the first stage, followed by optimizing the UE pairs over RBs in the second stage. Simulation results show that the proposed twostage algorithm outperforms the (one-stage) IRM method as well as the heuristic method based on simple relaxation, especially when the number of RBs is limited.

## 2. SYSTEM MODEL

We consider a single-cell wireless system consisting of one FD BS and M HD UEs. The BS and each of the UEs are equipped with a single antenna. The FD BS is able to transmit and receive data simultaneously. We assume that there are B RBs available for data transmission. Over each RB, one UE (i.e., uplink UE) can commu-

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nicate with the BS, and the BS can send data to another UE (i.e., downlink UE) at the same time. We say that a pair of UEs (i, j) is allocated over RB *b* if UE *i* is a downlink UE while UE *j* is an uplink UE over RB *b*. Following the OFDMA principle, we limit only one pair of UEs to be allocated to each RB. Throughout the paper, we assume that  $M \leq 2B$ ; otherwise, there must exist one UE which can never be assigned to any RB.

Suppose that UE pair (i, j) is allocated over RB b. Then the signal received at downlink UE i is given by

$$y_i^b = \sqrt{P_0} h_i^b s_{0i}^b + \sqrt{P_j} f_{ji}^b s_{j0}^b + n_i^b,$$
(1)

where  $P_0 \ge 0$  and  $P_j \ge 0$  are the transmission powers of the BS and UE j, respectively;  $h_i^b \in \mathbb{C}$  is the downlink channel between the BS and UE i, and  $f_{ji}^b \in \mathbb{C}$  is the channel between UE j and UE i over RB b. Moreover,  $s_{0i}^b \in \mathbb{C}$  and  $s_{j0}^b \in \mathbb{C}$  are the information signals sent from the BS to UE i and from UE j to the BS, respectively. In (1),  $n_i^b \sim \mathcal{CN}(0, \sigma_i^2)$  represents the additive white Gaussian noise (AWGN) with mean zero and variance  $\sigma_i^2$ . As seen, the first term in the right-hand side (RHS) of (1) is the desired signal of UE i whereas the second term is the CCI caused by uplink UE j.

On the other hand, the signal received by the BS over RB b is given by

$$y_0^b = \sqrt{P_j} g_j^b s_{j0}^b + \sqrt{P_0} \Delta h_0^b s_{0i}^b + n_0^b,$$
(2)

where  $g_j^b \in \mathbb{C}$  is the uplink channel between UE j and the BS over RB b, and  $n_0^b \sim \mathcal{CN}(0, \sigma_0^2)$  is the AWGN at the BS. The second term in the RHS of (2) is caused by the SI due to the FD BS. Here we assume that the SI has been properly suppressed via some interference cancellation schemes [1,2]. Thus,  $\sqrt{P_0} \Delta h_0^b s_{0i}^b$  in (2) represents the residual SI, and  $\Delta h_0^b \in \mathbb{C}$  represents the SI channel estimation error.

Assume that the average powers of information signals are all equal to one. According to (1) and (2), when UE pair (i, j) is allocated to RB b, the achievable downlink rate of UE i is given by

$$R_{ij}^d(b) = \log_2\left(1 + \frac{P_0|h_i^b|^2}{P_j|f_{ji}^b|^2 + \sigma_i^2}\right).$$
(3)

Analogously, the achievable uplink rate of UE j over RB b is

$$R_{ij}^{u}(b) = \log_2\left(1 + \frac{P_j |g_j^0|^2}{P_0 \mathbb{E}\left[|\Delta h_0^b|^2\right] + \sigma_0^2}\right),\tag{4}$$

where  $\mathbb{E}[|\Delta h_0^b|^2]$  is the average power of the SI channel estimation error. One can see from (3) and (4) that the choice of UE pair (i, j)has a significant impact on the achievable (uplink/downlink) rates of UEs, since uplink UEs can interfere with downlink UEs, especially they are in close proximity. In the next section, we propose a formulation that optimally assigns the transmission direction and pairs UEs over the RBs.

# 3. PROPOSED UE PAIRING FORMULATION AND ALGORITHMS

In this section, we formally formulate the considered joint UE assignment and pairing problem. We assume that the transmission powers  $P_0$  and  $P_j$  are fixed. We will first show that the problem is in fact NP-hard, and then propose an efficient algorithm for handling the UE assignment and pairing problem.

# 3.1. Problem Formulation and Complexity Analysis

We use  $\mathcal{M} \triangleq \{1, 2, ..., M\}$  and  $\mathcal{B} \triangleq \{1, 2, ..., B\}$  to represent the sets of UEs and RBs, respectively. Assume that all UEs always have data to transmit and receive. Thus each UE can either work as an

uplink UE or a downlink UE. We use a binary variable  $\alpha_i \in \{0, 1\}$  to indicate whether UE *i* is assigned as an uplink UE ( $\alpha_i = 0$ ) or a downlink UE ( $\alpha_i = 1$ ). In addition, we use a binary variable  $x_{ij}^b \in \{0, 1\}$  to denote whether UE pair (i, j) is allocated over RB *b* ( $x_{ij}^b = 1$ ) or not ( $x_{ij}^b = 0$ ). In particular, if  $x_{ij}^b = 1$ , then UE *i* is assigned to be a downlink UE, UE *j* is assigned to be an uplink UE, and they are paired to occupy RB *b*. Obviously, UE assignment and pairing has to satisfy the following constraints:

 OFDMA Constraint: Under the OFDMA constraint, only one pair of UEs can be allocated to each RB, i.e.,

$$\sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{M}} x_{ij}^b = 1, \, \forall \, b \in \mathcal{B},$$
(5)

$$x_{ij}^b \in \{0,1\}, \ \forall \ i,j \in \mathcal{M}, \ \forall \ b \in \mathcal{B}.$$
 (6)

• <u>HD Transmission Constraint</u>: Note that one UE could be assigned to more than one RB. However, since the UEs are HD, if one UE is assigned for uplink transmission (downlink reception) in one RB, then it must also perform as an uplink (downlink) UE when assigned to other RBs. To ensure this, we impose the following constraints

$$x_{ij}^{o} \leq \alpha_{i}, \,\forall \, j \neq i, \,\forall \, i, j \in \mathcal{M}, \,\forall \, b \in \mathcal{B},$$
(7)

$$x_{ji}^{b} \leq 1 - \alpha_{i}, \ \forall \ j \neq i, \ \forall \ i, j \in \mathcal{M}, \ \forall \ b \in \mathcal{B},$$
 (8)

$$\alpha_i \in \{0, 1\}, \, \forall \, i \in \mathcal{M}. \tag{9}$$

Denote  $\{x_{ij}^b\}$  as the set of binary variables  $x_{ij}^b$  for all  $i \neq j, i, j \in \mathcal{M}$  and  $b \in \mathcal{B}$ . Then, given  $\{x_{ij}^b\}$ , the achievable rate of each UE i can be expressed as

$$R_i \triangleq \sum_{b \in \mathcal{B}} \sum_{j \in \mathcal{M}} \left( x_{ij}^b R_{ij}^d(b) + x_{ji}^b R_{ji}^u(b) \right).$$
(10)

Due to the OFDMA constraint and the HD transmission constraint above,  $R_i$  in (10) is either the uplink sum rate or the downlink sum rate of UE *i*.

In this paper, we are interested in optimally determining the uplink/downlink transmission directions  $\{\alpha_i\}$  for all UEs and the UE pairs  $\{x_{ij}^b\}$  for maximizing the MMF rate. Mathematically, it is formulated as the following optimization problem

$$(\mathsf{P}) \max_{\{x_{ij}^b\},\{\alpha_i\}} \left\{ \min_{i \in \mathcal{M}} R_i \right\}$$
(11a)

t. 
$$(5) - (9)$$
 (11b)

To achieve a non-zero MMF rate, every UE must be allocated to at least one RB and paired with some other UEs. This implies that the following constraints

s.

$$\sum_{b \in \mathcal{B}} \sum_{j \in \mathcal{M}} (x_{ij}^b + x_{ji}^b) \ge 1, \ \forall \ i \in \mathcal{M},$$
(12)

have to hold. In addition, as there should not exist more downlink (uplink) UEs than the number of RBs, the solution of (P) should automatically satisfy

$$\sum_{i \in \mathcal{M}} \alpha_i \le B \text{ and } \sum_{i \in \mathcal{M}} (1 - \alpha_i) \le B.$$
 (13)

By adding (12) and (13) to (P), and reformulating the resultant problem as an epigraph form, we obtain

$$\max_{\tau, \{x_{ij}^k\}, \{\alpha_i\}} \tau \tag{14a}$$

s.t. 
$$R_i \ge \tau, \,\forall \, i \in \mathcal{M},$$
 (14b)

$$(5) - (9), (12), (13).$$
 (14c)

We remark that adding constraints (12) and (13) to problem (P) seems redundant and unnecessary at the first glance. However, as will become clear shortly, we will relax the binary variables in (6) and (9) to approximately handling (14). In that case, adding these constraints can effectively reduce the feasible region of the relaxed problem and improve the algorithm performance in practice.

Problem (14) is an MILP which is difficult to solve in general. In fact, problem (14) is intrinsically difficult from the complexity theory point of view.

#### Theorem 1 Problem (14) (and (P)) is NP-hard.

The proof of Theorem 1 is based on showing that solving problem (14) is at least as difficult as solving the 3-dimensional matching problem, which is known as a NP-complete problem [13]. In fact, our analysis shows that, even if the uplink/downlink directions of UEs have been determined, the remaining UE pairing problem is still NP-hard in general. The details of the proof are omitted due to the space limitation. In view of the NP-hardness, we present efficient suboptimal algorithms for handling problem (14) in the next subsection.

# **3.2.** $\ell_q$ -Norm Approximation and IRM Algorithm

The challenges for solving problem (14) is mainly caused by the binary constraints (6) and (9). Since (5) and (6) constitute the OFD-MA constraints, they can be handled by the  $\ell_q$ -norm approximation method in [15]. Specifically, let us consider the following problem

$$\max_{\tau, \{x_{ij}^b\}, \{\alpha_i\}} \tau - \lambda \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{M}} \sum_{b \in \mathcal{B}} \left( x_{ij}^b + \epsilon \right)^q$$
(15a)

s.t. 
$$0 \le x_{ij}^b \le 1, \forall i, j \in \mathcal{M}, \forall b \in \mathcal{B},$$
 (15b)

$$(5), (7) - (9), (12), (13), (14b), (15c)$$

where  $\lambda > 0, q \in (0, 1)$ , and  $\epsilon > 0$  are some parameters. By comparing problem (15) with (14), constraint (6) is replaced by (15b) and a penalty term  $-\lambda \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{M}} \sum_{b \in \mathcal{B}} (x_{ij}^b + \epsilon)^q$  is added in (15a). It has been shown in [15, 16] that for sufficiently large  $\lambda$ , (15) can approximately have the same optimal solution as (14) and the approximate error goes to zero as  $\lambda$  goes to infinity. Therefore, we consider solving (15) instead.

Note that problem (15) is still difficult to solve due to the binary variables  $\{\alpha_i\}$  in (9) and the non-convex  $\ell_q$ -norm regularization in (15a). Since binary  $\{\alpha_i\}$  can be obtained once  $\{x_{ij}^b\}$  are binary, we simply relax (9) as

$$0 \le \alpha_i \le 1, \, \forall \, i \in \mathcal{M}. \tag{16}$$

Then we use the iterative reweighted minimization (IRM) algorithm in [14] to handle the non-convex  $\ell_q$ -norm term. The IRM method iteratively approximates the non-convex  $\ell_q$ -norm term by its firstorder approximation. In particular, at the r-th iteration of the IRM algorithm, we solve the following problem:

$$\max_{\tau, \{x_{ij}^b\}, \{\alpha_i\}} \tau - \lambda q \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{M}} \sum_{b \in \mathcal{B}} w_{ij}^b(r) x_{ij}^b$$
(17)  
s.t. (5), (7), (8), (12), (13), (14b), (15b),

where  $w_{ij}^b(r) \triangleq (x_{ij}^b(r-1) + \epsilon)^{q-1}$ , and  $x_{ij}^b(r-1)$  denotes the value of the variable  $x_{ij}^b$  at the (r-1)-th iteration. Problem (17) is an LP and thus can be efficiently solved by off-the-shelf LP solvers. The IRM method for solving (15) is shown in Algorithm 1, where

Algorithm 1 IRM Algorithm for Solving Problem (15)

# Initialization:

- 1: Given  $q \in (0, 1), \lambda, \epsilon \in (0, 1), \sigma_1 \in (0, 1), \sigma_2 \in (0, 1)$  and  $\kappa > 1.$
- 2: Given a feasible solution  $\{x_{ij}^b(0)\}$  of (15).
- 3: Set  $w_{ij}^b(1) = (x_{ij}^b(0) + \epsilon)^{q-1}$
- 4: repeat
- 5: Set r = 1.
- 6: repeat
- Obtain  $\tau(r)$ ,  $\{x_{ij}^b(r)\}$  and  $\{\alpha_i(r)\}$  by solving (17). 7:
- Update  $w_{ij}^b(r+1) = (x_{ij}^b(r) + \epsilon)^{q-1}, \forall i, j \in \mathcal{M}, \forall b \in \mathcal{B}$ 8:
- 9:
- $\begin{array}{l} \substack{\mathcal{L}:\\ r \leftarrow r+1.\\ \text{until} \sum_{i \in \mathcal{M}, \ j \in \mathcal{M}, \ b \in \mathcal{B}} |x_{ij}^b(r) x_{ij}^b(r-1)| \leq \sigma_1 \\ \lambda \leftarrow \kappa \lambda. \end{array}$ 10:

12: **until** 
$$\sum_{l \in \mathcal{B}} \left( \{ x_{i,i}^b(r) \}_{l \in \mathcal{D}} \right)$$

12: **until**  $\sum_{b \in \mathcal{B}} \left( \left\{ x_{ij}^b(r) \right\}_{ij} \right)_2 \leq \sigma_2$ **Output:** Round  $\{ x_{ij}^b \}$  and  $\{ \alpha_i \}$  to  $\{ 0, 1 \}$  and output the solution.

 $\left(\left\{x_{ij}^{b}(r)\right\}_{ij}\right)_{2} \text{ denotes the second largest element in the set } \left\{x_{ij}^{b}(r)\right\} \text{ for each } b \in \mathcal{B}.$ 

We have some remarks on Algorithm 1. Firstly, a simple way to obtain the initial  $\{x_{ij}^b(0)\}$  is to solve (14) with relaxed  $\{\alpha_i\}$  and  $\{x_{ij}^b\}$ . Secondly, in practice, it is better to adaptively adjust parameters  $\epsilon$  and  $\lambda$  with the iteration index r; see [15, 16] for the details.

# 3.3. Proposed Two-Stage IRM Algorithm

Intriguingly, the IRM algorithm presented in Algorithm 1 does not always yield satisfactory performance in practice, especially when M = 2B (i.e., each UE has to occupy exactly one RB; otherwise the MMF rate is zero). The reason is that the IRM algorithm does not globally solve (15) in general, and as a result cannot guarantee to return binary  $\{\alpha_i\}$  and  $\{x_{ij}^b\}$ . When M = 2B, simply rounding fractional  $\{\alpha_i\}$  and  $\{x_{ij}^b\}$  to the binary set may give solutions that do not satisfy (12), i.e., some UEs are not paired and assigned to any RB which leads to a zero MMF rate in (11a).

To overcome this practical issue, we propose to solve (14) in a two-stage manner. In particular, in the first stage, we focus on determining the uplink and downlink assignment of UEs, i.e., we first determine  $\{\alpha_i\}$ . Once  $\{\alpha_i\}$  is determined, we then focus on determining the UE pairs  $\{x_{ij}^b\}$ . To implement the first stage, let us consider the following problem originated from (14):

$$\max_{\tau, \{x_{ij}^b\}, \{\alpha_i\}, \{\beta_i\}} \tau - \sum_{i \in \mathcal{M}} \mu \left( (\alpha_i + \epsilon_1)^q + (\beta_i + \epsilon_1)^q \right)$$
(18a)

s.t. 
$$\alpha_i + \beta_i = 1, \ \forall \ i \in \mathcal{M},$$
 (18b)

$$0 \le \alpha_i \le 1, \ \forall \ i \in \mathcal{M}, \tag{18c}$$

$$0 \le \beta_i \le 1, \ \forall \ i \in \mathcal{M},\tag{18d}$$

$$0 \le x_{ij}^{\circ} \le 1, \ \forall \ i, j \in \mathcal{M}, \ \forall \ b \in \mathcal{B},$$
(18e)

$$(5), (7), (8), (12), (13), (14b).$$
 (18f)

In (18), we have introduced new variables  $\beta_i \in [0, 1]$  for all  $i \in \mathcal{M}$ , added constraint (18b), relaxed (6) to (18e), and added the  $\ell_q$ -norm regularization in (18a). As seen, in contrast to (15), problem (18) attempts to obtain binary (or close to binary) solutions of  $\{\alpha_i\}$  by using the  $\ell_q$ -norm penalty  $\sum_{i \in \mathcal{M}} \mu((\alpha_i + \epsilon_1)^q + (\beta_i + \epsilon_1)^q)$ . Like (15), an IRM method similar to Algorithm 1 can be used to handle (18). We denote  $\{\alpha_i^{\star}\}$  as the solution obtained by solving (18).

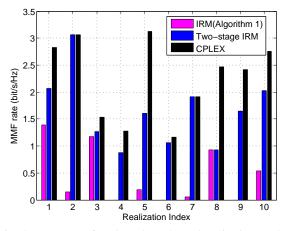


Fig. 1. The MMF rate for 10 random channel realizations and with B = 8, M = 16.

In the second stage, we fix  $\{\alpha_i\}$  in problem (15) with  $\{\alpha_i^{\star}\}$ , and only optimize (15) with respect to  $\{x_{ij}^b\}$  and  $\tau$ ; that is, we consider solving the following problem in the second stage

$$\max_{\tau, \{x_{ij}^b\}} \quad \tau - \lambda \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{M}} \sum_{b \in \mathcal{B}} \left(x_{ij}^b + \epsilon\right)^q \tag{19a}$$

s.t.  $x_{ij}^b \le \alpha_i^*, \ \forall \ j \ne i, \ \forall \ i, j \in \mathcal{M}, \ \forall \ b \in \mathcal{B},$  (19b)

$$x_{ji}^b \leq (1 - \alpha_i^\star), \ \forall \ j \neq i, \ \forall \ i, j \in \mathcal{M}, \ \forall \ b \in \mathcal{B},$$
 (19c)

Problem (19) can be handled by Algorithm 1. As  $\{\alpha_i\}$  are fixed in problem (19), solving problem (19) is more likely to return binary  $x_{ij}^b$  compared to solving (15). Simulation results in the next subsection will examine the performance of the two-stage IRM algorithm.

# 4. SIMULATION RESULTS

In this section, we consider the FD multi-user OFDMA system as described in Section 2. In our simulation, the transmission powers of all UEs are set to  $P_j = 23$  dBm, and the total transmission power of the BS is set to 43 dBm, which is equally allocated to the *B* RBs. The *M* UEs are randomly and uniformly located within a circle centered at the BS and with a radius 100 m. For the channel model, the path loss is set to  $140.7 + 36.7 \log_{10}(d)$  (dB), where *d* (km) denotes the distance between the transmitter and the receiver. The channel fading coefficients of all links are generated following the complex Gaussian with zero mean and unit variance. The averaged SI channel estimation error power  $\mathbb{E}[|\Delta h_0^b|^2]$  is set to be -110 dBm. Moreover, the noise power  $\sigma_0^2 = \sigma_i^2 = -90$  dBm for all  $i \in \mathcal{M}$ . The parameters used in Algorithm 1 are set to: q = 0.5,  $\epsilon(0) = \epsilon_1(0) = 0.1$ ,  $\sigma_1 = 10^{-3}$ ,  $\sigma_2 = 0.1$ ,  $\kappa = 1.1$ ,  $\lambda = \mu = 1$ .

We examine the performance of the following four algorithms, namely, (i) the (one-stage) IRM algorithm, i.e., Algorithm 1, (ii) the two-stage IRM algorithm, (iii) the exhaustive search method (by employing the MILP solver CPLEX [17]), and (iv) the simple relaxation method (which relaxes binary  $\{x_{ij}^b\}$  and  $\{\alpha_i\}$  to [0, 1] in (14) and rounds the solution to the binary set  $\{0, 1\}$ ).

Figure 1 displays the MMF rate achieved by the IRM algorithm (Algorithm 1), the two-stage IRM algorithm and the exhaustive search method (CPLEX), for 10 random channel realizations with B = 8 and M = 16. As expected, the exhaustive search method yields the highest MMF rate. Moreover, we can see from Fig. 1 that

**Table 1.** The percentage of channel realizations for which (12) is not satisfied for some  $i \in \mathcal{M}$ . The results are obtained over 50 channel realizations with B = 8 and various numbers of UEs.

No. of UEs $(M)$	6	8	10	12	14	16
Two-stage IRM	0%	0%	0%	0%	0%	0%
IRM (Algorithm 1)	0%	0%	8%	16%	26%	36%
Simple Relaxation	0%	2%	16%	44%	84%	98%

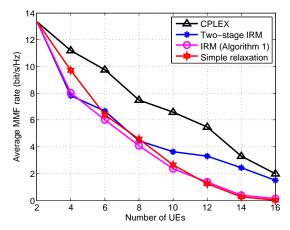


Fig. 2. Average MMF rate versus number of UEs, with B = 8.

the proposed two-stage IRM algorithm performs much better than Algorithm 1 most of the time. Specifically, one can observe that Algorithm 1 gives a zero MMF rate for the 4th, 6th, and 9th channel realizations. The reason for this is that the binary solution obtained from Algorithm 1 (after rounding) cannot satisfy constraint (12) very often, which means that there exist UEs who are not paired and not assigned to any RB.

Indeed, this undesirable phenomenon can occur more frequently when the number of UEs is close to two times of the number of RBs, i.e., M = 2B. To verify this, we show in Table 1 the percentage of channel realizations for which there exist unpaired UEs. The results are obtained by testing 50 channel realizations. We can observe from Table 1 that the proposed two-stage IRM method can successfully pair all the UEs from M = 6 to M = 16, whereas Algorithm 1 has 36% of realizations to yield zero MMF rate when M = 16. The results of the simple relaxation method are also shown in Table 1, which has an even worse performance.

In Figure 2, we further display the average MMF rates versus the number of UEs for 50 channel realizations. As seen from Fig. 2, while the average MMF rate decreases with the number of UEs, the proposed two-stage IRM algorithm performs very well, and outperforms Algorithm 1 and the simple relaxation method when M > 10.

In summary, we have studied the joint UE transmission direction assignment and UE pairing problem for maximizing the MMF rate in the FD OFDMA system. We have shown that the UE assignment and pairing problem is NP-hard in general and formulated the problem as an MILP. In light of the practically poor performance of the heuristic method by simple relaxation and rounding, we have proposed the two-stage IRM algorithm for achieving high-quality solutions. The presented simulation results have demonstrated that the two-stage IRM algorithm outperforms the one-stage IRM algorithm and the simple relaxation method, especially when the number of UEs is close to twice of the number of RBs.

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