STRUCTURED ESTIMATION OF TIME-VARYING NARROWBAND WIRELESS COMMUNICATION CHANNELS

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ABSTRACT

In this paper, the estimation of a narrowband time-varying channel under the practical assumptions of finite block length and finite transmission bandwidth is investigated. It is shown that the signal after passing through a time-varying narrowband channel, under these assumptions, reveals a particular low-rank structure. The rank in this structure is governed by the number of dominant paths in the channel. Moreover, it is shown that this low-rank structure can be represented as a summation of few rank-one atoms (matrix) that are fully described by the channel and leakage key parameters. To estimated the channel, a novel approach based on minimization of atomic norm using measurements of signal at time domain is proposed. Numerical results show that the performance of proposed algorithm is independent of the leakage effect and the new method can achieve significant gains over previously proposed methods.

Index Terms— Narrowband time-varying channels, delay-Doppler leakage estimation, low-rank matrix recovery, atomic norm, convex optimization.

1. INTRODUCTION

Wireless communications have enabled intelligent traffic safety [11, 3], automated robotic networks, underwater surveillance systems [8, 2], and many other useful technologies. In all of these systems, establishing a reliable, high data rate communication link between the source and destination is essential. To achieve this goal, the system requires accurate channel state information to equalize the distortion from the transmit signal and recover the massage with minimum error at the destination.

One of the well-known approaches to acquire the channel state information is to probe the channel in time/frequency with known signals and reconstruct the channel response from the output signals (see [10, 14] and references therein). Least-squares (LS) (or linear regression) and Wiener filters [9, 16] are classical examples of this approach. However, these methods do not take advantage of rich, intrinsic structures of wireless communication channels in their estimation process. More recent approaches have tried to exploit the simple structures of channels, such as sparsity [15, 1], group-sparsity or mixed/hybrid sparse and group sparsity structures [12, 13, 3] using compressed sensing/sparse approximation algorithms. However due to practical communication system constraints such as finite block length and finite transmission bandwidth, the inherent sparsity of the channel decreases dramatically. This effect is defined as the channel *leakage* in [15, 3]. It has been shown that the performance of CS methods are significantly degraded due to the leakage effect [3, 15].

In this work, we show that under above practical communication systems constraints, the transmit signal after passing thought a time-varying narrowband channel can be represented as summation of rank-one matrices with particular structure. Each rank-one matrix in this representation can be characterized by the channel and the pulse leakage key parameters. We define a set of atoms (normable functions) to describe the set of rank-one matrices in our channel estimation problem. Each element in this set is a rank-one matrix whose elements are described by unknown channel and leakage parameters. Utilizing this set of atoms, we show that the channel estimation problem can be stated as a parametric low-rank matrix recovery problem. In prior work, we have shown that this problem is, in general, a non-convex optimization problem [6]. However, in this work, motivated by the convex recovery for inverse problem via atomic norm heuristic [4, 5] , we develop a recovery algorithm based on the minimization of the atomic norm of proposed set of atoms to enforce the channel model and leakage structures via a convex optimization problem. We analyze the algorithm to show that the global optimum can be recovered in the absence of noise. Moreover, we propose a method to find the optimal solution of the channel estimation problem. Performance of the algorithm is demonstrated by numerical experiments.

The rest of this paper is organized as follows. Section 2 derives the communication system model which is used in Section 3 to derive the discrete time observation model and introduce the low-rank recovery problem using atomic norm heuristic. Section 4 describes the proposed algorithm using semidefinite optimization to find the optimal solution of channel estimation problem. Section 5 is devoted to discussion and numerical results, and finally Section 6 concludes the paper.

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2. SYSTEM MODEL

We assume that the transmitted signal $x(t)$[1] is generated by the modulation of a pilot sequence $x[n]$ onto the transmit pulse $p_t(t)$, $x(t) = \sum_{n=-\infty}^{+\infty} x[n]p_t(t - nT_s)$, where T_s is the sampling period. The signal $x(t)$ is transmitted over a linear, time-varying channel. The received signal $y(t)$ can be written as,

$$y(t) = \int_{-\infty}^{+\infty} h\left(t, \tau\right) x(t - \tau)\, d\tau + z(t). \qquad (1)$$

Here, $h(t, \tau)$ is the channel's time-varying impulse response, and $z(t)$ is a Gaussian noise. A common model for the narrowband time-varying impulse response is as follows,

$$h(t, \tau) = \sum_{k=1}^{p_0} \eta_k \delta(\tau - t_k) e^{j2\pi\nu_k t}, \qquad (2)$$

where p_0 denotes the number of dominant path in the channel, η_k, t_k, and ν_k denote the kth channel path's attenuation factor (gain), delay, and Doppler shift, respectively. At the receiver, $y(t)$ is converted into a discrete-time signal using an anti-aliasing filter $p_r(t)$. That is,

$$y[n] = \int_{-\infty}^{+\infty} y(t) p_r(nT_s - t)\, dt. \qquad (3)$$

We assume that $p_t(t)$ and $p_r(t)$ are causal with support $[0, T_{\text{supp}})$. Under the reasonable assumption $\nu_{\max} T_{\text{supp}} \ll 1$, where $\nu_{\max} = \max\left(\nu_1, \ldots, \nu_{p_0}\right)$ denotes the Doppler spread of the channel [11], and if we let $p(t) = p_t(t) * p_r(t)$, we can write the received signal after filtering and sampling as [3],

$$y[n] = \sum_{k=1}^{p_0} \sum_{m=0}^{m_0-1} \eta_k p(mT_s - t_k) e^{-j2\pi\nu_k t_k} x[n - m] e^{j2\pi\nu_k(n-m)T_s} + z[n] \qquad (4)$$

for $n \in \{m_0, \cdots, n_T + m_0 - 1\}$, where m_0 denotes the maximum discrete delay spread of the channel, *i.e.*, $m_0 = \left\lceil \frac{\tau_{\max}}{T_s} \right\rceil$ where $\tau_{\max}$ is the delay spread of the channel, and n_T denotes the total number of training samples. Here $z[n]$[2] is a sequence of *i.i.d* circularly symmetric complex Gaussian random variables with variance σ_z^2. Note that pulse leakage effect is due to the non-zero support of pulse $p(\cdot)$. This will increase the number of nonzero coefficients of observed leaked channel at receiver side (for more details, see [3, 15]).

3. CHANNEL ESTIMATION

In the time-varying doubly selective channels estimation problem, the main goal is the estimation of the attenuation, delay, and Doppler parameters of the channel, *i.e.*, η_k, t_k, ν_k, respectively for $1 \leq k \leq p_0$, in order to equalize their effect on the transmitted signal. These parameters are estimated via measurement the model derived in (4), where both the transmitted signal $x[n]$ and the received signal $y[n]$ are known during the training signal transmission.

3.1. Problem Statement

Define $f_k(t) = p(t - t_k)e^{-j2\pi\nu_k t}$, then we can rewrite (4) as

$$\begin{aligned} y[n] &= \sum_{k=1}^{p_0} \bar{\eta}_k \left(\sum_{m=0}^{m_0-1} f_k(mT_s) x[n - m] \right) e^{j2\pi\nu_k nT_s} \\ &= \sum_{k=1}^{p_0} \bar{\eta}_k g_k[n] e^{j2\pi\bar{\nu}_k n}, \end{aligned} \qquad (5)$$

where $\bar{\eta}_k = \eta_k e^{-j2\pi\nu_k t_k}$, $\bar{\nu}_k = \nu_k T_s \in [0, 1)$, and

$$g_k[n] = \sum_{m=0}^{m_0-1} f_k(mT_s) x[n - m] = \boldsymbol{x}_n^T \boldsymbol{f}_k, \qquad (6)$$

where

$$\boldsymbol{f}_k = \left[f_k\left(0T_s\right), \cdots, f_k\left((m_0 - 1)T_s\right)\right]^T \qquad (7)$$

and

$$\boldsymbol{x}_n = \left[x\left[n\right], x\left[n - 1\right] \cdots, x\left[n - (m_0 - 1)\right]\right]^T. \qquad (8)$$

Now, if we stack $g_k[n]$ for $m_0 \leq n \leq n_T + m_0 - 1$ in a vector as $\boldsymbol{g}_k = \left[g_k[m_0], \cdots, g_k[n_T + m_0 - 1]\right]^T$, we can write

$$\boldsymbol{g}_k = \mathbf{X} \boldsymbol{f}_k, \qquad (9)$$

where $\mathbf{X}$ is a n_T-by-m_0 matrix and its i-th row equals to $\boldsymbol{x}_{i+m_0-1}^T$ defined in Equation (8). In wireless communication systems typically $n_T > m_0$ or, that is, all $\boldsymbol{g}_k$ live in a common low-dimensional subspace spanned by the columns of a known $n_T \times m_0$ matrix $\mathbf{X}$ with $n_T > m_0$. We assume that $\|\boldsymbol{f}_k\|_2 = 1$ without loss of generality. Using Equation (6), recovery of $\boldsymbol{g}_k$ is guaranteed if $\boldsymbol{f}_k$ can be recovered. Therefore, the number of degrees of freedom in (5) becomes $O(m_0 p_0)$, which is smaller than the number of samples n_T when $p_0, m_0 \ll n_T$. Applying Equation (6), we can rewrite (5) as

$$y[n] = \sum_{k=1}^{p_0} \bar{\eta}_k e^{j2\pi n\bar{\nu}_k} \boldsymbol{x}_n^T \boldsymbol{f}_k. \qquad (10)$$

Defining $\boldsymbol{a}(\nu) = \begin{bmatrix} e^{-j2\pi m_0 \nu} & \cdots & e^{-j2\pi(n_T+m_0-1)\nu} \end{bmatrix}^T$, we have

$$\begin{aligned} \boldsymbol{y}[n] &= \sum_{k=1}^{p_0} \bar{\eta}_k \boldsymbol{a}(\nu_k)^H \boldsymbol{e}_{n-m_0+1} \boldsymbol{x}_n^T \boldsymbol{f}_k \\ &= \operatorname{trace}\left(\boldsymbol{e}_{n-m_0+1} \boldsymbol{x}_n^T \sum_{k=1}^{p_0} \bar{\eta}_k \boldsymbol{f}_k \boldsymbol{a}(\nu_k)^H \right) \\ &= \left\langle \sum_{k=1}^{p_0} \bar{\eta}_k \boldsymbol{f}_k \boldsymbol{a}(\nu_k)^H, \boldsymbol{x}_n \boldsymbol{e}_{n-m_0+1}^T \right\rangle, \end{aligned} \qquad (11)$$

[1] Note that this signal model is quite general, and encompasses OFDM signals as well as single-carrier signals.

[2] Without loss of generality, if we assume that $p_r(t)$ has a root-Nyquist spectrum with respect to the sample duration T_s, which implies that $z[n]$ is a sequence of *i.i.d* circularly symmetric complex Gaussian random variables.

for $n = m_0, \cdots, n_T + m_0 - 1$, where we have defined $\langle \mathbf{X}, \mathbf{Y} \rangle = \text{trace}(\mathbf{Y}^H \mathbf{X})$ and $\boldsymbol{e}_n$, $1 \le n \le n_T$, are a canonical basis for $\mathbb{R}^{n_T \times 1}$. We see that (11) leads to a parametrized rank-p_0 matrix recovery problem, which we write as

$$\boldsymbol{y} = \Pi(\mathbf{H}_o),$$

where the linear operator $\Pi : \mathbb{C}^{m_0 \times n_T} \to \mathbb{C}^{n_T \times 1}$ is defined as $[\Pi(\mathbf{H}_o)]_n = \langle \mathbf{H}_o, \boldsymbol{x}_n \boldsymbol{e}_n^T \rangle$, with $\mathbf{H}_o = \sum_{k=1}^{p_0} \bar{\eta}_k \boldsymbol{f}_k \boldsymbol{a}(\nu_k)^H$.

3.2. Structured Estimation

Since the number of terms in Equation (10), p_0 (the number of dominant paths in the channel), is small. We use the atomic norm to promote this structure. Define the atomic norm associated with the following set of atoms

$$\mathcal{A} = \left\{ \boldsymbol{f} \boldsymbol{a}(\nu)^H : \nu \in [0,1), \|\boldsymbol{f}\|_2 = 1, \boldsymbol{f} \in \mathbb{C}^{m_0 \times 1} \right\}$$

as

$$\begin{aligned} \|\mathbf{H}_o\|_{\mathcal{A}} &= \inf \left\{ t > 0 : \mathbf{H}_o \in t\text{conv}(\mathcal{A}) \right\} \qquad (12) \\ &= \inf_{\bar{\eta}_k, \nu_k, \|\boldsymbol{f}_k\|_2 = 1} \left\{ \sum_k |\bar{\eta}_k| : \mathbf{H}_o = \sum_k \bar{\eta}_k \boldsymbol{f}_k \boldsymbol{a}(\nu_k)^H \right\}. \end{aligned}$$

Remark 1. The atomic representation in Equation (12) for matrix $\mathbf{H}_o$, *i.e.*, $\mathbf{H}_o = \sum_k \bar{\eta}_k \mathbf{A}(\boldsymbol{f}_k, \nu_k) = \sum_k \bar{\eta}_k \boldsymbol{f}_k \boldsymbol{a}(\nu_k)^H$, not only captures the functional forms of its elements, but also enforces the rank-one constraint on each terms in the summation, *i.e.*, $\text{rank}(\mathbf{A}(\boldsymbol{f}_k, \nu_k)) = 1$.

To enforce the sparsity of the atomic representation or low-rank representation of received signal, we solve

$$\underset{\mathbf{H}}{\text{minimize}} \ \|\mathbf{H}\|_{\mathcal{A}} \ \text{ s.t. } \ \boldsymbol{y} = \Pi(\mathbf{H}). \qquad (13)$$

3.3. Optimality and Uniqueness

The dual of the optimization problem in (13), using standard Lagrangian analysis can be written as

$$\underset{\boldsymbol{\lambda}}{\text{maximize}} \ \text{Re}\left\{\{\langle \boldsymbol{\lambda}, \boldsymbol{y} \rangle\}\right\} \quad \text{ s.t. } \|\Pi^*(\boldsymbol{\lambda})\|_{\mathcal{A}}^* \le 1, \quad (14)$$

where $\Pi^*(\boldsymbol{\lambda}) = \sum_k \boldsymbol{\lambda}(k) \boldsymbol{x}_k \boldsymbol{e}_{k-m_0+1}$ is the adjoint operator of Π and $\|\cdot\|_{\mathcal{A}}^*$ denotes the dual norm of the atomic norm. Therefore, we have

$$\begin{aligned} \|\Pi^*(\boldsymbol{\lambda})\|_{\mathcal{A}}^* &= \sup_{\|\boldsymbol{\Theta}\|_{\Pi} \le 1} \text{Re}\left\{ \langle \Pi^*(\boldsymbol{\lambda}), \boldsymbol{\Theta} \rangle \right\} \qquad (15) \\ &= \sup_{\nu \in [0,1), \|\boldsymbol{f}\|_2 = 1} \text{Re}\left\{ \langle \Pi^*(\boldsymbol{\lambda}), \boldsymbol{f} \boldsymbol{a}(\nu)^H \rangle \right\}. \end{aligned}$$

Equality in (15) holds, since the set $\left\{ \boldsymbol{f} \boldsymbol{a}(\nu)^H \right\}_{\nu, \boldsymbol{f}}$ covers all the extremal points of atomic norm unit ball, *i.e.*, $\{\boldsymbol{\Theta} : \|\boldsymbol{\Theta}\|_{\mathcal{A}} \le 1\}$. Now if we define $\boldsymbol{\mu}(\nu) = \Pi^*(\boldsymbol{\lambda}) \boldsymbol{a}(\nu)$, we have

$$\begin{aligned} \|\Pi^*(\boldsymbol{\lambda})\|_{\mathcal{A}}^* &= \sup_{\nu \in [0,1), \|\boldsymbol{f}\|_2 = 1} \text{Re}\left\{ \boldsymbol{f}^H \boldsymbol{\mu}(\nu) \right\} \\ &\le \sup_{\nu \in [0,1)} \|\boldsymbol{\mu}(\nu)\|_2. \qquad (16) \end{aligned}$$

Now, if we consider the following condition that

$$\|\boldsymbol{\mu}(\nu)\|_2 \le 1, \qquad \text{(C-1)}$$

then we can rewrite the optimization problem in (14) as follows:

$$\underset{\boldsymbol{\lambda}}{\text{maximize}} \ \text{Re}\left\{ \langle \boldsymbol{\lambda}, \boldsymbol{y} \rangle \right\} \quad \text{ subject to } \|\boldsymbol{\mu}(\nu)\|_2 \le 1, \ (17)$$

where

$$\boldsymbol{\mu}(\nu) = \Pi^*(\boldsymbol{\lambda}) \boldsymbol{a}(\nu) = \sum_{n=m_0-1}^{n_T+m_0-1} \boldsymbol{\lambda}(n - m_0 + 1) e^{j2\pi n \nu} \boldsymbol{x}_n. \qquad (18)$$

Similarly, we have

$$\begin{aligned} \text{Re}\left\{\{\langle \boldsymbol{\lambda}, \boldsymbol{y} \rangle\}\right\} &= \text{Re}\left\{ \langle \Pi^*(\boldsymbol{\lambda}), \mathbf{H} \rangle \right\} \\ &= \text{Re}\left\{ \left\langle \Pi^*(\boldsymbol{\lambda}), \sum_k \bar{\eta}_k \boldsymbol{f}_k \boldsymbol{a}(\nu_k)^H \right\rangle \right\} \\ &= \sum_k \text{Re}\left\{ \bar{\eta}_k^* \boldsymbol{f}_k^H \boldsymbol{\mu}(\nu_k) \right\}. \qquad (19) \end{aligned}$$

Now, if we also assume that

$$\boldsymbol{\mu}(\nu_k) = \text{sign}(\bar{\eta}_k) \boldsymbol{f}_k, \qquad \text{(C-2)}$$

for $k \in \{1, \cdots, p_0\}$, then we have $\text{Re}\{\{\langle \boldsymbol{\lambda}, \boldsymbol{y} \rangle\}\} = \sum_k |\bar{\eta}_k| \ge \|\mathbf{H}\|_{\mathcal{A}}$. Moreover, using the Hölder inequality and Equation (19) we know that

$$\text{Re}\left\{\{\langle \boldsymbol{\lambda}, \boldsymbol{y} \rangle\}\right\} \le \|\Pi^*(\boldsymbol{\lambda})\|_{\mathcal{A}}^* \|\mathbf{H}\|_{\mathcal{A}} \le \|\mathbf{H}\|_{\mathcal{A}}.$$

Therefore, if condition (C-2) holds, then $\text{Re}\{\{\langle \boldsymbol{\lambda}, \boldsymbol{y} \rangle\}\} = \|\mathbf{H}\|_{\mathcal{A}}$. In other words, under conditions (C-1) and (C-2) the solution of the primal (Equation (13)) and dual (Equation (14)) optimization problems introduce zero duality gap. Thus, $\mathbf{H}_o$ and $\boldsymbol{\lambda}$ are optimal solutions of the primal and dual optimization problem. Furthermore, using proof by contradiction, we can see that condition (C-2) ensures the uniqueness of optimal solution. Suppose $\hat{\mathbf{H}} = \sum_k \hat{\eta}_k \hat{\boldsymbol{f}}_k \boldsymbol{a}(\hat{\nu}_k)^H$ is another optimal solution. Since $\hat{\mathbf{H}}$ and $\mathbf{H}_o$ are different, there are some $\hat{\nu}_k$ that are not in support of $\mathbf{H}_o$. Define $\mathcal{V}_o$ as the support of $\mathbf{H}_o$. Then, we have

$$\begin{aligned} &\text{Re}\left\{\{\langle \boldsymbol{\lambda}, \boldsymbol{y} \rangle\}\right\} = \text{Re}\left\{ \left\langle \Pi^*(\boldsymbol{\lambda}), \hat{\mathbf{H}} \right\rangle \right\} \\ &= \text{Re}\left\{ \left\langle \Pi^*(\boldsymbol{\lambda}), \sum_k \hat{\eta}_k \hat{\boldsymbol{f}}_k \boldsymbol{a}(\hat{\nu}_k)^H \right\rangle \right\} \\ &= \sum_{k \in \mathcal{V}_o} \text{Re}\left\{ \bar{\eta}_k^* \boldsymbol{f}_k^H \boldsymbol{\mu}(\nu_k) \right\} + \sum_{k \notin \mathcal{V}_o} \text{Re}\left\{ \hat{\bar{\eta}}_k^* \hat{\boldsymbol{f}}_k^H \boldsymbol{\mu}(\hat{\nu}_k) \right\} \\ &\le \sum_{k \in \mathcal{V}_o} \bar{\eta}_k^* \left\| \boldsymbol{f}_k^H \right\|_2 \|\boldsymbol{\mu}(\nu_k)\|_2 + \sum_{k \notin \mathcal{V}_o} \hat{\bar{\eta}}_k^* \left\| \hat{\boldsymbol{f}}_k^H \right\|_2 \|\boldsymbol{\mu}(\hat{\nu}_k)\|_2 \\ &< \sum_{k \in \mathcal{V}_o} \bar{\eta}_k^* + \sum_{k \notin \mathcal{V}_o} \hat{\bar{\eta}}_k^* = \left\| \hat{\mathbf{H}} \right\|_{\mathcal{A}}, \end{aligned}$$

which is in contradiction of the optimality of $\hat{\mathbf{H}}$.

4. PROPOSED ESTIMATION ALGORITHM

We observed that considering conditions (C-1) and (C-2), the optimization problems in (13) and (17) both recover the optimal solution of our channel estimation problem. Hence, after we evaluate the dual parameters $\boldsymbol{\lambda}$ by solving one of these optimization problems, we can construct the function $\boldsymbol{\mu}(\nu)$ in (18). Then, we can use it to estimate the Doppler parameters by enforcing condition (C-2), as we know that $|\boldsymbol{\mu}(\nu_k)| = 1$ for $k \in \{1, \cdots, p_0\}$. Towards this goal, we need to find the roots of the following polynomial

$$Q(\nu) = 1 - \|\boldsymbol{\mu}(\nu)\|_2^2 = 1 - \boldsymbol{\mu}(\nu)^H \boldsymbol{\mu}(\nu), \quad (20)$$

which are equal to $\{\nu_k\}_{k=1}^{p_0}$. After estimation of $\{\nu_k\}_{k=1}^{p_0}$, we can substitute them in (10) to achieve a linear system of equations to evaluate $\{\bar{\eta}_k \boldsymbol{f}_k\}_{k=1}^{p_0}$. Note that we do not need to evaluate the values of $\bar{\eta}_k$ and $\boldsymbol{f}_k$ separately in order to equalize the channel effect. As seen in (10), to construct an equalizer we just require $\bar{\eta}_k \boldsymbol{f}_k$ for $1 \leq k \leq p_0$.

4.1. Solving the Optimization Problem in (13)

From [4], we know that the convex hull of the set of atoms $\mathcal{A}$ can be characterized by a semidefinite program. Therefore $\|\mathbf{H}\|_{\mathcal{A}}$ admits an equivalent SDP representation.

Proposition 1 (see *i.e.*, [4]). *For any* $\mathbf{H} \in \mathbb{C}^{m_0 \times n_T}$,

$$\|\mathbf{H}\|_{\mathcal{A}} = \inf_{\boldsymbol{z}, \mathbf{W}} \left\{ \frac{1}{2n_T} \operatorname{trace}\left(\operatorname{Toep}(\boldsymbol{z}) + n_T \mathbf{W}\right) : \middle| \; \begin{bmatrix} \operatorname{Toep}(\boldsymbol{z}) & \mathbf{H}^H \\ \mathbf{H} & \mathbf{W} \end{bmatrix} \succeq 0 \right\}, \quad (21)$$

where $\boldsymbol{z}$ *is a complex vector whose first element is real,* $\operatorname{Toep}(\boldsymbol{z})$ *denotes the* $n_T \times n_T$ *Hermitian Toeplitz matrix whose first column is* $\boldsymbol{z}$*, and* $\mathbf{W}$ *is a Hermitian* $m_0 \times m_0$ *matrix.*

Therefore, we can use an efficient available SDP solver software such as CVX [7], to solve the optimization problem in (13) via the above SDP representation and also evaluate the dual parameter $\boldsymbol{\lambda}$. For noisy measurements, we consider

$$\underset{\mathbf{H}}{\text{minimize}} \;\; \|\mathbf{H}\|_{\mathcal{A}} \;\; \text{s.t.} \;\; \|\boldsymbol{y} - \Pi(\mathbf{H})\|_2 \leq \sigma_z^2.$$

5. NUMERICAL SIMULATIONS

In this section, we perform several numerical experiments to validate the performance of the proposed channel estimation algorithm. We construct a narrowband time-varying channel based on the model given in (2). We first generate the channel delay, Doppler, and attenuation parameters randomly. In all of these experiments, we consider $p_0 = 3$ and $m_0 = 10$. The delay and Doppler parameters are generated via uniform random variables and channel attenuation parameters are generated using a Rayleigh random variable. The transmit training signal $\boldsymbol{x} = [x[1], x[2], \cdots, x[n_T + n_0 - 1]]^T$ is generated using a normal random variable with zero mean and unit variance. Moreover, the transmit and received pulse shapes are considered as Gaussian pulses with 50% window's support. Results in Fig.1 depict the normalized mean squared error (NMSE) correspond to the estimation of the leaked channel's gains, *i.e.*, $\{\bar{\eta}\boldsymbol{f}_k\}_{k=1}^{p_0}$ using the proposed algorithm for $n_T = 48, 64,$ and 80. It is clear that by increasing the number of measurements the performance of the proposed algorithm gets better. In Fig.1, there is another curve, which present the performance of l_1-based sparse approximation (SA) method for the estimation of channel with $n_T = 80$ (see [1, 3]). In Fig.1, we observe that our proposed method achieves much better results, due to exploiting the most of channel structures compared with l_1-based SA method that only consider element-wise sparsity of channel coefficients. In Fig.2, the results show that the NMSE of our proposed algorithm for estimation of Doppler parameters. Results in both Figures 1 and 2 indicate that our algorithm is robust to the noise. In Fig. 2, for $SNR \geq 5$ dB, we see that for all values of n_T, the proposed algorithm can estimate the Doppler parameters with at most 0.01 (normalized) error.

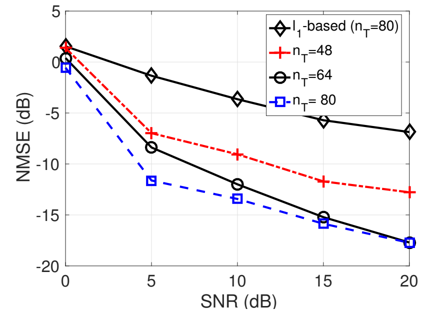

Fig. 1. NMSE vs. SNR - Leaked Channel gains $\{\bar{\eta}_k \boldsymbol{f}_k\}_{k=1}^{p_0}$

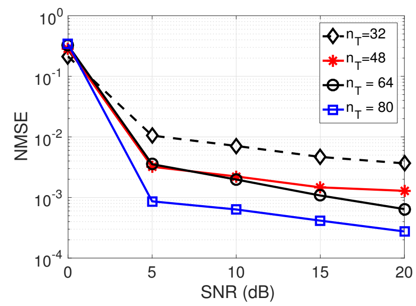

Fig. 2. NMSE vs. SNR - Channel Doppler parameters

6. CONCLUSIONS

In this work, we have proposed a new method to estimate the time-varying narrowband channels by promoting most of received signal structures via a proper set of atoms. We showed that the received signal measurements in time domain follow a parametric low-rank structure due to the pulse shape leakage and the channel model. We proposed a convex optimization problem to estimate the channel by promoting the low-rank structures via minimization of the atomic norm. Numerical results showed that the proposed algorithm can provides a performance (in SNR sense) with $5 - 8$ dB improvement in average in SNR> 5 compared to l_1-based SA method.

7. REFERENCES

[1] Waheed U Bajwa, Jarvis Haupt, Akbar M Sayeed, and Robert Nowak. Compressed channel sensing: A new approach to estimating sparse multipath channels. 98(6):1058–1076, 2010.

[2] Sajjad Beygi and Urbashi Mitra. Multi-scale multi-lag channel estimation using low rank structure of received signal. In *2014 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pages 4299–4303, 2014.

[3] Sajjad Beygi, Urbashi Mitra, and Erik G Ström. Nested sparse approximation: Structured estimation of V2V channels using geometry-based stochastic channel model. *IEEE Transactions on Signal Processing*, 63(18):4940–4955, 2015.

[4] Venkat Chandrasekaran, Benjamin Recht, Pablo A Parrilo, and Alan S Willsky. The convex geometry of linear inverse problems. *Foundations of Computational mathematics*, 12(6):805–849, 2012.

[5] Yuejie Chi. Guaranteed blind sparse spikes deconvolution via lifting and convex optimization. *IEEE Journal of Selected Topics in Signal Processing*, 10(4):782–794, 2016.

[6] Sunav Choudhary, Sajjad Beygi, and Urbashi Mitra. Delay-Doppler Estimation via Structured Low-Rank Matrix Recovery. In *2016 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, March 2016. to appear.

[7] Michael Grant and Stephen Boyd. CVX: Matlab software for disciplined convex programming, version 2.1. `http://cvxr.com/cvx`, March 2014.

[8] Geoffrey A Hollinger, Sunav Choudhary, Parastoo Qarabaqi, Christopher Murphy, Urbashi Mitra, Gaurav S Sukhatme, Milica Stojanovic, Hanumant Singh, and Franz Hover. Underwater data collection using robotic sensor networks. *IEEE Journal on Selected Areas in Communications*, 30(5):899–911, 2012.

[9] Shashwat Jnawali, Sajjad Beygi, and Hamid-Reza Bahrami. RF impairments compensation and channel estimation in MIMO-OFDM systems. In *2011 IEEE Vehicular Technology Conference (VTC Fall)*, pages 1–5, 2011.

[10] Weichang Li and J.C. Preisig. Estimation of Rapidly Time-Varying Sparse Channels. 32(4):927–939, October 2007.

[11] Gerald Matz, H Bolcskei, and Franz Hlawatsch. Time-frequency foundations of communications: concepts and tools. 30(6):87–96, 2013.

[12] N. Michelusi, U. Mitra, A.F. Molisch, and M. Zorzi. UWB Sparse/Diffuse Channels, Part I: Channel Models and Bayesian Estimators. 60(10):5307–5319, October 2012.

[13] N. Michelusi, U. Mitra, A.F. Molisch, and M. Zorzi. UWB Sparse/Diffuse Channels, Part II: Estimator Analysis and Practical Channels. 60(10):5320–5333, October 2012.

[14] Andreas F. Molisch. *Wireless Communications*. Wiley, 2 edition, December 2010.

[15] Georg Taubock, Franz Hlawatsch, Daniel Eiwen, and Holger Rauhut. Compressive estimation of doubly selective channels in multicarrier systems: Leakage effects and sparsity-enhancing processing. 4(2):255–271, 2010.

[16] Guanghan Xu, Hui Liu, Lang Tong, and T. Kailath. A Least-Squares Approach to Blind Channel Identification. 43(12):2982–2993, 1995.