On Cognitive Radio Systems with Directional Antennas and Imperfect Spectrum Sensing

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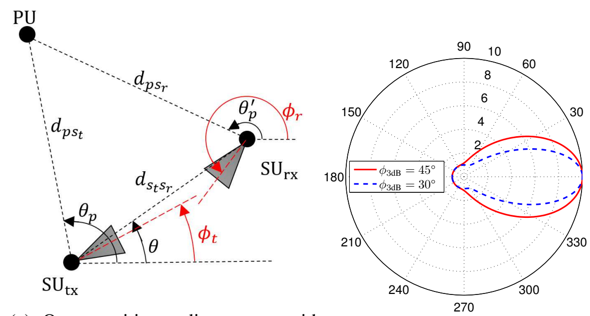
***Abstract*—In this paper, we consider a cognitive radio system, consisting of a primary user (PU), a secondary user (SU) transmitter, and a SU receiver. The SUs are equipped with directional antennas. The SU transmitter first performs spectrum sensing (with errors) and then transmits data. We assume the SU and PU can coexist and the SU transmits at two power levels, according to the result of spectrum sensing (i.e., whether the spectrum is sensed idle or busy). We establish a lower bound on the ergodic capacity of the channel between SU transmitter and receiver, and study how spectrum sensing errors affect the bound. Furthermore, we explore the optimal SU transmit power levels and the optimal directions of SU transmit and receive antennas, such that the lower bound is maximized, subject to average transmit power and average interference power constraints. Through numerical simulations, we show that (compared with the case when the SUs use omni-directional antennas) directional antennas can significantly improve the lower bound in the presence of spectrum sensing errors, subject to the constraints.**

***Index Terms*—capacity maximization, cognitive radio, directional antenna, imperfect spectrum sensing.**

I. Introduction

The communication paradigm of cognitive radios can alleviate spectrum scarcity problem, via allowing an unlicensed (cognitive or secondary) user (SU) to access the underutilized licensed bands opportunistically, in such a way that its imposed interference on the licensed (primary) users (PUs) does not exceed the maximum allowed interference power level [1]. There is a rich collection of elegant results on optimizing transmission strategies for opportunistic spectrum access of SUs, in the presence of a PU activities [2]–[8]. The majority of these works assume the SUs are equipped with *omni-directional antennas* [2]–[8] and they transmit data only when the spectrum is sensed idle. Another commonly adopted assumption is that *the result of spectrum sensing is perfect* [4] (i.e., when the SU senses the spectrum is (not) occupied by the PU and is busy (idle), the spectrum is truly busy (idle)). However, all spectrum sensing methods (e.g., matched filter detection, energy detection) are prone to errors, quantified in terms of false alarm and detection probabilities, due to several sources of uncertainties including noise.

In this work, we assume the SUs are equipped with *directional antennas*. The directional antennas can identify and enable transmission and reception across *spatial domain* [9]–[12] and further increase spectrum utilization, compared with omni-directional antennas. Also, we assume the *SUs and PU can coexist* and the SU can adapt its transmission



(a) Our cognitive radio system with directional antennas.

(b) Antenna pattern

Fig. 1: Our system model and antenna gain

power, according to the result of spectrum sensing (i.e., whether the spectrum is sensed idle or busy). We study how *spectrum sensing errors* affect the ergodic capacity of the channel between SU transmitter and receiver. Furthermore, we explore the optimal SU transmit power and optimal orientations of SU transmit and receive antennas, such that the channel capacity lower bound is maximized, subject to average transmit power and average interference power constraints. *To the best of our knowledge, this is the first work that combines the notions of directional antennas and imperfect spectrum sensing for cognitive radio systems.*

II. System Model

Fig. 1a depicts the cognitive radio system under consideration. The system consists of a PU, a SU transmitter (SU$_{\rm tx}$) and a SU receiver (SU$_{\rm rx}$). Suppose the SUs are equipped with steerable directional antennas and the main-lobes of SU$_{\rm tx}$ and SU$_{\rm rx}$ in their local coordination are centered on the angles ϕ_t and ϕ_r, respectively. The directions of PU and SU$_{\rm rx}$ with respect to SU$_{\rm tx}$ in azimuth plane are denoted by θ_p and θ, receptively. The direction of PU with respect to SU$_{\rm rx}$ is denoted by θ_p'. We assume the locations of PU and SUs are known and hence θ_p, θ and θ_p' are known. The angles ϕ_t and ϕ_r, however, are unknown and will be optimized.

A. Spectrum Sensing

We formulate the spectrum sensing problem at the SU$_{\rm tx}$ as a binary hypothesis testing problem, where H_1 denotes that the spectrum is occupied by the PU and thus is truly busy, and H_0 indicates the spectrum is not occupied by the PU and hence is truly idle. Let $\pi_1 = \Pr\{H_1\}$ and

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$\pi_0 = \Pr\{H_0\}$, respectively, represent the probabilities that the spectrum is truly busy and truly idle. Let $\hat{H}_1$ and $\hat{H}_0$, respectively, denote that the result of spectrum sensing is busy and idle. The accuracy and reliability of any spectrum sensing method can be characterized in terms of false alarm and detection probabilities, defined as $P_f = \Pr\{\hat{H}_1|H_0\}$ and $P_d = \Pr\{\hat{H}_1|H_1\}$. Clearly, $\Pr\{\hat{H}_0|H_1\} = 1 - P_d$ and $\Pr\{\hat{H}_0|H_0\} = 1 - P_f$. Let $\hat{\pi}_1 = \Pr\{\hat{H}_1\}$ and $\hat{\pi}_0 = \Pr\{\hat{H}_0\}$, respectively, show the probabilities that the spectrum is sensed busy and idle. It is easy to verify $\hat{\pi}_1 = \pi_0 P_f + \pi_1 P_d$ and $\hat{\pi}_0 = \pi_0(1-P_f) + \pi_1(1-P_d)$. In this work, we assume π_0, P_f, P_d are known. Also, when the spectrum is truly busy, the PU transmission power level is σ_p^2, although $\mathrm{SU_{tx}}$ is unaware of this value. Transmit power level of $\mathrm{SU_{tx}}$ depends on the result of spectrum sensing. When the spectrum is sensed idle and busy, $\mathrm{SU_{tx}}$ uses transmit power $P^{(0)}$ and $P^{(1)}$, respectively.

B. Data Communication Channel

Let $s_c[m]$ denote the discrete-time symbol transmitted by $\mathrm{SU_{tx}}$ and $r[m]$ represent the corresponding discrete-time signal received by $\mathrm{SU_{rx}}$. We assume a block transmission/reception model where the SUs transmit and receive several consecutive blocks of M symbols. We assume that during each block we have the following relationship

$$r[m] = h_{s_t s_r}\sqrt{G(\theta, \phi_t, \phi_r)}s_c[m] + n[m], \quad \text{for } m = 1, ..., M$$

where $G(\theta, \phi_t, \phi_r) = A(\phi_t - \theta)A(\phi_r - \pi - \theta)$ is the product of $\mathrm{SU_{tx}}$ and $\mathrm{SU_{rx}}$ antennas' gain. The term $h_{s_t s_r}$ is the fading coefficient between $\mathrm{SU_{tx}}$ and $\mathrm{SU_{rx}}$. The term $n[m]$ is the additive noise at $\mathrm{SU_{rx}}$ and is modeled as Gaussian $n[m] \sim N(0, \sigma_n^2)$. The transmitted symbols $s_c[m]$ are digitally modulated signals and have the average power $P^{(0)}$ or $P^{(1)}$ when the spectrum is sensed idle or busy, respectively. We model the antenna gain $A(\phi)$ as a function of direction ϕ as $A(\phi) = A_1 + A_0 \ \exp\big(-B\big(\frac{\phi}{\phi_{\mathrm{3dB}}}\big)^2\big)$ [11] where ϕ_{3dB} is the 3dB beam-width of antenna, $B = \ln(2)$, A_1 and A_0 are two constant parameters and A_1 is the minimum antenna gain. This model is an approximation of a real antenna pattern used in [11], [12]. In Fig. 1b the antenna pattern is depicted for $A_1 = 1$, $A_0 = 9$ and $\phi_{\mathrm{3dB}} = 30°, 45°$. Let d_{ps_t}, d_{ps_r} and $d_{s_t s_r}$ be the distances between PU and $\mathrm{SU_{tx}}$, PU and $\mathrm{SU_{rx}}$, and $\mathrm{SU_{tx}}$ and $\mathrm{SU_{rx}}$, respectively. All the fading coefficients include path loss and have Rayleigh distribution with variance $\sigma_h^2 = (\frac{d_0}{d})^\nu$, where d_0 is the reference distance, d is the distance between users (SU or PU), and ν is the path loss exponent.

III. Ergodic Capacity Maximization

we first characterize the ergodic capacity of the channel between $\mathrm{SU_{tx}}$ and $\mathrm{SU_{rx}}$, incorporating the fact that the result of spectrum sensing is imperfect, in terms of four optimization parameters: the antenna angles ϕ_t, ϕ_r and transmit power levels $P^{(0)}$ and $P^{(1)}$. Next, we study optimal ϕ_t, ϕ_r, $P^{(0)}$ and $P^{(1)}$, such that the channel capacity is maximized, subject to average transmit power constraint and average interference power constraint. For the clairvoyant scenario when spectrum sensing is perfect the maximum rate that the channel can support is $C = \mathbb{E}\{c_{0,0} + c_{1,1}\}$, where

$$c_{0,0} = \log_2\left(1 + \frac{|h_{s_t s_r}|^2 \ G(\theta, \phi_t, \phi_r) \ P^{(0)}}{\sigma_n^2}\right), \quad (1)$$

$$c_{1,1} = \log_2\left(1 + \frac{|h_{s_t s_r}|^2 \ G(\theta, \phi_t, \phi_r) \ P^{(1)}}{\sigma_n^2 + \sigma_p^2 \ |h_{ps_r}|^2 \ A(\phi_r - \theta'_p)}\right). \quad (2)$$

$\mathbb{E}\{.\}$ is the expectation operator and the expectation is taken with respect to random fading coefficients. The terms $c_{0,0}$ and $c_{1,1}$ are channel capacities when the spectrum is idle and busy, respectively and h_{ps_r} is the fading coefficient from PU to $\mathrm{SU_{rx}}$. The term $\sigma_p^2|h_{ps_r}|^2 A(\phi_r - \theta'_p)$ in (2) captures the interference on $\mathrm{SU_{rx}}$ due to PU activities. However, when spectrum sensing is imperfect, depending on the true status of the PU and the spectrum sensing result, two extra terms $c_{0,1}$ and $c_{1,0}$ would be added to the capacity expression where

$$c_{0,1} = \log_2\left(1 + \frac{|h_{s_t s_r}|^2 \ G(\theta, \phi_t, \phi_r) \ P^{(1)}}{\sigma_n^2}\right), \quad (3)$$

$$c_{1,0} = \log_2\left(1 + \frac{|h_{s_t s_r}|^2 \ G(\theta, \phi_t, \phi_r) \ P^{(0)}}{\sigma_n^2 + \sigma_p^2 \ |h_{ps_r}|^2 \ A(\phi_r - \theta'_p)}\right), \quad (4)$$

and $c_{i,j}$, for $i, j \in \{0, 1\}$ is instantaneous capacity corresponding to H_i and $\hat{H}_j$ with the corresponding probability $\Pr\{H_i, \hat{H}_j\}$. Thus, the ergodic capacity can be written as $C = \mathbb{E}\{\alpha_0 \ c_{0,0} + \beta_0 \ c_{1,0} + \alpha_1 \ c_{0,1} + \beta_1 \ c_{1,1}\}$ where

$$\alpha_0 = \Pr\{H_0, \hat{H}_0\} = \pi_0(1-P_f), \quad \alpha_1 = \Pr\{H_0, \hat{H}_1\} = \pi_0 P_f,$$
$$\beta_0 = \Pr\{H_1, \hat{H}_0\} = \pi_1(1-P_d), \quad \beta_1 = \Pr\{H_1, \hat{H}_1\} = \pi_1 P_d.$$

Let $\bar{I}_{\mathrm{av}}$ denote the maximum allowed interference power level. To satisfy the average interference power constraint, we have

$$\mathbb{E}\Big\{\Big[\beta_0 \ P^{(0)} + \beta_1 \ P^{(1)}\Big] \ |h_{s_t p}|^2 \ A(\phi_t - \theta_p)\Big\} \le \bar{I}_{\mathrm{av}}. \quad (5)$$

Let $\bar{P}_{\mathrm{av}}$ indicate the maximum average transmit power of $\mathrm{SU_{tx}}$. To satisfy the average transmit power constraint, we find

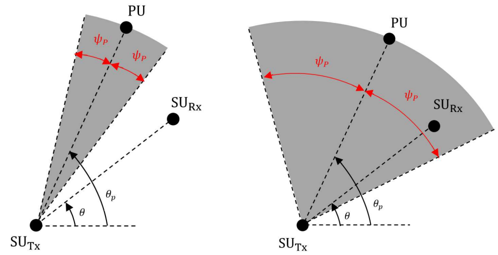
$$\hat{\pi}_0 \ P^{(0)} + \hat{\pi}_1 \ P^{(1)} \le \bar{P}_{\mathrm{av}}. \quad (6)$$

Since $\mathrm{SU_{tx}}$ is unaware of σ_p^2 value, we lump the effects of PU interference on $\mathrm{SU_{rx}}$ in (3) and (4) and the additive noise at $\mathrm{SU_{rx}}$ into one Gaussian noise term with power σ^2 [4], [8]. Consequently, we can rewrite c_{00}, c_{10}, c_{01} and c_{11} as

$$c_{0,0} = c_{1,0} = \log_2\left(1 + \frac{|h_{s_t s_r}|^2 \ G(\theta, \phi_t, \phi_r) \ P^{(0)}}{\sigma^2}\right), \quad (7)$$

$$c_{0,1} = c_{1,1} = \log_2\left(1 + \frac{|h_{s_t s_r}|^2 \ G(\theta, \phi_t, \phi_r) \ P^{(1)}}{\sigma^2}\right), \quad (8)$$

and C reduces to $C = \mathbb{E}\big\{\hat{\pi}_0 \ c_{0,0} + \hat{\pi}_1 \ c_{0,1}\big\}$. Next, we find $\mathbb{E}\{c_{0,0}\}$ and $\mathbb{E}\{c_{0,1}\}$. If random variable x has a exponential distribution with parameter λ, we have $\mathbb{E}\big\{\ln\big(1 + \rho x\big)\big\} = -e^{\frac{\lambda}{\rho}} \ \mathrm{Ei}\big(\frac{-\lambda}{\rho}\big)$ where $\mathrm{Ei}(-z) = -\int_z^\infty e^{-t} \ t^{-1} dt$ [13]. In our



(a) when $|\theta_p - \theta| > \psi_p$ (b) when $|\theta_p - \theta| < \psi_p$

Fig. 2: Illustration of ϕ_t^{opt} for $0 < Z \leq 1$

problem $|h_{i,j}|^2$ has an exponential distribution with parameter $\lambda_{i,j} = (1 - \frac{\pi}{4})/\sigma^2_{h_{i,j}}$, where i and j can be SU$_{\text{tx}}$, SU$_{\text{rx}}$ or PU. Thus, we can write $\mathbb{E}\Big\{ \ln\Big(1 + \frac{|h_{s_t s_r}|^2 \ GP^{(0)}}{\sigma^2}\Big) \Big\} = -e^{\frac{\sigma^2}{aP^{(0)}}} \ \text{Ei}\Big(\frac{-\sigma^2}{aP^{(0)}}\Big)$ where $G \triangleq G(\theta, \phi_t, \phi_r)$ and $a = G/\lambda_{s_t s_r}$. Since the analytical maximization of the capacity is infeasible, instead, we establish a lower bound on the capacity, denoted as C^{LB}, and maximize C^{LB}. Using the inequality $\frac{1}{2} \ \ln\big(1 + \frac{2}{x}\big) < -e^{x} \ \text{Ei}(-x) < \ln\big(1 + \frac{1}{x}\big)$ for $x > 0$ [14] we can write $\mathbb{E}\big\{ \ln\big(1 + \frac{|h_{s_t s_r}|^2 \ GP^{(0)}}{\sigma^2}\big) \big\} > \frac{1}{2} \ln\big(1 + \frac{2aP^{(0)}}{\sigma^2}\big)$ and obtain

$$C^{\text{LB}} = \frac{\hat{\pi}_0}{2} \log_2\left(1 + \frac{2aP^{(0)}}{\sigma^2}\right) + \frac{\hat{\pi}_1}{2} \log_2\left(1 + \frac{2aP^{(1)}}{\sigma^2}\right). \quad (9)$$

In the following, we address maximization of C^{LB} with respect to four optimization parameters ϕ_t, ϕ_r, $P^{(0)}$, $P^{(1)}$, subject to two constraints in (5) and (6). Let ϕ_t^{opt}, ϕ_r^{opt}, $P^{(0)}_{\text{opt}}$, $P^{(1)}_{\text{opt}}$ be the optimal solutions. This optimization problem is convex with respect to ϕ_r, $P^{(0)}$, $P^{(1)}$, but not with respect to ϕ_t. Since ϕ_t lies in interval $[0, 2\pi]$, ϕ_t^{opt} can be obtained using one-dimensional exhaustive search, i.e., we can consider an initial value for ϕ_t and solve the problem with respect to ϕ_r, $P^{(0)}$, $P^{(1)}$. Then, we find the value of ϕ_t which maximizes the C^{LB} [15]. Given ϕ_t, the constrained maximization of C^{LB} with respect to ϕ_r, $P^{(0)}$, $P^{(1)}$ can be solved using the Lagrange multiplier method. By applying the Karush-Kuhn-Tucker (KKT) conditions, the optimum value for ϕ_r can be obtained as $\phi_r^{\text{opt}} = \pi + \theta$. We define

$$\begin{aligned} b_0 &= A(\phi_t - \theta_p)\ \beta_0/\lambda_{s_t p}, & b_1 &= A(\phi_t - \theta_p)\ \beta_1/\lambda_{s_t p} \\ \Sigma &= b_1\hat{\pi}_0 - b_0\hat{\pi}_1, & \Upsilon &= b_1\hat{\pi}_0^2 + b_0\hat{\pi}_1^2 \\ \Psi &= -b_1\bar{P}_{\text{av}} + \hat{\pi}_1\bar{I}_{\text{av}}, & \Pi &= b_0\bar{P}_{\text{av}} - \hat{\pi}_0\bar{I}_{\text{av}}. \end{aligned}$$

The parameter Σ can be simplified as $\Sigma = \frac{\pi_0\pi_1}{\lambda_{s_t p}}(P_d - P_f)A(\phi_t - \theta_p)$. Assuming $P_d \gg P_f$ we conclude that $\Sigma > 0$. Solving the KKT conditions with regard to $P^{(0)}$ and $P^{(1)}$, we obtain the optimal transmit power under $\hat{H}_0$ and $\hat{H}_1$

$$P^{(0)} = \begin{cases} \frac{\bar{I}_{\text{av}}}{b_0}, & \Delta_2 < 0,\ \Pi \geq 0, & \text{[case I]} \\ \bar{P}_{\text{av}}, & \Delta_0 \leq 0, & \text{[case II]} \\ \frac{\Delta_1}{2ab_0}, & \Delta_3 \leq 0,\ \Delta_2 > 0, & \text{[case III]} \\ \frac{-\Psi}{\Sigma}, & \Delta_3 > 0,\ \Delta_0 > 0,\ \Pi < 0,\ \Psi < 0 & \text{[case IV]} \end{cases} \quad (10)$$

$$P^{(1)} = \begin{cases} 0, & \Delta_2 < 0,\ \Pi \geq 0, & \text{[case I]} \\ \bar{P}_{\text{av}}, & \Delta_0 \leq 0, & \text{[case II]} \\ \frac{\Delta_2}{2ab_1}, & \Delta_3 \leq 0,\ \Delta_2 > 0, & \text{[case III]} \\ \frac{-\Pi}{\Sigma}, & \Delta_3 > 0,\ \Delta_0 > 0,\ \Pi < 0,\ \Psi < 0 & \text{[case IV]} \end{cases} \quad (11)$$

where $\Delta_0 = (b_0 + b_1)\bar{P}_{\text{av}} - \bar{I}_{\text{av}}$, $\Delta_1 = 2a\hat{\pi}_0\bar{I}_{\text{av}} + \sigma^2\Sigma$ and

$$\begin{aligned} \Delta_2 &= 2a\hat{\pi}_1\bar{I}_{\text{av}} - \sigma^2\Sigma \\ \Delta_3 &= \sigma^2\Sigma^2 + 2a\Upsilon\bar{I}_{\text{av}} - 2a\bar{P}_{\text{av}}b_0b_1. \end{aligned}$$

In order to reduce the computational complexity of one-dimensional exhaustive search for finding ϕ_t^{opt} in the interval $[0, 2\pi]$, in the following we find a narrower interval to which ϕ_t^{opt} belongs to, i.e., we find ϕ_t^{L} and ϕ_t^{U} such that $\phi_t^{\text{opt}} \in \big[\phi_t^{\text{L}}, \phi_t^{\text{U}}\big]$. The maximum imposed interference on PU would occur when SU$_{\text{tx}}$ always transmits data with maximum allowable transmit power $P^{(0)} = P^{(1)} = \bar{P}_{\text{av}}$ without considering the spectrum sensing result. In this case, considering (5) we obtain $A(\phi_t - \theta_p) \leq \frac{\lambda_{s_t p}\ \bar{I}_{\text{av}}}{\pi_1 \bar{P}_{\text{av}}}$. We define $Z = (\lambda_{s_t p}\ \bar{I}_{\text{av}})/(\pi_1 A_0 \bar{P}_{\text{av}}) - A_1/A_0$. From (5) we have $\exp\big(-B\big(\frac{\phi_t - \theta_p}{\phi_{3\text{dB}}}\big)^2\big) \leq Z$. If $Z > 1$, it means that PU can tolerate an interference power that is larger than the interference power imposed by SU$_{\text{tx}}$ and (5) holds true for every value of ϕ_t and it is obvious that $\phi_t^{\text{opt}} = \theta$ maximizes C^{LB}. For $0 < Z \leq 1$, we define $\psi_p = \phi_{3\text{dB}}\sqrt{\frac{-1}{B}\ln(Z)}$ and consider two cases. When $|\theta_p - \theta| > \psi_p$, ϕ_t^{opt} has to lie outside the shaded area shown in Fig. 2a. The unshaded area in Fig. 2a includes the line of sight (LOS) between SU$_{\text{tx}}$ and SU$_{\text{rx}}$ and it is clear that $\phi_t^{\text{opt}} = \theta$. When and $|\theta_p - \theta| < \psi_p$, which is shown in Fig. 2b, ϕ_t^{opt} lies in the following interval

$$\begin{cases} \phi_t^{\text{opt}} \in [\theta_p - \psi_p, \theta], & \text{if } \theta_p > \theta \\ \phi_t^{\text{opt}} \in [\theta, \theta_p + \psi_p], & \text{if } \theta_p < \theta \end{cases} \quad (12)$$

If $Z \leq 0$, we cannot find a narrower interval and the entire interval $[0, 2\pi]$ should be considered in one-dimensional exhaustive search. The following table summarizes our proposed approach to find the optimal solutions ϕ_t^{opt}, ϕ_r^{opt}, $P^{(0)}_{\text{opt}}$ and $P^{(1)}_{\text{opt}}$.

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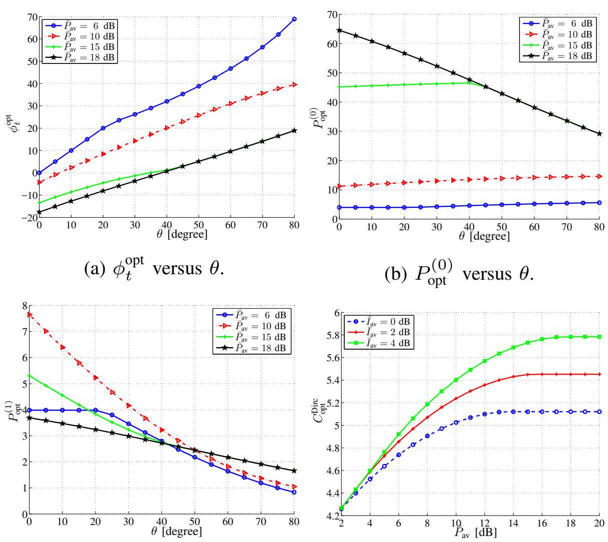
1) $\phi_r^{\text{opt}} = \pi + \theta$,
2) Calculate Z,
3) Calculate the interval which contains ϕ_t^{opt},
- **Case a.** If $Z > 1$ or if $Z < 1$ and $|\theta_p - \theta| > \psi_p$, then $\phi_t^{\text{opt}} = \theta$, i.e., no further optimization over ϕ_t is needed,
- **Case b.** If $0 < Z < 1$ and $|\theta_p - \theta| < \psi_p$, then ϕ_t^{opt} lies in the interval mentioned in (12),
- **Case c.** If $Z \leq 0$, then ϕ_t^{opt} lies in the interval $[0, 2\pi]$,

4) For case **a**, calculate $P^{(0)}_{\text{opt}}$, $P^{(1)}_{\text{opt}}$ using (10) and (11). For cases **b** and **c**, given a ϕ_t value within the obtained interval, calculate $P^{(0)}$, $P^{(1)}$ using (10) and (11),
5) For cases **b** and **c**, substitute $P^{(0)}$, $P^{(1)}$, ϕ_t, ϕ_r^{opt} in (9),
6) $\Big[\phi_t^{\text{opt}}, P^{(0)}_{\text{opt}}, P^{(1)}_{\text{opt}}\Big] = \text{argmax}\ \big\{C^{\text{LB}}\big\}$.

IV. Numerical Results and Conclusion

Using Matlab simulations, we illustrate how directional antennas can improve the channel capacity when considering imperfect spectrum sensing. Assume $\sigma^2 = 1$, $\phi_{3\text{dB}} = 45°$, $A_0 = 9$, $A_1 = 1$ and $\pi_1 = 0.3$. Let $\nu = 2$, $d_{ps_t} = 4$ m and $d_{s_t s_r} = 2$ m. Suppose $C_{\text{opt}}^{\text{Dirc}}$ denote C^{LB} in (9), evaluated at the optimal solutions ϕ_t^{opt}, ϕ_r^{opt}, $P_{\text{opt}}^{(0)}$ and $P_{\text{opt}}^{(1)}$. We compare $C_{\text{opt}}^{\text{Dirc}}$ with the lower bound on the channel capacity when SU_{tx} and SU_{rx} have omni-directional antennas with the antenna gain $A(\phi) = 10$ for all ϕ, and only transmit powers $P^{(0)}$ and $P^{(1)}$ are optimized subject to the transmit power and interference constraints, which we denote as $C_{\text{opt}}^{\text{Omni}}$. Furthermore, to quantify the advantage of optimizing the angles of SU_{tx} and SU_{rx} directional antennas, we compare $C_{\text{opt}}^{\text{Dirc}}$ with C^{LB} in (9), evaluated at $\phi_t = \theta$, $\phi_r = \pi + \theta$ (the antennas of SU_{tx} and SU_{rx} are exactly pointed at each other), $P^{(0)}$ and $P^{(1)}$ obtained from (10) and (11), which we denote as $C_{\text{opt}}^{\text{LOS}}$. For fair comparisons, we consider a fixed spectrum sensing method with $P_d = 0.9$ and $P_f = 0.1$. Note that $P_{\text{opt}}^{(0)}$ and $P_{\text{opt}}^{(1)}$ are constant for all θ when SUs use omni-directional antenna and $C_{\text{opt}}^{\text{Omni}}$ is independent of θ.

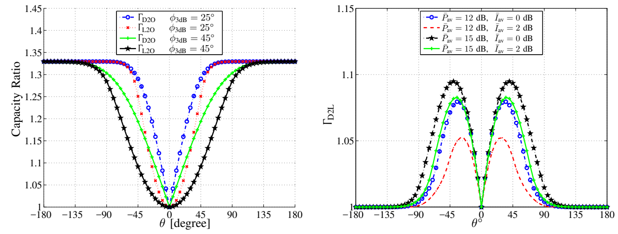


(a) ϕ_t^{opt} versus θ. (b) $P_{\text{opt}}^{(0)}$ versus θ.

(c) $P_{\text{opt}}^{(1)}$ versus θ. (d) $C_{\text{opt}}^{\text{Dirc}}$ versus $\bar{P}_{\text{av}}$.

Fig. 3: Optimal solutions versus θ.

Fig. 3a to Fig. 3c present the optimal ϕ_t^{opt}, $P_{\text{opt}}^{(0)}$ and $P_{\text{opt}}^{(1)}$ versus angle θ, when $\theta_p = 90°$, $\bar{P}_{\text{av}} = 6, 10, 15, 18$ dB and $\bar{I}_{\text{av}} = 0$ dB. Fig. 3a shows that as θ increases, ϕ_t^{opt} increases and approaches to θ_p. When $\bar{P}_{\text{av}} = 6$ dB, for $\theta \leq 20°$ the interference which is imposed on the PU is less than $\bar{I}_{\text{av}}$ and SU_{tx} transmits by it's maximum allowable power, i.e., $P^{(0)} = P^{(1)} = \bar{P}_{\text{av}}$ and $\phi_t^{\text{opt}} = \theta$ (case II in equations (10) and (11)). As θ increases beyond $20°$, case IV happens. Since b_0 and b_1 increase equally and $P^{(0)} = (\bar{P}_{\text{av}} - \frac{\hat{\pi}_1}{b_1}\bar{I}_{\text{av}})/(\hat{\pi}_0 - \frac{b_0}{b_1}\hat{\pi}_1)$ we can conclude that $P_{\text{opt}}^{(0)}$ increases. With the same argument



(a) Γ_{D2O} and Γ_{L2O} versus θ. (b) Γ_{D2L} versus θ.

Fig. 4: Behavior of three capacity ratios versus θ.

we conclude that $P_{\text{opt}}^{(1)}$ decreases. When $\bar{P}_{\text{av}} = 10, 15, 18$ dB, for all $\theta \in [0°, 80°]$, we have $\Delta_0 \geq 0$. When $\bar{P}_{\text{av}} = 10$ dB, case IV in (10) and (11) happens. Thus, as θ increases, b_0 and b_1 increase. As a result, $P_{\text{opt}}^{(0)}$ increases, while $P_{\text{opt}}^{(1)}$ decreases. When $\bar{P}_{\text{av}} = 15$ dB, for $\theta \leq 42°$, case IV happens. However, for $\theta > 42°$ case III happens. Since in case III b_0 and b_1 are in the denominator, both $P_{\text{opt}}^{(0)}$ and $P_{\text{opt}}^{(1)}$ decrease by increasing θ. When $\bar{P}_{\text{av}} = 18$ dB, for all $\theta \in [0°, 80°]$ case III happens and $P_{\text{opt}}^{(0)}$ and $P_{\text{opt}}^{(1)}$ decrease as θ increases. Fig. 3d shows the $C_{\text{opt}}^{\text{Dirc}}$ versus $\bar{P}_{\text{av}}$ for different values of $\bar{I}_{\text{av}}$. We observe that by increasing $\bar{P}_{\text{av}}$ or $\bar{I}_{\text{av}}$, $C_{\text{opt}}^{\text{Dirc}}$ increases.

We define three capacity ratios $\Gamma_{\text{D2O}} = C_{\text{opt}}^{\text{Dirc}}/C_{\text{opt}}^{\text{Omni}}$, $\Gamma_{\text{L2O}} = C_{\text{opt}}^{\text{LOS}}/C_{\text{opt}}^{\text{Omni}}$ and $\Gamma_{\text{D2L}} = C_{\text{opt}}^{\text{Dirc}}/C_{\text{opt}}^{\text{LOS}}$. Fig. 4a plot Γ_{D2O} and Γ_{L2O} versus θ when $\theta_p = 0°$, $\bar{P}_{\text{av}} = 15$ dB and $\bar{I}_{\text{av}} = 0$ dB for $\phi_{3\text{dB}} = 25°, 45°$. It can be seen that when $\theta = 0$, $C_{\text{opt}}^{\text{Dirc}} \approx C_{\text{opt}}^{\text{LOS}}$ and as $|\theta - \theta_p|$ increases, $C_{\text{opt}}^{\text{Dirc}} > C_{\text{opt}}^{\text{LOS}}$. We observe that $C_{\text{opt}}^{\text{Dirc}} > C_{\text{opt}}^{\text{LOS}}$ for $|\theta - \theta_p| < 135°$ and for $|\theta - \theta_p| > 135°$, $C_{\text{opt}}^{\text{Dirc}} \approx C_{\text{opt}}^{\text{LOS}}$ when $\phi_{3\text{dB}} = 45°$. Since $C_{\text{opt}}^{\text{Omni}}$ is independent of θ, by comparing Γ_{D2O} to Γ_{L2O} in Fig. 4a, we can see that the trend of the curves in two figures are different. In fact, the shape of the curves for Γ_{D2O} is similar to letter **V**. However, this shape is similar to letter **U** for Γ_{L2O}. It means that Γ_{D2O} increases sharper (has a larger slope) with respect to θ, compared with Γ_{L2O}. This effect is also shown in Fig. 4b where the performance gain of $C_{\text{opt}}^{\text{Dirc}}$ against $C_{\text{opt}}^{\text{LOS}}$ is plotted. Also, in Fig. 4a we see that for a fixed value of θ, as the beam-width decreases, the capacity gain increases. In other words, as half power beam-width decreases, the directional antenna can cancel more interference power imposed on or by the PU. Thus, the optimal capacity can reach to its maximum value for smaller $|\theta - \theta_p|$.

In summary, we considered a cognitive radio system, where the SUs are equipped with directional antennas and spectrum sensing is imperfect. We explored the optimal SU transmit power levels and the optimal directions of SU antennas, such that the channel capacity lower bound is maximized, subject to average transmit power and average interference power constraints. Through simulations, we showed that directional antennas significantly enhance the lower bound.

Acknowledgment

This research is supported by NSF under grant ECCS-1443942.

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