MULTIPLE SUBSPACE MATCHING PURSUIT FOR SPECTRUM SENSING

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ABSTRACT

Spectrum sensing is used to perceive the spectral environment over a wide frequency band. The multiple measurement vector (MMV) model can be applied to the spectrum sensing scenario since it enables jointly sparse signal recovery. In this paper, a novel spectrum sensing algorithm, referred to as multiple subspace matching pursuit (MSMP), is proposed to reduce the miss detection and false alarm events in the spectrum sensing. Numerical simulations demonstrate that the proposed algorithm shows the outstanding recovery performance with the reduction of the incorrect spectrum decisions.

Index Terms— Spectrum sensing, multiple measurement vector (MMV), miss detection, false alarm, spectrum utilization.

1. INTRODUCTION

The problem of recovering sparse vector (i.e., vector with a few non-zero elements) from small number of linear measurements arises in various applications [1], [2]. Basic premise of this new paradigm, often called compressed sensing (CS), is that the sparse signal vector can be reconstructed from handful of incoherent linear measurements [3]. As a generalization of this so called single measurement vector (SMV) problem, multiple measurement vector (MMV) problem has received much attention in recent years [2], [4]. The goal of MMV problem is to recover a common support of the sparse vectors $x_1, \cdots, x_d$ from the group of measurement vectors $y_1, \cdots, y_d$. If we denote $\mathbf{Y} = [y_1 \cdots y_d]$ and $\mathbf{X} = [x_1 \cdots x_d]$, then the MMV system model is given by

$$\mathbf{Y} = \mathbf{\Phi X} + \mathbf{W} \tag{1}$$

where $\mathbf{\Phi} \in \mathbb{R}^{m \times n}$ $(m \ll n)$ is the sensing matrix and $\mathbf{W} \in \mathbb{R}^{m \times d}$ is the measurement noise matrix. The basic principle of the MMV-based CS is that when the signal matrix is row sparse (i.e., when $\mathbf{X}$ has a few nonzero rows), it can be uniquely determined via the identification of its support - a set of indices corresponding to nonzero rows in the signal matrix $\mathbf{X}$. Once the support is determined, the problem to estimate the signal matrix can be converted into an overdetermined linear inverse problem. After this conversion, one can recover the desired signal matrix $\mathbf{X}$ using a simple pseudo-inverse.

The MMV problem can be applied to the spectrum sensing techniques in cognitive radio (CR) networks as a means to improve the overall spectrum efficiency. CR technique offers a new way of exploiting temporarily available frequency resources. Specifically, when the primary users do not use the spectrum, the secondary users may access it in such a way that they do not cause interference to the primary users. Early works focused on the energy detection [5] and feature detection [6], which are not quite desirable since it takes too much time to process a whole spectrum or it is too expensive in terms of cost and power consumption. Another line of researches popularly studied in recent years is CS-based spectrum sensing technique. Major benefits of this approach are to alleviate the sampling rate issue of ADC and the cost issue of RF circuitry [7], [8].

An aim of this paper is to propose an efficient and robust spectrum sensing algorithm projecting the multiple subspaces into the signal subspace. From the CS perspective, the spectrum sensing problem can be translated into the MMV problem to find the spectral support in the muti-band signal model [7]. The proposed algorithm, referred to as multiple subspace matching pursuit (MSMP), improves the reconstruction quality of the spectral support by preventing the miss detection and false alarm events. Since MSMP finds the indices as the spectral support after double-checking and updates the residual subspace with the overlapped indices, the chances of selecting true spectral support and reducing the incorrect spectrum decisions are increased significantly.

We briefly summarize the notations used in this paper. $\Sigma = \{1, 2, \cdots, n\}$ is the set of column indices of $\mathbf{\Phi}$ and Ω denotes the set of nonzero row indices of $\mathbf{X}$ The submatrix of $\mathbf{\Phi}$ with columns indexed by $D \subseteq \Sigma$ is denoted by $\mathbf{\Phi}_D$. The submatrix of $\mathbf{\Phi}$ with rows indexed by $J \subseteq \Sigma$ is denoted

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by $\mathbf{\Phi}^J$. $\Sigma \setminus D$ is the set of all elements contained in Σ but not in D. $\mathcal{R}(\mathbf{\Phi}_D)$ is the span of columns in $\mathbf{\Phi}_D$. $\mathbf{\Phi}_D^{'}$ is the transpose of the matrix $\mathbf{\Phi}_D$ and $\mathbf{\Phi}_D^{\dagger} = (\mathbf{\Phi}_D \mathbf{\Phi}_D^{'})^{-1} \mathbf{\Phi}_D^{'}$ is the pseudoinverse of $\mathbf{\Phi}_D$. $\mathbf{P}_D = \mathbf{\Phi}_D \mathbf{\Phi}_D^{\dagger}$ is the orthogonal projection onto $\mathcal{R}(\mathbf{\Phi}_D)$. $\mathbf{P}_D^{\perp} = \mathbf{I} - \mathbf{P}_D$ is the orthogonal projection onto the orthogonal complement of $\mathcal{R}(\mathbf{\Phi}_D)$.

2. SYSTEM MODEL FOR SPECTRUM SENSING AND CONVENTIONAL ALGORITHM

Spectrum sensing techniques identify the active frequencies and then determine the spectral support where the primary user signals are occupied. In CR networks, secondary users are allowed to use empty subbands of primary users opportunistically as long as they do not interfere with the primary users. Clearly, key to the success of the CR technology is the accurate sensing of the spectrum to ensure that the secondary users can safely use the spectrum without hindering the operation of primary users. As shown in Fig. 1, each spectral subband corresponds to one of the four types of spectrum decision: spectrum hole, correct decision, false alarm, and miss detection. Miss detection and false alarm refer to the failure of detecting the existing signal and the mistake of detecting the unused spectrum, respectively.

2.1. System model for spectrum sensing

In [7], blind multi-band signal reconstruction scheme has been proposed to perform the wideband spectrum sensing. If the multi-band signals are sparse over a wideband spectrum, meaning that a small number of the spectral subbands contain the signal energies, then CS techniques can be applied to the spectral support recovery. To describe a multi-band signal, we consider N disjoint continuous signals, expressed as

$$x(t) = \sum_{i=1}^{N} g_i(t) \cos(2\pi f_i t) + n(t), \qquad (2)$$

where $g_i(t)$ is the baseband signal with the bandwidth B_i, f_i is the carrier frequency, and $n(t)$ is the Gaussian noise in the system. When the bandwidth of signal is smaller than that of each subband, the multi-band signals can be expressed in a linear formula as [7]

$$\mathbf{y}(f) = \mathbf{\Phi}\mathbf{z}(f), \quad f \in \mathcal{F}_s, \qquad (3)$$

where $\mathbf{y}(f)$ is a vector of length m whose elements are the DTFT (discrete-time Fourier transform) of the sub-Nyquist sampled sequence $y[n]$ in the range $\mathcal{F}_s = [-f_s/2, +f_s/2]$ and the unknown vector $\mathbf{z}(f)$ is of length L (the number of subbands). Since $\mathbf{y}(f)$ and $\mathbf{z}(f)$ are continuous functions with regard to f, $\mathbf{y}(f)$ and $\mathbf{z}(f)$ have the infinite vectors. This model, called the infinite measurement vector (IMV) model, can be converted into the MMV model as

$$\mathbf{V} = \mathbf{\Phi}\mathbf{U} \qquad (4)$$

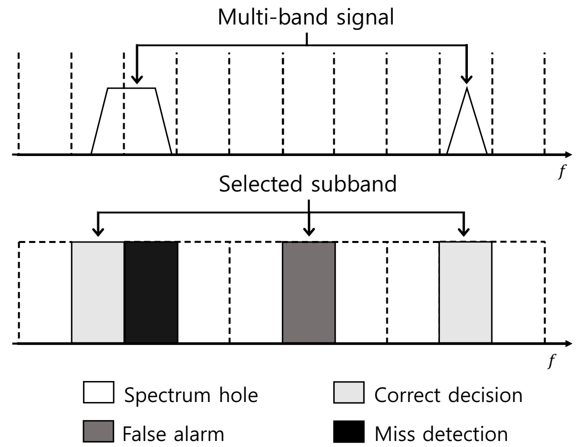

Fig. 1. Illustration of classifying spectral subbands into four types of spectrum decision in a multi-band signal model.

where $\mathbf{V}$ is the measurement matrix and $\mathbf{U}$ is the signal matrix [9]. From [9, Preposition 2], if the integral satisfying the following equality exists

$$\mathbf{Q} = \int_{f \in \mathcal{F}_s} \mathbf{y}(f)\, \mathbf{y}^H(f)\, df = \sum_{n=-\infty}^{+\infty} \mathbf{y}[n]\, \mathbf{y}^T[n], \qquad (5)$$

then $\mathbf{V}$, the matrix square root of $\mathbf{Q}$ (i.e. $\mathbf{Q} = \mathbf{V}\mathbf{V}^H$), has the same column space with $\mathbf{y}(f)$ and $\mathbf{U}$ has the same support with $\mathbf{z}(f)$. Thus, the problem of identifying the spectral support via spectrum sensing in the wideband boils down to the MMV problem.

2.2. Recovery of spectral support via existing algorithm

By extending the orthogonal matching pursuit (OMP) algorithm [10], [11] into the MMV scenario, we obtain the simultaneous orthogonal matching pursuit (SOMP) [12], which can be used to reconstruct the multi-band signals [7]. Generally speaking, SOMP identifies the support iteratively by finding the index i such that

$$\underset{i \in \Sigma}{\operatorname{argmax}} \, \|\mathbf{\Phi}_i^T \mathbf{R}\|_2, \qquad (6)$$

where $\mathbf{R}$ is the residual matrix.

When performing the spectrum sensing, SOMP selects the dominant subband (frequency) where the signal energy is presumably existing. Moreover, the adjacent subband, having small signal energy, is added to the spectral support because it is possible that the multi-band signal can be located over two adjacent subbands. Often, the adjacent subband selection in SOMP can cause undesirable frequent false alarms. For example, in case of the triangular signal in Fig. 1, one of the neighbor bands should be added to the spectral support although the triangular signal accounts for only one subband.

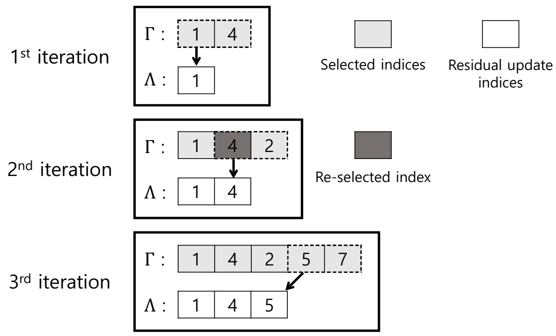

Fig. 2. Relationship between the pre-selection set Γ and the residual update set Λ.

This means that false alarms are inevitable if the bandwidth of multi-band signal is smaller than that of the subband.

While SOMP is simple to implement and efficient to compute, it will also increase the miss detection probability in selecting the spectral support. At each iteration, SOMP selects one spectral support and then updates the residual signal. If an incorrect spectral support is chosen in the iteration (i.e., false alarm occurs), other miss detected support and false alarm might happen due to the erroneous update of the residual. Therefore, it is desired to use better recovery algorithm in view of identifying the accurate spectral support.

3. MSMP ALGORITHM FOR MULTIPLE MEASUREMENT VECTOR PROBLEM

In this section, we propose the MSMP algorithm to recover the spectral support efficiently. The key idea to improve the recovery performance is to select the double-checked indices as the spectral support and as a result to perform the precise residual update in each iteration. In doing so, the probabilities of the miss detection and false alarm events are reduced in the spectrum sensing. The proposed MSMP algorithm is summarized in Algorithm 1. In essence, MSMP consists of three key steps: 1) signal subspace estimation, 2) index selection for a pre-selection set Γ and a residual update set Λ, and 3) the estimation of support and signal matrix, respectively.

Step 1 of MSMP is to estimate a signal subspace, denoted as $\hat{S}$, spanned by the r dominant eigenvectors of $YY^{\dagger}$. The estimated signal subspace $\hat{S}$ is close to an r-dimensional subspace of $S \triangleq \mathcal{R}(Y)$. Step 2 of MSMP is to find indices for a pre-selection set Γ and a residual update set Λ before determining the spectral support. In the i-th iteration, MSMP selects two indices α, β that represent the two closest column vectors of Φ to the $\mathcal{R}(P_{\Lambda}^{\perp}\hat{S})$ that satisfy the following equation

$$\frac{\left|\left|P_{\mathcal{R}(P_{\Lambda}^{\perp}\hat{S})}\phi_{\alpha}\right|\right|_2}{\left|\left|P_{\Lambda}^{\perp}\phi_{\alpha}\right|\right|_2} \geq \frac{\left|\left|P_{\mathcal{R}(P_{\Lambda}^{\perp}\hat{S})}\phi_{\beta}\right|\right|_2}{\left|\left|P_{\Lambda}^{\perp}\phi_{\beta}\right|\right|_2}. \qquad (7)$$

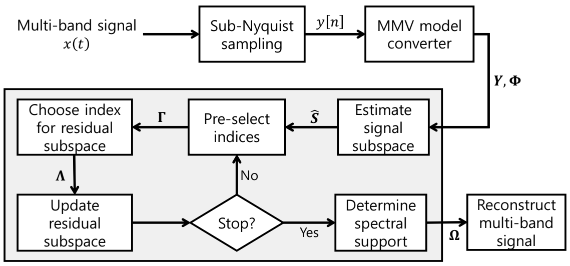

Fig. 3. Overall process for spectrum sensing based on the proposed MSMP algorithm.

Then, MSMP organizes the pre-selection set Γ and the residual update set Λ. Fig. 2 illustrates the selection procedure of the pre-selection set Γ and residual update set Λ in MSMP. At this point, the re-selected index is highly likely to be a spectral support since it is chosen twice in the consecutive iterations. If the residual update set Λ has a number of the re-selected indices, each iteration may produce the residual subspace $P_{\Lambda}^{\perp}\hat{S}$ precisely. Step 3 of MSMP is to estimate the support Ω and signal matrix $\mathbf{X}$ as $\hat{\Omega}$ and $\hat{\mathbf{X}}$.

We now describe how the proposed algorithm can be applied to the reconstruction of spectral support in spectrum

Algorithm 1 MSMP algorithm

Input: $\hat{S} \in \mathbb{R}^{m\times r}$, $Y \in \mathbb{R}^{m\times l}$, $\Phi \in \mathbb{R}^{m\times n}$, $k \in \mathbb{N}$
Output: $\hat{\Omega} \subseteq \Sigma$, $\hat{X} \in \mathbb{R}^{n\times l}$

$\Omega_c \leftarrow \emptyset$
$\Gamma \leftarrow \emptyset$
$\Lambda \leftarrow \emptyset$
for $i = 1$ to k **do**
  Select two indices α, β corresponding to 2 largest entries in
$$\left\{ \frac{\left|\left|P_{\mathcal{R}(P_{\Lambda}^{\perp}\hat{S})}\phi_l\right|\right|_2}{\left|\left|P_{\Lambda}^{\perp}\phi_l\right|\right|_2} \middle| l \in \Sigma \setminus \Lambda \right\}$$
  if $\alpha \in \Gamma$ **then**
    $\Lambda \leftarrow \Lambda \cup \{\alpha\}$
  else if $\beta \in \Gamma$ **then**
    $\Lambda \leftarrow \Lambda \cup \{\beta\}$
  else
    $\Lambda \leftarrow \Lambda \cup \{\alpha\}$
  end if
  $\Gamma \leftarrow \Gamma \cup \{\alpha, \beta\}$
end for
$\bar{X} \leftarrow (\Phi_{\Gamma})^{\dagger} Y$
for $l \in \Gamma$ **do**
  $\zeta_l \leftarrow \|\bar{X}^{\{l\}}\|_2$
end for
$\hat{\Omega} \leftarrow$ {indices of the k-largest ζ_l's}
$\hat{X} \leftarrow \left(\Phi_{\hat{\Omega}}\right)^{\dagger} Y$
return $\hat{\Omega}$, $\hat{X}$

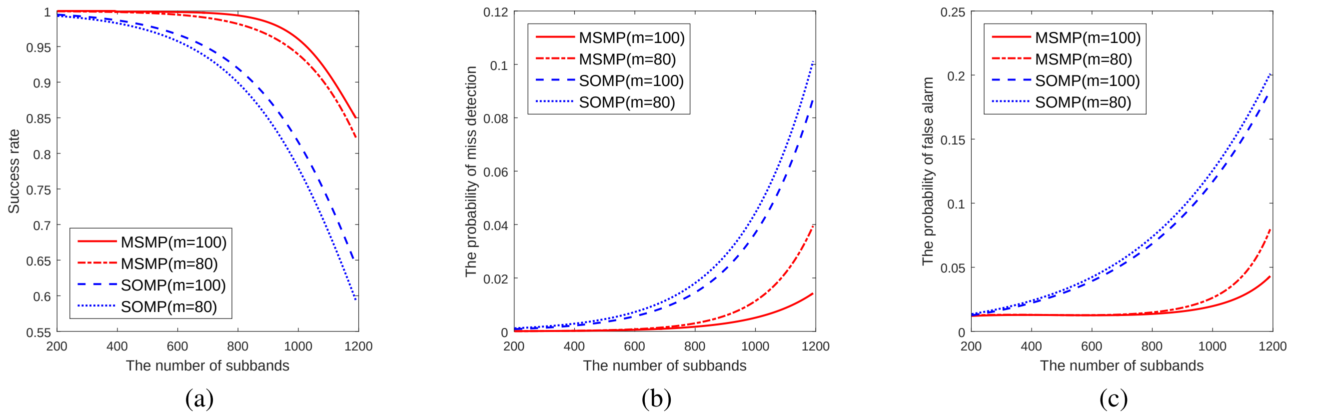

Fig. 4. Performances of the proposed MSMP algorithm and the SOMP algorithm: (a) success rate of spectral support recovery, (b) probability of miss detection, (c) probability of false alarm.

sensing. At each iteration, MSMP selects the multiple subbands, which already contain not only the dominant subband but also the adjacent subband having signal energy. Since the examination of surrounding subbands is unnecessary, MSMP can reduce the false alarm events substantially.

Furthermore, a number of elements in Λ are double-checked indices because the re-selected indices would appear in Γ frequently. By using most of the double-checked elements in Λ, reliability of the residual subspace can be improved, resulting in accurate spectral support selection. Therefore, the chance of miss detection can be reduced by the double-checking of the spectral subbands. The complete procedure for spectrum sensing by using the MSMP algorithm is shown in Fig. 3.

4. NUMERICAL EXPERIMENTS AND RESULTS

In this section, we compare the proposed MSMP algorithm and the SOMP algorithm in terms of spectrum sensing performance. As performance metrics, we use the success rate of spectral support recovery, the probability of miss detection, and the probability of false alarm. The probability of miss detection is examined by computing the number of miss detected subbands over the number of selected subbands. In the same way, the probability of false alarm is computed by the number of false alarms over the number of selected subbands.

In order to test MSMP and SOMP in the multi-band signal model over a wide spectrum, we set 5GHz the wideband range (from 0 to 5GHz) and generate 3 pairs of noisy signal, where $g_i(t) = \sqrt{B_i}\text{sinc}\,(B_i t)$ in (2) and B_i is the 97 percent bandwidth of subband. $n(t)$ in (2) is a standard Gaussian noise process scaled so that the test signal has the desired signal-to-noise ratio (SNR), where the SNR is defined as $10\log(\|\mathbf{\Phi X}\|_2^2/\|\mathbf{W}\|_2^2)$. All signal carriers are chosen uniformly random in the wideband range. For each algorithm, we perform 2,000 independent trials with two different lengths of sampled sequence ($m = 80, 100$), when SNR = 10dB.

In Fig. 4 (a), we provide the success rate of the spectral support recovery as a function of the number of subbands L. High success rate with a large number of subbands implies that it is possible to detect the multi-band signal precisely. Although the bandwidth of multi-band signal is relatively small, the spectral support can be selected accurately by using MSMP. We see that the success rate decreases as L increases (i.e., the bandwidth of subband gets smaller) overall, the success rate of MSMP is higher than that of SOMP at the same L value. In other words, when compared to SOMP, MSMP is less sensitive to the number of subbands L. The result shows that the success rate over 0.95 is accomplished when L is smaller than 1000.

Fig. 4 (b) and Fig. 4 (c) represent the probabilities of miss detection and false alarm as a function of L. Low probabilities of miss detection and false alarm directly imply that the spectrum resources can be utilized efficiently since more unused band can be available. We see that the probabilities of miss detection and false alarm of MSMP are much smaller than that of SOMP. These results demonstrate that MSMP is effective in recovering spectral support for spectrum sensing.

5. CONCLUSIONS

In this paper, we proposed the MSMP algorithm for spectrum sensing. The MSMP algorithm reconstructs the spectral support more accurately due to the substantial reduction in the miss detection and false alarm. This is because MSMP selects multiple indices at each iteration and updates the residual subspace with the double-checked indices. Numerical simulations showed that the proposed algorithm has effectiveness on the spectrum utilization and the superiority on the spectral support recovery.

6. REFERENCES

[1] E. J. Candes and M. Wakin, "An introduction to compressive sampling," *IEEE Signal Process. Mag.*, vol. 25, no. 2, pp. 21–30, Mar. 2008.

[2] J. Choi, B. Shim, Y. Ding, B. Rao, and D. Kim, "Compressed sensing for wireless communications: Useful tips and tricks," *submitted to IEEE Commun. Surveys Tuts.*

[3] D. L. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289–1306, Apr. 2006.

[4] S. F. Cotter, B. D. Rao, K. Engan, and K. Kreutz-Delado, "Sparse solutions to linear inverse problems with multiple measurement vectors," *IEEE Trans. Signal Process.*, vol. 53, no. 7, pp. 2477–2488, July 2005.

[5] W. Zhang, R. Mallik, and K. Letaief, "Optimization of cooperative spectrum sensing with energy detection in cognitive radio networks," *IEEE Trans. on wireless Commun.*, vol. 8, no. 12, pp. 5761–5766, Dec. 2009.

[6] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Select. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[7] M. Mishali and Y. C. Eldar, "From theory to practice: Sub-Nyquist sampling of sparse wideband analog signals," *IEEE J. Sel. Topics Signal Process.*, vol. 4, no. 2, pp. 375–391, Apr. 2010.

[8] J. A. Tropp, J. N. Laska, M. F. Duarte, J. K. Romberg, and R. G. Baraniuk, "Beyond Nyquist: Efficient sampling of sparse bandlimited signals," *IEEE Trans. Inf. Theory*, vol. 56, no. 1, pp. 520–544, Jan. 2010.

[9] M. Mishali and Y. C. Eldar, "Reduce and boost: Recovering arbitrary sets of jointly sparse vectors," *IEEE Trans. Signal Process.*, vol. 56, no. 10, pp. 4692–4702, Oct. 2008.

[10] Y. C. Pati, R. Rezaiifar, and P.S. Krishnaprasad, "Orthogonal matching pursuit: Recursive function approximation with applications to wavelet decomposition," in *Conf. Rec. 27th Asilomar Conf. Signals, Syst. Comput.* IEEE, Nov. 1993, pp. 40–44.

[11] J. Wang, S. Kwon, and B. Shim, "Generalized orthogonal matching pursuit," *IEEE Trans. Signal Process.*, vol. 60, no. 12, pp. 6202–6216, Sept. 2012.

[12] J. A. Tropp, A. C. Gilbert, and M. J. Strauss, "Algorithms for simultaneous sparse approximation. Part I Greedy pursuit," *Signal Process.*, vol. 86, no. 3, pp. 572–588, Mar. 2006.