ENERGY-EFFICIENT DESIGN FOR NON-REGENERATIVE MIMO RELAY NETWORKS

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ABSTRACT

We consider the global energy efficiency (GEE) maximization problem for the general non-regenerative MIMO relay network, under the maximum power constraints for each user and each relay. The problem is reformulated through the fractional optimization technique, and the mean square error receiver filter is considered. By applying the alternating minimization method, we simplify the problem into several convex quadratic constrained quadratic programming subproblems, and solve the subproblems by the feasible shrinkage method combined with the sequential quadratic programming method. Because the result highly depends on the initialization, we design a deterministic initialization by introducing an auxiliary power minimization problem. Simulation results show that our proposed algorithm can achieve more than 10 times higher GEE than the previous works which are not tailored for GEE maximization.

***Index Terms*—** MIMO relay, energy efficiency, deterministic initialization, sequential quadratic programming

1. INTRODUCTION

Non-regenerative MIMO relay networks have been widely studied, as one extension of MIMO networks. With the assistance of relays, networks are more reliable and more performing. Fundamental studies for MIMO amplify-and-forward (AF) relay networks have investigated different aspects, such as resource allocation and sum rate maximization. For power allocation problems, different models and algorithms are summarized in [1], which analyzes the diagonalization of channel matrices. For sum rate maximization problems, a weighted mean square error minimization (WMMSE) model is proposed, and the precoders and the relay AF matrices with the MMSE receiver filter are designed [2]. Another approach is to approximate the sum rate maximization by the total signal to total interference plus noise ratio (TSTINR) maximization [3–6]. It supports multiple data stream transmission by introducing orthogonality constraints, and proposes algorithms to determine the numbers of transmit data streams as preprocess; distributed algorithms with local channel state information (CSI) are designed as well [7].

Energy efficiency (EE) is another effective way to balance quality of service and power consumption. The next generation of cellular networks (5G) require a 1000x increase of data-rate to support the exponentially growing amount of connected devices. However, due to sustainable growth concerns, this increase must be obtained at half of the energy consumption of today's networks. Consequently this requires a 2000x increase of the bit-per-Joule energy efficiency [8]. Thus the "Green Communication" concept is attracting more and more attention [9–12]. Some works have explored the algorithms to solve the EE problem for MIMO relay networks [13–19]. In [13] a low-complexity algorithm to jointly allocate source and relay power is proposed; the energy-efficient power allocation problem is considered in [14], where a high Signal-to-Noise-Ratio (SNR) approximation is employed; the authors in [15] design the relay AF matrix by linearizing the nonconvex part in the optimization problem, for a two-user network. In [9], interference neutralization (IN) technique is introduced and a non-cooperative game is applied, where both the individual EE and the global EE (GEE) are maximized. Due to the introduction of EE, the nonlinear fractional optimization technique is widely used [16]. In [17] the sequential quadratic approximation technique is applied to simplify the problem. For the asymmetric two-way MIMO relay network with statistical CSI, the EE and the spectral efficiency problem are solved by [18]. For DF relay-aided MIMO-OFDMA networks, the interference alignment technique is applied by [19] which optimizes EE in a distributed fashion.

Most of the above mentioned studies only consider networks assisted by one single relay, while the general two-hop multi-relay MIMO interference network has not been investigated so far. In this paper, we propose an efficient algorithm to solve the GEE problem for general MIMO relay AF networks. In line with previously mentioned works, we consider a MIMO half-duplex amplify-and-forward relay network, making the following main contributions:
1) the formulation of the GEE maximization model for multi-hop multi-link MIMO interference networks;
2) the design of the precoding matrices and relay AF matrices based on the fractional optimization technique with MSE receiver filter, and the introduction of the deterministic initialization;
3) the numerical assessment of the achievable GEE with 10-fold performance gain compared to the state of the art.
The rest of the paper is organized as follows. The system model and the corresponding optimization problem is introduced in Section 2. Section 3 shows the algorithm to tackle the GEE problem. Simulation results in Section 4 show the efficiency of the proposed algorithm compared to the previous works.

Notation: $\mathbb{C}$ represents the complex domain. Re means the real part. $\mathbf{I}_d$ represents the $d \times d$ identity matrix. $\otimes$ represents the Kronecker product. $\mathrm{vec}(\mathbf{A})$ is a column vector consisting of the columns of $\mathbf{A}$. $\mathcal{K}$ and $\mathcal{R}$ represent the set of the user indices $\{1, 2, \ldots, K\}$ and that of relay indices $\{1, 2, \ldots, R\}$, respectively.

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2. SYSTEM MODEL

A two-hop half-duplex MIMO interference channel with K user pairs and R relays is considered. The transmitted signal vector of User k is denoted as $\mathbf{s}_k \in \mathbb{C}^{d_k \times 1}$, where d_k is the number of transmit data streams, and $\mathbb{E}(\mathbf{s}_k\mathbf{s}_k^H) = \mathbf{I}_{d_k}$. Suppose Transmitter k, Receiver k and Relay r have M_k, N_k and L_r antennas, respectively, for all $k \in \mathcal{K}$ and $r \in \mathcal{R}$. We assume that there are no direct links between users, and perfect CSI is available at a central controller.

The transmission process consists of two phases. In the first phase, each transmitter transmits its precoded signal to all relays. Relay r receives the signal $\mathbf{x}_r = \sum_{k\in\mathcal{K}} \mathbf{G}_{rk}\mathbf{U}_k\mathbf{s}_k + \mathbf{n}_r$, where $\mathbf{U}_k \in \mathbb{C}^{M_k\times d_k}$ is the precoding matrix of User k, $\mathbf{G}_{rk} \in \mathbb{C}^{L_r\times M_k}$ is the channel matrix between Transmitter k and Relay r, and $\mathbf{n}_r$ is the local noise at Relay r, with zero mean and variance matrix $\sigma_r^2\mathbf{I}_{L_r}$. In the second phase, Relay r multiplies the received signal by its AF matrix $\mathbf{W}_r \in \mathbb{C}^{L_r\times L_r}$, and obtains $\mathbf{t}_r = \mathbf{W}_r\mathbf{x}_r$. Then $\mathbf{t}_r$ is transmitted to all receivers. Receiver k multiplies the received signal by its decoding matrix $\mathbf{V}_k \in \mathbb{C}^{N_k\times d_k}$, and finally obtains:

$$\begin{aligned}\tilde{\mathbf{y}}_k \;=\; & \underbrace{\mathbf{V}_k^H\mathbf{T}_{kk}\mathbf{s}_k}_{\text{desired signal}} + \underbrace{\sum_{q\in\mathcal{K},q\neq k}\mathbf{V}_k^H\mathbf{T}_{kq}\mathbf{s}_q}_{\text{interference}} \\ & + \underbrace{\sum_{r\in\mathcal{R}}\mathbf{V}_k^H\mathbf{H}_{kr}\mathbf{W}_r\mathbf{n}_r + \mathbf{V}_k^H\mathbf{z}_k}_{\text{noise}}.\end{aligned}$$

Let $\mathbf{T}_{kq} = \sum_{r\in\mathcal{R}} \mathbf{H}_{kr}\mathbf{W}_r\mathbf{G}_{rq}\mathbf{U}_q$ be the effective channel from Transmitter q to Receiver k. Here $\mathbf{H}_{kr} \in \mathbb{C}^{N_k\times L_r}$ is the channel matrix between Relay r and Receiver k, and $\mathbf{z}_k$ is the noise at Receiver k, with zero mean and variance matrix $\mu_k^2\mathbf{I}_{N_k}$. It consists of the desired signal, the interference from other users and the noise including relay enhanced noise and the local noise.

Let us define $\{\mathbf{W}_{-r}\} = \{\mathbf{W}_1, \ldots, \mathbf{W}_{r-1}, \mathbf{W}_{r+1}, \ldots, \mathbf{W}_R\}$, $\{\mathbf{U}_{-k}\}$, $\bar{\mathbf{G}}_{rk} = \mathbf{G}_{rk}\mathbf{U}_k$, $\bar{\mathbf{W}}_{rk} = \mathbf{W}_r\mathbf{G}_{rk}$, $\bar{\mathbf{H}}_{kr} = \mathbf{H}_{kr}\mathbf{W}_r$, and $\bar{\mathbf{V}}_{kr} = \mathbf{V}_k^H\mathbf{H}_{kr}$. Motivated by the need for energy efficiency improvement, we want to design the precoding matrices $\{\mathbf{U}\} = \{\mathbf{U}_k, k \in \mathcal{K}\}$ and the relay AF matrices $\{\mathbf{W}\} = \{\mathbf{W}_r, r \in \mathcal{R}\}$, for global energy efficiency maximization, defined as the network sum-rate over the network total power consumption [9]:

$$\text{GEE} = \frac{R_{\text{sum}}}{P^U + P^R + P^C}. \tag{1}$$

The GEE in (1) is measured in bit-per-Joule and represents the network benefit-cost ratio in terms of amount of information reliably transmitted per Joule of consumed energy. Here $R_{\text{sum}} = \frac{1}{2}\sum_{k\in\mathcal{K}} \log_2\det(\mathbf{I}_{N_k} + \mathbf{F}_k^{-1}\mathbf{T}_{kk}\mathbf{T}_{kk}^H)$ is the sum rate of the network, and $\mathbf{F}_k = \sum_{q\neq k,q\in\mathcal{K}} \mathbf{T}_{kq}\mathbf{T}_{kq}^H + \sum_{r\in\mathcal{R}} \sigma_r^2\bar{\mathbf{H}}_{kr}\bar{\mathbf{H}}_{kr}^H + \mu_k^2\mathbf{I}_{N_k}$. $P^U = \sum_{k\in\mathcal{K}} P_k^U$ and $P^R = \sum_{r\in\mathcal{R}} P_r^R$ are the total transmit power of users and relays, respectively, where $P_k^U = \|\mathbf{U}_k\|_F^2$ and $P_r^R = \sum_{k\in\mathcal{K}} \|\mathbf{W}_r\mathbf{G}_{rk}\mathbf{U}_k\|_F^2 + \sigma_r^2\|\mathbf{W}_r\|_F^2$. P^C is the circuit power of the network, which is assumed as a constant.

Considering the individual user and individual relay power constraints, we formulate the GEE maximization model as the following optimization problem:

$$\begin{aligned}\max_{\{\mathbf{U}\},\{\mathbf{W}\}} \quad & \text{GEE} = \frac{R_{\text{sum}}}{P^U + P^R + P^C} && \text{(2a)}\\ \text{s. t.} \quad & \|\mathbf{U}_k\|_F^2 \le p_k^U, k \in \mathcal{K}, && \text{(2b)}\\ & \sum_{k\in\mathcal{K}} \|\mathbf{W}_r\mathbf{G}_{rk}\mathbf{U}_k\|_F^2 + \sigma_r^2\|\mathbf{W}_r\|_F^2 \le p_r^R, r \in \mathcal{R}, && \text{(2c)}\end{aligned}$$

where p_k^U and p_r^R are the power budgets for User k and Relay r, respectively. As a highly nonlinear nonconvex matrix optimization problem, it is very difficult to solve problem (2) optimally.

3. APPROXIMATION MODEL AND ALGORITHM

In this section, we first approximate the GEE model as a more tractable optimization problem, and then propose an efficient algorithm to tackle the problem.

3.1. Problem reformulation

In order to handel the fractional function in (2a), we apply the fractional optimization technique [20], introduce a new parameter C, and reformulate (2) as:

$$\min_{\{\mathbf{U}\},\{\mathbf{W}\}} \; C(P^U + P^R) - R_{\text{sum}} \;\text{ s. t. }\; (2b) - (2c). \tag{3}$$

Here the constant CP^C is omitted in the objective function. The parameter C is updated as

$$C = \frac{R_{\text{sum}}}{P^U + P^R + P^C} = \frac{R_{\text{sum}}}{P_{\text{sum}}}. \tag{4}$$

Similar to [3, Theorem 2], providing the update strategy of C as (4), we can prove that all the KKT points of problem (3) are the KKT points of problem (2). Next, we consider solving (3) instead of (2).

At each receiver, we use the linear MMSE filter, which minimizes the mean square error between the transmit data vector $\mathbf{s}_k$ and its soft estimate at the receiver. Defining $\bar{\mathbf{F}}_k = \mathbf{T}_{kk}\mathbf{T}_{kk}^H + \mathbf{F}_k$, the linear MMSE receiver for user k is written as

$$\mathbf{V}_k = \bar{\mathbf{F}}_k^{-1}\mathbf{T}_{kk}. \tag{5}$$

Then the MSE matrix at Receiver k is obtained: $\mathbf{E}_k^{\text{MMSE}} = \mathbf{V}_k^H\bar{\mathbf{F}}_k\mathbf{V}_k - \mathbf{V}_k^H\mathbf{T}_{kk} - \mathbf{T}_{kk}^H\mathbf{V}_k + \mathbf{I}_{d_k}$. Furthermore, by introducing auxiliary weight matrix variables $\mathbf{S}_k$ for all $k \in \mathcal{K}$, we can show that the following problem has the same KKT points as (3) [2]:

$$\begin{aligned}\min_{\substack{\{\mathbf{U}\},\{\mathbf{W}\}\\ \{\mathbf{V}\},\{\mathbf{S}\}}} \quad & C(P^U + P^R) + \sum_{k\in\mathcal{K}} \big[\text{tr}\big(\mathbf{S}_k\mathbf{E}_k^{\text{MMSE}}\big) - \log_2\det(\mathbf{S}_k)\big]\\ \text{s. t.} \quad & (2b) - (2c).\end{aligned} \tag{6}$$

Due to the KKT conditions, the weight matrix $\mathbf{S}_k$ is set as

$$\mathbf{S}_k = (\mathbf{E}_k^{\text{MMSE}})^{-1} = \mathbf{I}_{d_k} + \mathbf{T}_{kk}^H\mathbf{F}_k^{-1}\mathbf{T}_{kk}. \tag{7}$$

It holds that $\sum_{k\in\mathcal{K}} \big[\text{tr}\big(\mathbf{S}_k\mathbf{E}_k^{\text{MMSE}}\big) - \log_2\det(\mathbf{S}_k)\big] = -R_{\text{sum}}$. In this case, (3) and (6) have the same objective function values.

3.2. Alternating minimization

Here we apply the alternating minimization method to tackle problem (6). First, in each iteration, the decoding matrix $\mathbf{V}_k$ and the weight matrix $\mathbf{S}_k$ are updated by (5) and (7), respectively, for all $k \in \mathcal{K}$. Then, fixing $\{\mathbf{U}\}$, $\{\mathbf{V}\}$, $\{\mathbf{S}\}$ and $\{\mathbf{W}_{-r}\}$, the subproblem for the relay AF matrix $\mathbf{W}_r$ is

$$\begin{aligned}\min_{\mathbf{X}\in\mathbb{C}^{L_r\times L_r}} \quad & \text{tr}\big[\mathbf{X}\mathbf{A}_r\mathbf{X}^H\big(\sum_{k\in\mathcal{K}} \bar{\mathbf{V}}_{kr}^H\mathbf{S}_k\bar{\mathbf{V}}_{kr} + C\mathbf{I}_{L_r}\big)\big]\\ & -2\text{Re}\big[\sum_{k\in\mathcal{K}}\sum_{q\in\mathcal{K}}\sum_{\substack{l\neq r,\\ l\in\mathcal{R}}} \text{tr}\big(\mathbf{X}\bar{\mathbf{G}}_{rq}\bar{\mathbf{G}}_{lq}^H\mathbf{W}_l^H\bar{\mathbf{V}}_{kl}^H\mathbf{S}_k\bar{\mathbf{V}}_{kr}\big)\big]\\ \text{s.t.} \quad & \text{tr}\big(\mathbf{X}\mathbf{A}_r\mathbf{X}^H\big) \le p_r^R,\end{aligned}$$

where $\mathbf{X}$ represents $\mathbf{W}_r$, $\mathbf{A}_r = \sum_{k\in\mathcal{K}} \bar{\mathbf{G}}_{rk}\bar{\mathbf{G}}_{rk}^H + \sigma_r^2\mathbf{I}_{L_r}$. Its equivalent form is:

$$\min_{\mathbf{x}} \; \mathbf{x}^H\bar{\mathbf{B}}_1\mathbf{x} + \bar{\mathbf{b}}^H\mathbf{x} + \mathbf{x}^H\bar{\mathbf{b}} \;\text{ s.t. }\; \mathbf{x}^H\mathbf{x} \le p_r^R. \tag{8}$$

Here $\mathbf{x} = \mathbf{Q} \cdot \text{vec}(\mathbf{X})$, $\bar{\mathbf{B}}_1 = \mathbf{Q}^{-H}\mathbf{B}_1\mathbf{Q}^{-1}$ and $\bar{\mathbf{b}} = \mathbf{Q}^{-1}\mathbf{b}$; $\mathbf{b} = \sum_{k\in\mathcal{K}}\sum_{q\in\mathcal{K}}\sum_{l\neq r,l\in\mathcal{R}} \bar{\mathbf{V}}_{kr}^H\mathbf{S}_k\bar{\mathbf{V}}_{kl}\mathbf{W}_l\bar{\mathbf{G}}_{lq}\bar{\mathbf{G}}_{rq}^H$, $\mathbf{B}_1 = \mathbf{A}_r \otimes (\sum_{k\in\mathcal{K}} \bar{\mathbf{V}}_{kr}^H\mathbf{S}_k\bar{\mathbf{V}}_{kr}+C\mathbf{I}_{L_r})$; $\mathbf{Q} \succ 0$ is computed by the eigenvalue decomposition of $\mathbf{B}_2 = \mathbf{A}_r \otimes \mathbf{I}_{L_r}$ as $\mathbf{B}_2 = \mathbf{Q}^H\mathbf{Q}$. An efficient algorithm to solve problem (8) optimally is shown in [21, Chapter 6.1.1].

If we fix $\{\mathbf{V}\}$, $\{\mathbf{W}\}$, $\{\mathbf{S}\}$ and $\{\mathbf{U}_{-k}\}$, the subproblem for the precoding matrix $\mathbf{U}_k$ is:

$$\begin{aligned}\min_{\mathbf{X}\in\mathbb{C}^{M_k\times d_k}} \quad & \text{tr}\Big\{\mathbf{X}^H\big[C(\mathbf{I}_{M_k}+\sum_{r\in\mathcal{R}}\bar{\mathbf{W}}_{rk}^H\bar{\mathbf{W}}_{rk})+\sum_{q\in\mathcal{K}}\mathbf{J}_{qk}^H\mathbf{S}_q\mathbf{J}_{qk}\big]\mathbf{X}\Big\}\\ & -\text{tr}(\mathbf{S}_k\mathbf{J}_{kk}\mathbf{X}+\mathbf{X}^H\mathbf{J}_{kk}^H\mathbf{S}_k)\\ \text{s.t.} \quad & \|\mathbf{X}\|_F^2 \leq p_k^U,\\ & \text{tr}(\mathbf{X}^H\bar{\mathbf{W}}_{rk}^H\bar{\mathbf{W}}_{rk}\mathbf{X}) \leq \eta_r, r\in\mathcal{R}. \end{aligned} \tag{9}$$

Here $\mathbf{X}$ represents $\mathbf{U}_k$, $\mathbf{J}_{qk} = \sum_{r\in\mathcal{R}}\bar{\mathbf{V}}_{qr}\bar{\mathbf{W}}_{rk}$ and $\eta_r = p_r^R - \sigma_r^2\|\mathbf{W}_r\|_F^2 - \sum_{q\neq k,q\in\mathcal{K}}\|\bar{\mathbf{W}}_{rq}\mathbf{U}_q\|_F^2$. As a special structured convex quadratic constrained quadratic programming (QCQP) problem, it can be solved optimally by the feasible shrinkage (FS) method combining the sequential quadratic programming (SQP) method. This hybrid algorithm is proposed in [3, Section B-(3)-c], which performs much faster than the interior point method such like Sedumi and CVX. The FS method works as the efficient initialization for the SQP method, and its idea is to use one quadratic constraint to approximate the original quadratic constraints. The main idea of the SQP method is to solve a quadratic programming (QP) subproblem in each iteration, where the objective function is the second order Tailor expansion of the Lagrangian function and the constraints are the linearizations of the original constraints [21, Alg 12.2.2]. The detailed description of the method is omitted due to the space limit.

3.3. Deterministic initialization

Due to the nonconvexity of problem (6), the achieved GEE by alternatively updating the variables is highly dependent on the initialization. It will lead to poor performances if we simply randomly generate initial variables and scale them to be feasible. Empirically we have observed that, if a large power is used as initialization, the power level does not change significantly during the execution of the algorithm. To overcome the disadvantage caused by random initializations, we introduce an auxiliary problem. It minimizes the total transmit power of users and relays while requiring that the achieved sum rate is above a prefixed threshold:

$$\min_{\{\mathbf{U}\},\{\mathbf{W}\}} \quad P^U + P^R \quad \text{s.t.} \quad R_{\text{sum}} \geq r_0. \tag{10}$$

Here r_0 is a constant. The detailed setting of r_0 is shown in Section 4. By solving this problem suboptimally, we can obtain a good initialization for problem (6), and further yield to better performances.

First, by applying the MMSE receiver filters $\{\mathbf{V}\}$, and introducing the weight matrices $\{\mathbf{S}\}$, we obtain the following problem:

$$\begin{aligned}\min_{\substack{\{\mathbf{U}\},\{\mathbf{W}\}\\ \{\mathbf{V}\},\{\mathbf{S}\}}} \quad & P^U + P^R \\ \text{s.t.} \quad & \sum_{k\in\mathcal{K}}\big[\text{tr}\big(\mathbf{S}_k\mathbf{E}_k^{\text{MMSE}}\big) - \log_2\det(\mathbf{S}_k)\big] + r_0 \leq 0. \end{aligned} \tag{11}$$

If we update the decoding matrices $\{\mathbf{V}\}$ and the weight matrices $\{\mathbf{S}\}$ by (5) and (7), respectively, we can show that (10) and (11) have the same KKT points [2]. Fixing the other variables, the relay AF matrix $\mathbf{W}_r$ is updated by the following subproblem:

$$\begin{aligned}\min_{\mathbf{x}\in\mathbb{C}^{L_r^2\times 1}} \quad & \mathbf{x}^H\mathbf{B}_2\mathbf{x}\\ \text{s.t.} \quad & \mathbf{x}^H\mathbf{B}_3\mathbf{x}+\mathbf{b}^H\mathbf{x}+\mathbf{x}^H\mathbf{b}+c\leq 0, \end{aligned} \tag{12}$$

where $\mathbf{x} = \text{vec}(\mathbf{W}_r)$, $\mathbf{B}_3 = \mathbf{A}_r \otimes (\sum_{k\in\mathcal{K}}\bar{\mathbf{V}}_{kr}^H\mathbf{S}_k\bar{\mathbf{V}}_{kr})$, $c = \sum_{k\in\mathcal{K}}\big\{\text{tr}\big[(\mathbf{V}_k\mathbf{S}_k\mathbf{V}_k^H)\big(\sum_{l\neq r,l\in\mathcal{R}}\sigma_r^2\bar{\mathbf{H}}_{kl}\bar{\mathbf{H}}_{kl}^H+\sum_{q\in\mathcal{K}}\mathbf{T}_{kq}^r(\mathbf{T}_{kq}^r)^H +\eta_k^2\mathbf{I}_{N_k}\big)-\mathbf{S}_k\mathbf{V}_k^H\mathbf{T}_{kq}^r-\mathbf{S}_k(\mathbf{T}_{kq}^r)^H\mathbf{V}_k+\mathbf{S}_k\big]-\log_2\det(\mathbf{S}_k)\big\}+r_0$ and $\mathbf{T}_{kq}^r = \sum_{l\neq r,l\in\mathcal{R}}\mathbf{H}_{kl}\mathbf{W}_l\mathbf{G}_{lq}\mathbf{U}_q$; $\mathbf{A}_r$, $\mathbf{B}_2$ and $\mathbf{b}$ are defined under problem (8). Similarly, the precoding matrix $\mathbf{U}_k$ is updated by the subproblem below:

$$\begin{aligned}\min_{\mathbf{x}\in\mathbb{C}^{N_kd_k\times 1}} \quad & \mathbf{x}^H\mathbf{L}_2\mathbf{x}\\ \text{s.t.} \quad & \mathbf{x}^H\mathbf{L}_1\mathbf{x}+\mathbf{l}^H\mathbf{x}+\mathbf{x}^H\mathbf{l}+e\leq 0, \end{aligned} \tag{13}$$

where $\mathbf{x} = \text{vec}(\mathbf{U}_k)$, $\mathbf{L}_1 = \sum_{q\in\mathcal{K}}\mathbf{J}_{qk}^H\mathbf{S}_q\mathbf{J}_{qk}$, $\mathbf{L}_2 = \mathbf{I}_{M_k} + \sum_{r\in\mathcal{R}}\bar{\mathbf{W}}_{rk}^H\bar{\mathbf{W}}_{rk}$, $\mathbf{l} = \mathbf{J}_{kk}^H\mathbf{S}_k$ and $e = r_0 - \sum_{q\neq k,q\in\mathcal{K}}(\mathbf{S}_q\mathbf{V}_q^H\mathbf{T}_{qq}+\mathbf{S}_q\mathbf{T}_{qq}^H\mathbf{V}_q)+\sum_{q\in\mathcal{K}}\Big\{\text{tr}\big[(\mathbf{V}_q\mathbf{S}_q\mathbf{V}_q^H)(\eta_q^2\mathbf{I}_{N_q}+\sum_{p\neq k,p\in\mathcal{K}}\mathbf{T}_{qp}\mathbf{T}_{qp}^H+\sum_{r\in\mathcal{R}}\sigma_r^2\bar{\mathbf{H}}_{qr}\bar{\mathbf{H}}_{qr}^H)+\mathbf{S}_q\big]-\log_2\det(\mathbf{S}_q)\Big\}$. As $\mathbf{B}_3$ and $\mathbf{L}_1$ are both positive definite, subproblems (12) and (13) can be transformed into equivalent problems similar to (8), and we apply the same method to solve them optimally. To save computational cost, we only update all the variables in problem (11) for once, and then use them as the initialization of problem (6).

3.4. Algorithmic framework

Combining the above three subsections, we summarize the algorithm to tackle the GEE maximization problem (2) as Algorithm 1.

Input : Randomly generated $\{\mathbf{U}\}$ and $\{\mathbf{W}\}$
Output: $\{\mathbf{U}\}$, $\{\mathbf{V}\}$, $\{\mathbf{W}\}$ and $\{\mathbf{S}\}$
1. Calculate $\{\mathbf{V}\}$ and $\{\mathbf{S}\}$ according to (5) and (7), respectively;
2. Update $\mathbf{W}_r$ by solving (12), for all $r\in\mathcal{R}$;
3. Update $\mathbf{U}_k$ by solving (13), for all $k\in\mathcal{K}$;
repeat
  1. Update $\mathbf{V}_k$ and $\mathbf{S}_k$ according to (5) and (7), respectively, for all for all $k\in\mathcal{K}$;
  2. Update $\mathbf{W}_r$ by solving (8), for all $r\in\mathcal{R}$;
  3. Update $\mathbf{U}_k$ by solving (9), for all $k\in\mathcal{K}$;
  4. Update the parameter C according to (4);
until *convergence*;

Algorithm 1: GEE Algorithm

Proposition 1 *In Algorithm 1, the achieved GEE converges.*

Proof: In each iteration, each subproblem is solved optimally. Thus the objective function of (6) and (3) reduces. Let $\{\mathbf{X}\}$ represent the set of the iterative points $\{\{\mathbf{U}\},\{\mathbf{V}\},\{\mathbf{W}\},\{\mathbf{S}\}\}$, and define the objective function of (3) is $f(\{\mathbf{X}\};C)$. Suppose $\{\mathbf{X}^i\}$ are the feasible points achieved from the ith iteration. Then the expression of parameter C used in the ith iteration as well as the GEE achieved in the $(i-1)$th iteration is:

$$C^i = \text{GEE}^{i-1} = \frac{R_{\text{sum}}(\{\mathbf{X}^{i-1}\})}{P_{\text{sum}}(\{\mathbf{X}^{i-1}\})}.$$

Since in the ith iteration the objective function of (3) reduces, it holds that

$$f(\{\mathbf{X}^i\};C^i)=C^iP_{\text{sum}}(\{\mathbf{X}^i\})-R_{\text{sum}}(\{\mathbf{X}^i\})\le f(\{\mathbf{X}^{i-1}\};C^i)=0,$$

Then $\text{GEE}^i = \frac{R_{\text{sum}}(\{\mathbf{X}^i\})}{P_{\text{sum}}(\{\mathbf{X}^i\})} \ge C^i = \text{GEE}^{i-1}$. Thus the achieved GEE monotonically increases and converges. $\square$

4. NUMERICAL SIMULATIONS

In this section, we evaluate the performance of the proposed algorithm by numerical tests. Each element of the channel matrices $\mathbf{G}_{rk}$ is the product of a zero-mean, unit variance Gaussian random variable multiplying the coefficient $\text{PL}(r,k) = \text{PL}_0(\frac{s(r,k)}{s_0})^4$, which accounts for the propagation pathloss. Here PL_0 is the free-space attenuation at the distance $s_0 = 100$ meters with a carrier frequency of 1800 MHz, and $s(r,k)$ is the distance between Transmitter k and Relay r, which is randomly generated in between $[100, 500]$ meters. The channel matrices $\mathbf{H}_{kr}$ are generated in the same way, for all $k \in \mathcal{K}$ and $r \in \mathcal{R}$. The total circuit power $P_c = 10$dBm. The noise variances $\sigma_r^2 = \mu_k^2 = \sigma^2 = FN_0W$, with the receiver noise figure $F = 3$dB, the receive power spectral density $N_0 = -174$dBm/Hz and the communication bandwidth $W = 180$KHz.

In Algorithm 1, r_0 is set in the following way. First we randomly generate the initial variables and scale them to satisfy the constraints (2b) and (2c), and calculate the corresponding sum rate value R_0. Then we let $r_0 = 0.1R_0$. Thus problem (10) is always feasible. For each plotted point in the figures, 100 random realizations of different channel coefficients are generated to show the average performance.

First, we compare our algorithm with the cooperative GEE algorithm proposed in [9]. The latter algorithm serves for one-relay networks. It applies the IN technique to obtain the relay AF matrix. The $(2\times 2, 1)^K + 16^1$ network is considered [1]. The maximum transmit power for each transmitter and the relay are 0dBW and 10dBW, respectively. With the number of user pairs, K, varying from 2 to 10, the curves representing the achieved GEE of the two algorithms are plotted in Fig. 1. Notice that the GEE definition in this paper is different from [9], where the latter omits the relay transmit power. Because our algorithm considers the transmit power for both users and relays, it achieves much higher GEE than [9]. The achieved GEE of our proposed algorithm increases with the increment of K. This is reasonable, because the relay is better utilized when the number of user pairs increases.

Second, our algorithm is applied to the $(3\times 3, 2)^3 + 3^3$ network. The maximum transmit power of each relay is 5dBW. We compare our algorithm with the sum rate maximization algorithm [2]. Both algorithms use the WMMSE technique to reformulate the problems. The achieved GEE and sum rate of both algorithms with respect to different p^U are shown in Fig. 2, where p^U is the maximum transmit power of each transmitter. Compared to the algorithm in [2], which aims to maximize the sum rate directly, our algorithm significantly improves the achieved GEE, and achieves moderate sum rate. When p^U increases, although the achieved sum rate of our algorithm increases, the achieved GEE reduces eventually. It saves much resources by solving the GEE problem, which addresses the green communication challenge in 5G and beyond networks.

The performances of our algorithm show that our algorithm is effective to tackle the GEE problems, and that our deterministic initialization works well.

[1]Denote $(N\times M, d)^K + L^R$ as the network with K user pairs and R relays, where each transmitter, each receiver and each relay have M, N and L antennas, respectively, and each user pair transmits d data streams.

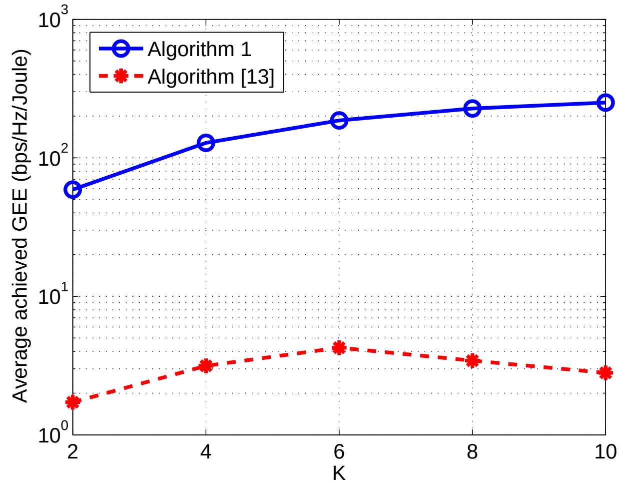

Fig. 1 Achieved GEE for the $(2\times 2, 1)^K + 16^1$ network

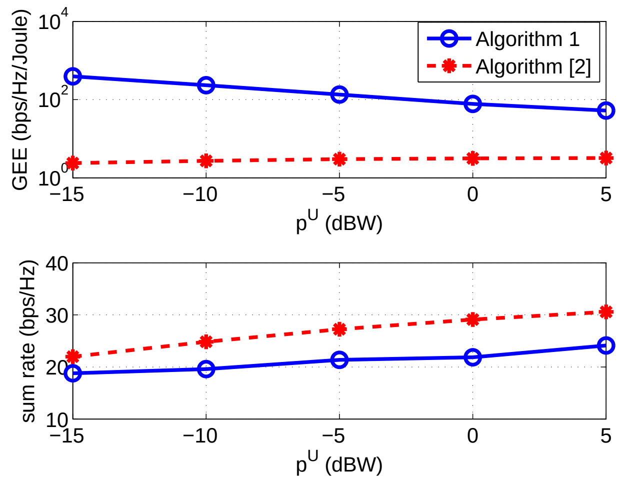

Fig. 2 Achieved GEE and sum rate for the $(3\times 3, 2)^3 + 3^3$ network

5. CONCLUSION

For the general relay-assisted MIMO interference network, we set up a model to maximize the global energy efficiency of the network, with the individual user and individual relay power constraints. We proposed an algorithm to solve the problem by the following approaches: first, the fractional optimization technique was applied to reformulate the fractional objective function; second, we introduced the MSE receiver filter and transformed the sum rate expression as the weighted MSE; third, the alternating minimization method was applied, and the problem was simplified to several convex QCQP subproblems; fourth, we introduced an auxiliary power minimization problem to provide good initializations. Simulation results show that our proposed algorithm achieves high GEE in different scenarios, and the deterministic initialization technique is efficient.

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