OPTIMAL ACHIEVABLE RATE TRADE-OFF IN COOPERATIVE COGNITIVE RADIO SYSTEMS

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ABSTRACT

A relay-aided cooperative underlay cognitive radio system with random number of secondary users (SU), and an interference constraint at the primary receiver (PR) is considered. When the primary (PU) is active, those SUs whose interference at the PR is above a threshold will amplify-and-forward PU's signal to enhance the PU's performance, and those SUs below this threshold will proceed with their own transmission in an underlay fashion. As the mean number of relay SUs increases, the PU's performance is improved but causes degradation of SU system performance. We study the optimal trade-off between the achievable rate of the SU and PU systems in the large mean number of SUs regime. This is then followed by a derivation of the optimal ratio between the mean number of relays and underlay SUs. Monte-Carlo simulations are used to verify our analytical results.

Index Terms— Cognitive radio, opportunistic relay selection, random number of users

1. INTRODUCTION

Cognitive radio (CR) is a promising technology to overcome the scarcity in the available spectrum [1]. In overlay CR systems, SUs sense the PU's activity and only transmit when the PU is idle [1, 2]. In underlay CR systems, which is the focus of this paper, SUs transmit simultaneously with the PU provided that its interference at the primary receiver is below some threshold [3, 4]. However, SUs will transmit with limited power in order to satisfy the interference constraint.

Cooperative spectrum sharing systems, where SUs assist the PU's transmission has been considered recently in the literature. Cooperation between the PU and the SU networks has been studied in an information theoretic framework in [5, 6]. Physical layer spectrum sharing protocols based on amplify-and-forward (AF) and decode-and-forward (DF) relaying techniques are proposed in [7] and [8] where single transmit/receive pair of PUs and SUs are considered with outage probability as a performance metric. Reference [9] considers the case of multiple SUs with opportunistic SU relay selection [10, 11], and studies *numerically* the choice of number of potential SU relays. Opportunistic relay selection based on the highest end-to-end relay gain has been recently studied in the cognitive radio setup [12–14] to achieve spatial diversity, where the relay cooperation is exclusively *among* either PU or SU networks.

In this paper, we consider a cooperative underlay CR system with single PU and random number of SUs, where the cooperation is *between* the PU and SUs. Unlike [9], we aim to characterize the optimal rate trade-off *analytically* for the first time in the literature using a novel combined sum rate metric of the PU and SU systems. The total $\mathcal{L}$ SUs are split into two groups based on whether they satisfy a predefined interference threshold at the primary receiver (PR). Also unlike the exiting literature where SUs continuously adjust their transmission powers to satisfy the interference constraint, we assume each SU operates with a fixed power level that significantly simplifies the transmitter. The number of SUs that is above the threshold is random at each time instance due to the random nature of the interference channel. These $\mathcal{M} \leq \mathcal{L}$ SUs below the threshold enter the underlay mode. Multi-user diversity (MUD) is used where the best SU is selected among the $\mathcal{M}$ SUs. The remaining $\mathcal{N} = \mathcal{L} - \mathcal{M}$ SUs whose interference exceed the threshold will amplify and relay the PU's signal to mitigate the limited interference caused by the selected underlay SU. AF relaying is assumed due to its practical simplicity and to maintain the PU's privacy by keeping the SU from decoding the PU's signal. With opportunistic relay selection, we assume that one out of $\mathcal{M}$ SUs with the best end-to-end channel will relay the PU's signal in exchange for spectrum access. The average achievable rates of both PU and SU are studied. The sum of these achievable rates is dependent on the ratio $t = \mathrm{E}[\mathcal{N}] / \mathrm{E}[\mathcal{L}]$. When $\mathrm{E}[\mathcal{N}]$ increases, the desired SU relay can be selected from a larger number of potential relay candidates, which improves the PU's average achievable rate. Meanwhile this reduces the average number of underlay SUs from which the best SU is chosen, and so deteriorates the underlay SU's average transmission rate. We capture this performance trade-off between the primary and secondary systems with a single metric, which enables optimizing the combined system performance with respect to the ratio t in closed form. Compared with the existing literature, we study beyond the outage performance of [8, 9], and derive analytically the optimal value of t as the mean number of SUs $\lambda = \mathrm{E}[\mathcal{L}]$ grows. This optimal value converges to $t = 1/3$ due to the half-duplex nature of the cooperation and the reduced effect of the fading for large mean number of users. Numerical results show that this even holds for moderate λ, and also when there exists mutual interference at the SR from the primary network. Similar performance trade-off can be captured under the same analysis paradigm when the metric is average bit error rate rather than achievable rate [15].

The rest of the paper is organized as follows. Section 3.1 investigates scaling laws for achievable rates of both PU and the selected SU for large number of users and maximizes the system sum rates in the deterministic number of SUs case. Section 3.2 extends this study to a large class of user number

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distributions. Section 4 presents numerical results for achievable rate to corroborate our analytical results. Section 5 concludes our work.

2. SYSTEM MODEL

System model is shown in Figure 1, where dots stand for SU relays and stars stand for underlay SUs. We assume that at the beginning of each transmission block, SUs decide their access mode in the transmission phase by estimating their interference temperature at the PR with the knowledge of the interference channel through feedback [16], and comparing with a threshold. $\mathcal{N}$ SUs above the interference threshold will serve as relays to the PU in order to compensate for the limited interference caused by the selected underlay SU at the PR.

The opportunistic relay selection is employed by the PU and SUs. A single SU out of the $\mathcal{N}$ SUs is selected, depending on which SU relay provides the largest end-to-end path gain between the PU and the PR. We adopt the cooperative diversity model in [17] so that during the first half of the transmission block, the PU broadcasts its signal, and SUs and the PR receive it. In the second half, the selected SU relays the received signal to the PR, and the PR combines the two copies using maximum ratio combining (MRC). We assume all channels are Rayleigh fading.

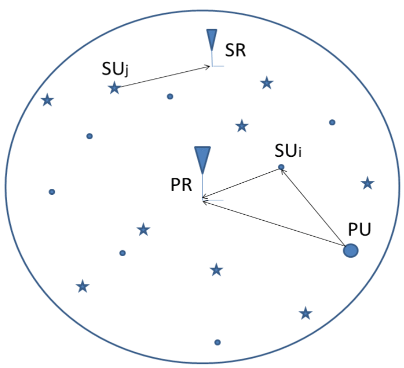

Fig. 1. Cooperative Cognitive Radio Systems

For the primary network, channel gains on the PU-PR link, the PU to the i^{th} SU relay link, and the i^{th} SU relay to the PR link are denoted as γ_{D}, $\gamma_{\mathrm{P,S}_i}$, and $\gamma_{\mathrm{S}_i,\mathrm{P}}$ respectively with average received SNR $\mathrm{E}[\gamma_{\mathrm{D}}] = \beta_1$, $\mathrm{E}[\gamma_{\mathrm{P,S}_i}] = \beta_2$ and $\mathrm{E}[\gamma_{\mathrm{S}_i,\mathrm{P}}] = \beta_3$. The additive white Gaussian noise (AWGN) at the PR and the i^{th} SU, and the transmitted signal at each user has zero mean and unit variance. Assuming that $A = \sqrt{1/(\beta_2\gamma_{\mathrm{P,S}_i} + 1)}$ is the amplification factor at the i^{th} SU, constant average output power is maintained [17], and the received SNR of the SR with the help of the opportunistic relay γ_{R}^* is derived as

$$\gamma_{\mathrm{R}}^* = \max_{1\le i\le \mathcal{N}} \frac{\gamma_{\mathrm{P,S}_i}\gamma_{\mathrm{S}_i,\mathrm{P}}}{1+\gamma_{\mathrm{P,S}_i}+\gamma_{\mathrm{S}_i,\mathrm{P}}}. \quad (1)$$

Due to the MUD scheme among $\mathcal{M}$ underlay SUs below the interference threshold, only a single underlay SU with the highest receiving SNR at the SR will be selected to transmit. The received SNR of the selected SU is denoted by

$$\gamma_{\mathrm{S}}^* = \max_{1\le j\le \mathcal{M}} \gamma_{\mathrm{S}_j}. \quad (2)$$

where γ_{S_j} denotes the individual underlay channel gain on the j^{th} SU to the SR link, and $\mathrm{E}[\gamma_{\mathrm{S}_j}] = \beta_4$.

3. COMBINED ACHIEVABLE SUM RATE

As described in Section 1, the primary and secondary network performance depend on the values of $\mathcal{M}$ and $\mathcal{N}$. In this section, we introduce our novel average combined sum rate metric, which enables us to mathematically quantify the performance trade-off between the PU and SU networks. We will show that achievable rates averaged across both fading and user distributions can be maximized with respect to the ratio $t = \mathrm{E}[\mathcal{N}]/\mathrm{E}[\mathcal{L}]$. This can be achieved by optimizing the combined sum rate in the deterministic number of users scenario first in Section 3.1, and followed by the random counterpart of this result in Section 3.2. The reason for studying the deterministic number of SUs case is twofold. First, the achievable rate trade-off has never been characterized in closed form with cooperation between PU and SU networks in existing literature. Second, it will be considered as a mathematical preliminary to the derivation in the random number of users scenario. Specifically, we will show that the scaling laws of the system sum rate in the random scenario converge to the deterministic case as mean number of SUs grows.

3.1. Optimal Ratio of t for Achievable Rate

When the PU transmits, among total (deterministic) number of $L = N + M$ SUs, N SUs serve as relays to the PU and M SUs communicate with the SR using MUD. In the primary network, the average achievable rate of the relay assisted PU can be expressed as

$$\overline{\mathrm{C}}_{\mathrm{P}}(N) = \frac{1}{2}\int_0^\infty \log(1+x)\, f_{\gamma_{\mathrm{P}}}(x)dx, \quad (3)$$

where $\overline{\mathrm{C}}_{\mathrm{P}}(N)$ is the achievable rate of the PU and $\gamma_{\mathrm{P}} = \gamma_{\mathrm{D}} + \gamma_{\mathrm{R}}^*$. The factor 1/2 is due to the fact that the PU only transmits during the first half of the transmission phase. Using asymptotic extreme value theory, the CDF and PDF of γ_{R}^* can be captured in the large N regime as [11]:

$$F_{\gamma_{\mathrm{R}}^*}(x) \approx \exp\left(-\exp\left(-\frac{x-\alpha_N}{c}\right)\right), \quad (4)$$

and

$$f_{\gamma_{\mathrm{R}}^*}(x) \approx \frac{1}{c}\exp\left(-\frac{x-\alpha_N}{c} + \exp\left(-\frac{x-\alpha_N}{c}\right)\right), \quad (5)$$

where $c = \beta_2\beta_3/(\sqrt{\beta_2}+\sqrt{\beta_3})^2$ and $\alpha_N = \log(N)/c + O(\log(\sqrt{\log(N)}))$.

Similarly, the average achievable rate of the selected SU with MUD can be expressed as

$$\overline{\mathrm{C}}_{\mathrm{S}}(M) = \int_0^\infty \log(1+x)\, f_{\gamma_{\mathrm{S}}^*}(x)dx. \quad (6)$$

As described in Section 1, there is a trade-off between the PU and the selected underlay SU performance when the metric of interest is the achievable rate. We define $t = N/L =$

$N/(N+M)$ where N and M are deterministic numbers, and use the combined sum rate metric

$$\overline{\mathrm{C}}_{\mathrm{all}}(N,M)=\overline{\mathrm{C}}_{\mathrm{P}}(N)+\overline{\mathrm{C}}_{\mathrm{S}}(M) \quad (7)$$

of the proposed system to capture the overall rate performance of the whole system. $\overline{\mathrm{C}}_{\mathrm{all}}(N,M)$ characterizes the total rate that the whole network can support. In this section, we will adjust the value of t to balance between the primary and secondary network performance in the achievable rate sense, and then aim to maximize the combined sum rate $\overline{\mathrm{C}}_{\mathrm{all}}(N,M)$ as a function of t in the large number of SUs regime, and obtain the following theorem:

Theorem 1. *When the number of SUs N and M are large and their ratio remain constant, the sum rate of the proposed system $\overline{\mathrm{C}}_{\mathrm{all}}(N,M)$ is concave in $t=N/L$ over $0\le t\le 1$ where $L=N+M$, and can be maximized at $t^*=1/3$.*

Proof. The sum rate of the proposed CR system can be expressed as

$$\overline{\mathrm{C}}_{\mathrm{all}}(N,M)=\overline{\mathrm{C}}_{\mathrm{P}}(N)+\overline{\mathrm{C}}_{\mathrm{S}}(M) \quad (8)$$

It has been shown in [18] that

$$\overline{\mathrm{C}}_{\mathrm{S}}(M)=\log(\log(M))+O\left((\log(M))^{-1/2}\right). \quad (9)$$

It has been proved in [19] that if a family of positive i.i.d. random variables $\{\mathcal{X}_N\}$ with finite mean ν_N and variance σ_N^2 satisfy $\nu_N\to\infty$ and $\frac{\sigma_N^2}{\nu_N}\to 0$ as $N\to\infty$, then we have

$$\mathrm{E}[\log(1+\mathcal{X}_N)]=\log(1+\nu_N)+o(\log(1+\nu_N)). \quad (10)$$

When N is large, from equation (1) it can be seen that the family of i.i.d. random variables γ_{P} have finite mean $\mathrm{E}[\gamma_{\mathrm{P}}]=\mathrm{E}[\gamma_{\mathrm{D}}]+\mathrm{E}[\gamma_{\mathrm{R}}^*]=\beta_1+c\log(N)$, and variance $\mathrm{var}[\gamma_{\mathrm{P}}]=\beta_1^2+\pi^2/6$, where c is derived in Section 3.1. Then, $\mathrm{E}[\gamma_{\mathrm{P}}]\to\infty$ and $\frac{\mathrm{var}[\gamma_{\mathrm{P}}]}{\mathrm{E}[\gamma_{\mathrm{P}}]}\to 0$ as $N\to\infty$. We apply (10) and obtain that

$$\overline{\mathrm{C}}_{\mathrm{P}}(N)=\frac{1}{2}\log(\log(N))+o(\log(\log(N))). \quad (11)$$

Hence,

$$\overline{\mathrm{C}}_{\mathrm{all}}(N,M)=\frac{1}{2}\log(\log(N))+\log(\log(M)) +O\left((\log(M))^{-1/2}\right)+o(\log(\log(N))). \quad (12)$$

Ignoring the non-dominant terms $O\left((\log(M))^{-1/2}\right)\to 0$ and $o(\log(\log(N)))$ in the second line of (12) and letting $t=\frac{N}{N+M}$, the optimization problem is equivalent to

$$\max_{0\le t\le 1}\ h(t)=\frac{1}{2}\log(\log(Lt))+\log(\log(L(1-t))). \quad (13)$$

Taking the first order derivative of $h(t)$ with respect to t and setting it to zero $h'(t)=0$, we have the following equation of the optimal ratio t^* as

$$\frac{3t^*-1}{2t\log(t)-(1-t^*)\log(1-t^*)}=\frac{1}{\log(L)}. \quad (14)$$

It is easily seen that as $L\to\infty$, $t^*\to 1/3$, which completes the proof. Moreover, it can be shown that $h'(t)$ is positive and negative when $t<t^*$ and $t>t^*$ respectively, hence $h(t)$ is unimodal over $0\le t\le 1$. □

Intuitively, when N and M are large, there exists some SUs with sufficiently good channels to be selected for relaying and underlay transmissions. Hence the effect of the fading on the achievable rates will diminish as N and M grow. The ratio t corresponds to the portion of the PU's rate out of the combined sum rate of the PU together with the selected underlay SU. Since PU is transmitting over half of the total transmission block (half-duplex), while the selected underlay SU is transmitting over the entire block (full-duplex), the optimal value of t follows that $t^*=(1/2)/(1/2+1)=1/3$. In Section 4, it will be shown numerically that this optimal value also holds when there exists mutual interference at the SR.

Recall that $t=0$ or $t=1$ stand for conventional cooperative and underlay CR systems respectively. Consequently, the fact that $0<t<1$ shows that that the cooperative CR system with cooperation between the PU and SU networks outperforms those with cooperation exclusively among PU and SU networks, and as well as the conventionally underlay CR system in the achievable rate sense when N and M are large. Moreover, we can generalize $\overline{\mathrm{C}}_{\mathrm{all}}(N,M)$ to any weighted linear combinations of $\overline{\mathrm{C}}_{\mathrm{P}}(N)$ and $\overline{\mathrm{C}}_{\mathrm{S}}(M)$, which can be maximized using the same method as Theorem 1, but would yield a result depending on the weights.

3.2. Sum Achievable Rate in General User Distributions

In this section, we show that as the mean value of total number of SUs $\mathrm{E}[\mathcal{L}]=\mathrm{E}[\mathcal{M}+\mathcal{N}]\to\infty$, $\mathrm{E}_{\mathcal{N},\mathcal{M}}[\overline{\mathrm{C}}_{\mathrm{all}}(\mathcal{N},\mathcal{M})]$, which denotes the combined sum rate averaged across both fading and user distributions, can be maximized over $0\le t\le 1$ as $\mathcal{M}$ and $\mathcal{N}$ are drawn from a large class of user distributions at t^*=1/3, same as the deterministic case of Theorem 1.

Theorem 2. *Assuming that $\mathcal{M}$ and $\mathcal{N}$ are positive random variables with mean values $\lambda(1-t)$ and λt, and variances $\sigma_{\mathcal{M}}^2$ and $\sigma_{\mathcal{N}}^2$, the sum achievable rate of the PU and the selected underlay SU $E_{\mathcal{N},\mathcal{M}}[\overline{C}_{\mathrm{all}}(\mathcal{N},\mathcal{M})]$ averaged across fading and user distributions can be maximized at $t=t^*$, provided that: $(a)Pr[\mathcal{M}=0]=o(1/\log\log\lambda(1-t))$, $Pr[\mathcal{N}=0]=o(1/\log\log\lambda t)$; $(b)\sigma_{\mathcal{M}}^2=o(\lambda^2(1-t)^2)$, $\sigma_{\mathcal{N}}^2=o(\lambda^2t^2)$ as $\lambda\to\infty$. Moreover, $t^*\to 1/3$ as $\lambda\to\infty$.*

Proof. Firstly, for the PU's rate, we derive the scaling laws of $\mathrm{E}_{\mathcal{N}}[\overline{\mathrm{C}}_{\mathrm{P}}(\mathcal{N})]$ as $\mathrm{E}[\mathcal{N}]\to\infty$. It can be shown that $\overline{\mathrm{C}}_{\mathrm{P}}(\mathrm{E}[\mathcal{N}])$ is concave in $\mathcal{N}$, and due to the Jensen's inequality, the achievable rate of the PU $\mathrm{E}_{\mathcal{N}}[\overline{\mathrm{C}}_{\mathrm{P}}(\mathcal{N})]$ can be upper bounded as:

$$\mathrm{E}_{\mathcal{N}}[\overline{\mathrm{C}}_{\mathrm{P}}(\mathcal{N})]\le\overline{\mathrm{C}}_{\mathrm{P}}(\mathrm{E}[\mathcal{N}]), \quad (15)$$

which scales like $\frac{1}{2}\log(\log(\lambda t))$ as $\lambda\to\infty$ from (11). This upper bound can be interpreted as any randomization of the number of SUs always deteriorates the achievable rate performance of the selected user, and will be depicted numerically in Section 4. $\mathrm{E}_{\mathcal{N}}[\overline{\mathrm{C}}_{\mathrm{P}}(\mathcal{N})]$ is also lower bounded by removing the direct link from PU to the PR, and we have

$$\mathrm{E}_{\mathcal{N}}[\overline{\mathrm{C}}_{\mathrm{P}}(\mathcal{N})]\ge\mathrm{E}[\log(1+\gamma_{\mathrm{R}}^*)]. \quad (16)$$

When conditions (a) and (b) are satisfied, by using extreme value theory and bounding the probability generating function of γ_{R}^* [18], the $\mathrm{E}[\log(1+\gamma_{\mathrm{R}}^*)]$ can be shown to scale like $\frac{1}{2}\log(\log(\lambda t)) + O\left((\log(\lambda t))^{-1/2}\right)$ as $\lambda \to \infty$. Consequently, both the upper bound and lower bound on $\mathrm{E}_{\mathcal{N}}[\overline{\mathrm{C}}_{\mathrm{P}}(\mathcal{N})]$ converge to each other, and applying the squeeze theorem we have

$$\mathrm{E}_{\mathcal{N}}[\overline{\mathrm{C}}_{\mathrm{P}}(\mathcal{N})] = \frac{1}{2}\log(\log(\lambda t)) + O\left((\log(\lambda t))^{-1/2}\right) + o(\log(\log(\lambda t))) \quad (17)$$

as $\lambda \to \infty$. The convergence of these upper and lower bounds will also be illustrated numerically in Section 4.

Secondly, for the selected SU, we show the scaling laws of $\mathrm{E}_{\mathcal{M}}[\overline{\mathrm{C}}_{\mathrm{S}}(\mathcal{M})]$ as $\mathrm{E}[\mathcal{M}] \to \infty$. Similarly we have

$$\mathrm{E}_{\mathcal{M}}[\overline{\mathrm{C}}_{\mathrm{S}}(\mathcal{M})] = \log(\log(\lambda(1-t))) + O\left((\log(\lambda(1-t)))^{-1/2}\right). \quad (18)$$

Hence, combining (17) and (18), we have for large λ,

$$\mathrm{E}_{\mathcal{N},\mathcal{M}}[\overline{\mathrm{C}}_{\mathrm{all}}(\mathcal{N},\mathcal{M})] = \frac{1}{2}\log(\log(\lambda t)) + \log(\log(\lambda(1-t))) + O\left((\log(\lambda(1-t)))^{-\frac{1}{2}}\right) + o(\log(\log(\lambda(1-t)))). \quad (19)$$

The rest of the proof is very similar to the proof of Theorem 1 that minimizing $\mathrm{E}_{\mathcal{N},\mathcal{M}}[\overline{\mathrm{C}}_{\mathrm{all}}(\mathcal{N},\mathcal{M})]$ is equivalent to minimizing $\overline{\mathrm{C}}_{\mathrm{all}}(N,M)$ by replacing L with λ. □

This theorem implies that the random SUs case has the same optimal t^* solution as its deterministic counterpart when $\lambda \to \infty$ proved in Theorem 1. It can be verified that many well-known discrete random variables including Binomial, Poisson-binomial, Poisson, and negative binomial satisfy conditions (a) and (b), hence can be potential candidate user distributions of $\mathcal{M}$ and $\mathcal{N}$. For example, when each SU satisfies the interference constraint with a equal probability, $\mathcal{M}$ and $\mathcal{N}$ are both binomial distributed.

4. NUMERICAL RESULTS

In this section, we generate i.i.d. fading coefficients and number of SUs as random variables, and use Monte-Carlo simulations to plot the combined sum achievable rate derived in (19) to corroborate our analytical results. For all simulations, Rayleigh fading channels are assumed, and $\mathcal{M}$ and $\mathcal{N}$ are Poisson distributed.

In Fig. 2, the scaling laws for the achievable rate of the PU $\mathrm{E}_{\mathcal{N}}[\overline{\mathrm{C}}_{\mathrm{P}}(\mathcal{N})]$ in (17) is simulated as $\lambda \to \infty$. In the proof of Theorem 2, we provide an upper bound and a lower bound on $\mathrm{E}_{\mathcal{N}}[\overline{\mathrm{C}}_{\mathrm{P}}(\mathcal{N})]$ and show that both bounds will converge as $\mathrm{E}[\mathcal{N}] = \lambda t \to \infty$. It is seen that the upper bound and lower bound will converge to each other as λ increases, and that the upper bound is tighter.

In Fig. 3, system sum rate metrics $\mathrm{E}_{\mathcal{N},\mathcal{M}}[\overline{\mathrm{C}}_{\mathrm{all}}(\mathcal{N},\mathcal{M})]$ and $\overline{\mathrm{C}}_{\mathrm{all}}(N,M)$ under different user distributions and mutual interference scenarios are simulated. In Theorem 1 we show that $\overline{\mathrm{C}}_{\mathrm{all}}(N,M)$ is unimodal and can be maximized at

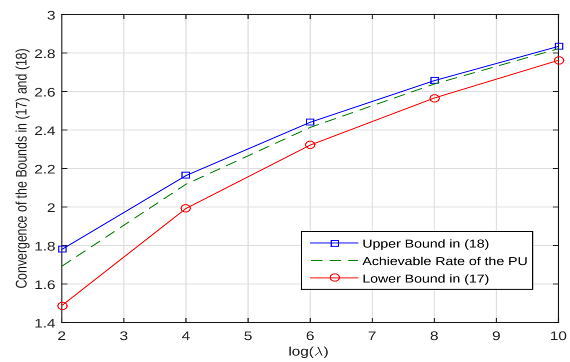

Fig. 2. Convergence of Upper and Lower Bounds on $\mathrm{E}_{\mathcal{N}}[\overline{\mathrm{C}}_{\mathrm{P}}(\mathcal{N})]$ as $\lambda \to \infty$.

t^* when N and M are large. We show in Theorem 2 that when $\mathcal{N}$ and $\mathcal{M}$ are random, similar optimal t^* can be obtained. In Figure 3, combined sum rate metrics $\mathrm{E}_{\mathcal{N},\mathcal{M}}[\overline{\mathrm{C}}_{\mathrm{all}}(\mathcal{N},\mathcal{M})]$ and $\overline{\mathrm{C}}_{\mathrm{all}}(N,M)$ of deterministic and Poisson distributed number of SUs with and without mutual interference at the SR are simulated. In the simulation, $L = 50$ and $\lambda = 50$, and the average SNR of all the fading links are normalized to 10 dB. It can be observed that even for moderate L and λ, sum rates in all scenarios can be maximized simultaneously at the same t^*. We have solved $t^* = 0.3444$ numerically which is very close to 1/3 as we derived in Theorem 1 and 2. The minor difference between 0.3444 and 1/3 is due to insufficiently large L and λ. Moreover, it has been stated in Section 3.2 that any randomization on the number of SUs will deteriorate the sum rates performance, which can be verified from the figure that deterministic cases are slightly above their random counterparts.

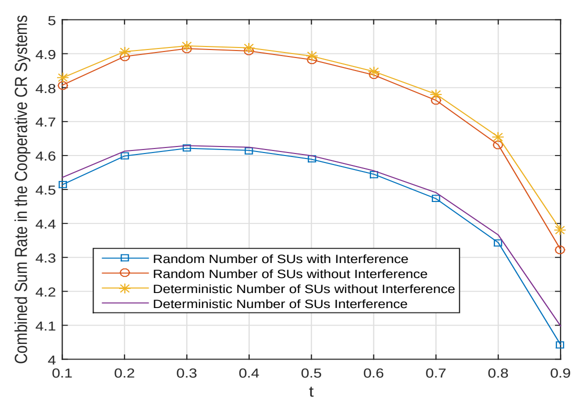

Fig. 3. Achievable Rate under Deterministic and Poisson User Distributions.

5. CONCLUSIONS

When there is cooperation between the PU and SU networks in CR systems, our novel combined sum rate metric enables us to characterize the achievable rate performance trade-off analytically. We show that as the number of SUs grows in both deterministic and random number of SUs cases, the effect of fading on the system sum rate will vanish, and the sum rate metric can be maximized at $t = 1/3$, which implies that the average number of SUs that serve as relays should be a third of the overall number of SUs available.

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