DISTRIBUTED LARGEST EIGENVALUE DETECTION

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ABSTRACT

Cognitive radio (CR) systems need to detect the presence of a primary user (PU) signal by continuously sensing the spectrum area of interest. Radiowave propagation effects like fading and shadowing often complicate sensing of spectrum holes because the PU signal can be weak in a particular area. Cooperative spectrum sensing is seen as a prospective solution to enhance the detection of PU signals. In this paper we study distributed spectrum sensing, based on the largest eigenvalue of adaptively estimated correlation matrices (CMs) of received signals. The PU signal is assumed to be temporally correlated. In this paper an Combine and Adapt (CTA) least mean square (LMS) diffusion based mean vector estimation scheme is proposed. No fusion center (FC) for estimation or detection is used. We analyse the resulting detection performance and verify the theoretical findings through simulations.

Index Terms— Cognitive radio, distributed estimation, diffusion LMS, distributed detection, Spectrum Sensing.

1. INTRODUCTION

In cognitive radio (CR) contexts we would like to avoid creating interference to the PU user and find free spectrum opportunities as fast as possible. On the other hand the active detection hypothesis may change during the processing time. Distributed, adaptive network learning methods are able to learn the statistical information based on observations received by the nodes in the network. These methods can react to possible changes in the properties of estimated statistics in real time. Cooperative spectrum sensing is seen as a prospective solution to address these problems and to enhance the detection of PU signals [1].

Depending on the signal model assumptions, several type of detectors for spectrum sensing have been proposed in the literature such as the Matched filter detector [2], the Energy Detector [2], [3], and the Cyclostationary detector [4]. A second large group of detectors are based on the properties of an estimated signal correlation matrix eigenvalues [5], [6], [7]. The Largest Eigenvalue (LE) method [5] uses *a priori* knowledge about the additive noise power to determine the detection threshold.

Several distributed adaptive estimation and detection schemes have been studied in the past. Consensus based schemes are analysed for example in [8], [9], [10], [11]. Least mean square (LMS) and recursive least squares (RLS) based estimation schemes in [12], [13], [14], [15]. Optimal, distributed MFD, based on diffusion type LMS and RLS estimation schemes, were studied in [16], where good properties of diffusion LMS algorithms where shown. In [17], [18] and [19] we proposed and analysed diffusion LMS based energy detectors in a CR network. In this paper we propose and study the performance of LE detection in a distributed CR network, based on adaptively, distributively estimated CMs, using the completely distributed CTA type of diffusion LMS strategy (with no central processing unit as a potential single point of failure). We make the assumption that the CR network does not have prior information about the waveform of the PU signal and about the channel gains in the secondary nodes except that the CM of the PU signal is low rank (due to temporal correlation). In the distributed CR network, every node acts as an independent detector in terms of detection decision making based on the available CM estimates.

We organize the remainder of the paper as follows. In section II we specify the system models for the LE detection method and derive an adaptive, distributed CM estimation algorithm based on the CTA diffusion LMS strategy. In section IV we analyse the performance of the proposed distributed CM estimation algorithm (using a common framework) and the detection performance of the distributed LE detection method. In section V we present our simulations results.

Notation. In the paper we use the following notations. Bold-face uppercase and lowercase letters denote matrices and vectors, respectively. $E[\cdot]$, $Cov[\cdot]$ denote expectation and covariance operators, respectively. $vec[\cdot]$ and $vec^{-1}[\cdot]$ denote conversion from matrix to vector and from vector to matrix. $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^c$ denote the vector or matrix transpose, the Hermitian transpose and the complex conjugate, respectively. \otimes denotes the Kronecker product.

2. DISTRIBUTED ADAPTIVE LARGEST EIGENVALUE DETECTION

2.1. Signal model and assumptions

Let the K CR nodes independently sense a communication band of a PU. Every CR node obtains individually a $M \times 1$ observation vector

$$\mathbf{y}_k(n) = \left[z_s(nT_s), z_s(nT_s - \delta_s), \dots, z_s(nT_s - (M-1)\delta_s)\right],$$

which contain a bunch of samples of the down converted continuous time signal $z_s(t)$, which are collected every T_s seconds with the sampling period $\delta_s < T_s$. Thus in general we have the following signal model under both detection hypotheses

$$H_0: \mathbf{y}_k(n) = \mathbf{v}_k(n), H_1: \mathbf{y}_k(n) = \alpha_k \mathbf{s}(n) + \mathbf{v}_k(n),$$
(2)

where k = 1, 2, ..., K is the node number, M is the length of observation vector, and n = 1, 2, ...N is the sample discrete time index. The primary signal $\mathbf{s}(n) \sim CN_M(0, \boldsymbol{\Sigma}_s)$, the noise $\mathbf{v}_k(n)$ and channel gains α_k at node k are assumed to be statistically independent. The additive noise $\mathbf{v}_k(n)$ is assumed to be independently and

identically distributed (*i.i.d*) Circularly Symmetric Complex Gaussian noise with zero mean and covariance $\Sigma_{v,k} = \sigma_{v,k}^2 \mathbf{I}_M$ and uncorrelated in time and space. We assume the noise power is known a *priori* and has the same power level over all the nodes in the CR network.

Each node in the CR network estimates the $M \times M$ CM \mathbf{R}_k as

$$\mathbf{R}_{k} = \mathbf{E}\left[\mathbf{y}_{k}(n)\mathbf{y}_{k}(n)^{H}\right] = \mathbf{R}_{s,k} + \boldsymbol{\Sigma}_{v,k}.$$
 (3)

We additionally assume that $\mathbf{R}_{s,k}$ has a low rank (see also [20], [21]), while $\boldsymbol{\Sigma}_{v,k} = \sigma_{v,k}^2 \mathbf{I}_M$. This property can be used for detecting a PU signal.

Let us define the eigenvalues of the estimate $\hat{\mathbf{R}}_k(n)$ of CM \mathbf{R}_k in non-increasing order as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$. Every node k detects the presence of a PU signal by independently determining the LE of the locally available estimate $\hat{\mathbf{R}}_k(n)$ and by performing the following detection test

$$\lambda_1 \left[\hat{\mathbf{R}}_k(n) \right] \stackrel{H_1}{\underset{H_0}{\gtrless}} \gamma_{LE},\tag{4}$$

using a threshold γ_{LE} , which is given by (24).

2.2. Adaptive, Distributed CM estimation and LE detection

CR nodes could cooperate via internal communication links to enhance the detection performance (of the PU signal(s)) at every node k. We assume, that the K nodes in the CR network can rely only on the subset of global information, that is available to them. The CR network topology is assumed to be fixed over the sensing time and strongly connected. We consider a linear, fixed combination of neighbour estimates and measurements at every node k.

We propose a global (theoretical) model for estimating the CM in a cooperative manner, where the CR nodes jointly estimate the network average CM, which is denoted as \mathbf{R}^{o} and in vectorized form defined as follows

$$\mathbf{r}^{o} = \frac{1}{K} \sum_{k=1}^{K} \operatorname{vec}(\mathbf{R}_{k}^{o}) = \frac{1}{K} \sum_{k=1}^{K} \operatorname{E}\left[\operatorname{vec}\left[\mathbf{y}_{k}(n)\mathbf{y}_{k}(n)^{H}\right]\right], \quad (5)$$

where the $M^2 \times 1 \mathbf{r}^o$ is the vectorized form of \mathbf{R}^o .

We can vectorize the observation $\mathbf{d}_{R,k}(n) = \operatorname{vec} [\mathbf{y}_k(n)\mathbf{y}_k(n)^H]$ at node k at time instant n and decompose it into the product of a $M^2 \times M^2$ constant (invertible) complex matrix **T** (whose elements take the values 0, 1 and $\pm i$, where i denotes the imaginary unit) and a $M^2 \times 1$ real vector $\mathbf{d}_k(n)$ as $\mathbf{d}_{R,k}(n) = \mathbf{T}\mathbf{d}_k(n)$, to keep the dimension of the estimated vector minimal in the adaptive recursions. We denote the estimate of the real valued $\operatorname{E} [\mathbf{d}_k(n)]$ as $\hat{\mathbf{p}}_k(n)$ and propose to relate the estimation of the \mathbf{R}_k^o and \mathbf{R}^o in (5) with the minimization of the following Mean Square Error (MSE) type of global cost function

$$\mathbf{p}^{o} = \operatorname{argmin}_{\mathbf{p}} \sum_{k=1}^{K} J_{k}(\mathbf{p}) = \operatorname{argmin}_{\mathbf{p}} \sum_{k=1}^{K} \operatorname{E} \|\mathbf{d}_{k}(n) - \mathbf{p}\|^{2}, \quad (6)$$

where $M^2 \times 1$ dimensional $\mathbf{p} \in \mathbb{R}^M$. By using standard derivation steps on (6) we get the optimal solution

$$\mathbf{p}^{o} = \frac{1}{K} \sum_{k=1}^{K} \mathbb{E}\left[\mathbf{d}_{k}(n)\right].$$
(7)

Thus with help of the transformation matrix \mathbf{T} , the previously introduced minimization framework can be used to re-define the \mathbf{R}_k^o and \mathbf{R}^o as follows

$$\mathbf{R}^{o} = \operatorname{vec}^{-1} \left[\mathbf{T} \mathbf{p}^{o} \right] \quad \text{and} \quad \mathbf{R}^{o}_{k} = \operatorname{vec}^{-1} \left[\mathbf{T} \mathbf{p}^{o}_{k} \right]. \tag{8}$$

We need to seek an iterative solution to estimate the \mathbf{p}_k^o and \mathbf{p}^o in a manner, which is adaptive in time and is fully distributed (cooperative).

2.3. Iterative Diffusion solutions

In this paper we skip the derivation details of the CTA type of diffusion LMS mean vector estimation algorithm (provided in [22], following the ideas of [13]). Let \mathcal{N}_k denote the neighbourhood group of node $k \in K$, i.e \mathcal{N}_k . Let μ_k be a positive step size of node k. We introduce the $K \times K$ matrix **C** with non-negative elements satisfying

$$c_{l,k} = 0$$
 if $l \notin \mathcal{N}_k$, $\mathbf{C1} = \mathbf{1}$. (9)

Similarly let the $K \times K$ matrix **A** satisfy

$$a_{l,k} = 0$$
 if $l \notin \mathcal{N}_k$, $\mathbf{1}^T \mathbf{A} = \mathbf{1}^T$. (10)

We summarize the CTA based CM estimation recursions and the detection step in a common form in Algorithm 1. The coefficients $c_{l,k}$

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Start with $\hat{\mathbf{p}}_k(0) = \mathbf{p}(0)$ for every k. Given non-negative real coefficients $a_{l,k}, c_{l,k}$ for every time instant $n \ge 1$ do for every node k = 1, ..., K do 1. CTA type of CM estimation recursions: $\hat{\psi}_k(n) = \sum_{l \in N_k} a_{l,k} \hat{\mathbf{p}}_l(n).$ $\hat{\mathbf{p}}_k(n+1) = \hat{\psi}_k(n)$ $+\mu_k \sum_{l \in N_k} c_{l,k} \left[\mathbf{d}_l(n) - \hat{\psi}_k(n) \right]$ 2. LE detection decision: $H_0 : \lambda_1 \left[\operatorname{vec}^{-1} \left[\mathbf{T} \hat{\mathbf{p}}_k(n+1) \right] \right] < \gamma_k$ or $H_1 : \lambda_1 \left[\operatorname{vec}^{-1} \left[\mathbf{T} \hat{\mathbf{p}}_k(n+1) \right] \right] > \gamma_k.$ (Refer to (24) for selecting the γ_k). end for

and $a_{l,k}$ define respectively how the neighbouring measurements $\mathbf{d}_{l}(n)$ and estimates $\hat{\mathbf{p}}_{l}(n)$ are (unidirectionally) available for the node k in the CR network. Thus after several iterations the adaptive estimate $\hat{\mathbf{R}}_{k}(n)$ of \mathbf{R}^{o} is available for every node in the CR network, while the FC is not used. The node k at time instant n can independently perform the LE detection based on the available matrix estimate $\hat{\mathbf{R}}_{k}(n) = \operatorname{vec}^{-1}[\mathbf{T}\hat{\mathbf{p}}_{k}(n)].$

As a result the proposed LE detection scheme is able to react to a possible change in the statistics of observations on line (i.e when the detection hypothesis changes during the observation time) and estimates the CMs in a cooperative manner with an averaging effect over the CR network.

3. PERFORMANCE ANALYSIS

The performance analysis of the proposed algorithm is divided into three parts: analysis of the moments of the adaptive CM estimates of recursions in Algorithm 1 in one framework, analysis of the statistical properties of the adaptive CM estimates and analysis of the detection performance of the LE of the adaptive CM estimates. Let us note that for the theoretical performance analysis of the LE detector, we need to know the values of the channel gains.

3.1. Moment analysis of adaptive CM estimates

Let us stack the $M^2 \times 1$ estimates and observations from all the nodes $k \in K$ into a $KM^2 \times 1$ column vector $\hat{\mathbf{p}}(n)|H_i = [\hat{\mathbf{p}}_1(n)|H_i \dots \hat{\mathbf{p}}_K(n)|H_i]^T$ and $\mathbf{d}(n)|H_i = [\mathbf{d}_1(n)|H_i \dots \mathbf{d}_K(n)|H_i]^T$ respectively, where i = 1 denotes the case when PU signal is present and i = 0 the case when PU signal is absent. For the positive step sizes we define additional $K \times K$ matrix $\mathcal{M} = \text{diag} \{\mu_1, \dots, \mu_K\}$. Let \otimes denote the Kronecker product. The $K \times K$ matrices $\mathbf{A}_1, \mathbf{A}_2$, \mathbf{C} and \mathcal{M} are in CR network extended to $KM^2 \times KM^2$ matrices $\overline{\mathbf{A}}_1 = \mathbf{A}_1^T \otimes \mathbf{I}_{M^2}, \overline{\mathbf{A}}_2 = \mathbf{A}_2^T \otimes \mathbf{I}_{M^2}, \overline{\mathbf{C}} = \mathbf{C}^T \otimes \mathbf{I}_{M^2}$ and $\overline{\mathcal{M}} = \mathcal{M} \otimes \mathbf{I}_{M^2}$. Then we can write the CM estimation recursion in the following general form

$$\hat{\mathbf{p}}(n+1)|H_i = \overline{\mathbf{A}}_2 \left(\mathbf{I} - \overline{\mathbf{\mathcal{M}}} \right) \overline{\mathbf{A}}_1 \hat{\mathbf{p}}(n) |H_i + \overline{\mathbf{A}}_2 \overline{\mathbf{\mathcal{MC}}} \mathbf{d}(n)|H_i.$$
(11)

For example for CTA algorithm we take $\overline{\mathbf{A}}_1 = \mathbf{A}_{\text{diff}}^T \otimes \mathbf{I}_{M^2}, \overline{\mathbf{A}}_2 = \mathbf{I}_K \otimes \mathbf{I}_{M^2}, \overline{\mathbf{C}} = \mathbf{I}_K \otimes \mathbf{I}_{M^2}$ or $\overline{\mathbf{C}} = \mathbf{A}_{\text{diff}}^T \otimes \mathbf{I}_{M^2}$.

By denoting the conditional expectation of the observation vector as $E[\mathbf{d}(n)|H_i]$ for i = 0, 1, then based on (11), we have that

$$E\left[\hat{\mathbf{p}}(n+1)|H_i\right] = \mathbf{A}_2\left(\mathbf{I} - \mathcal{M}\right) \mathbf{A}_1 E\left[\hat{\mathbf{p}}(n)|H_i\right] + \overline{\mathbf{A}}_2 \overline{\mathcal{M}} \mathbf{C} E\left[\mathbf{d}(n)|H_i\right], \qquad (12)$$

for i = 0, 1, where the initial value is given as $E[\hat{\mathbf{p}}(0)|H_i]$. It can be shown that a sufficient condition for the algorithm to be stable is to select the step size for every $k = 1 \dots K$ as

$$0 < \mu_k < 2. \tag{13}$$

Similarly by denoting the conditional covariance of the observations and estimates under the hypothesis H_i , i = 0, 1 as $\text{Cov} [\mathbf{d}(n)|H_i]$ and $\text{Cov} [\hat{\mathbf{p}}(n+1)|H_i]$ we have

$$\operatorname{Cov}\left[\hat{\mathbf{p}}(n+1)|H_{i}\right] = \overline{\mathbf{A}}_{2}\left(\mathbf{I} - \overline{\mathcal{M}}\right)\overline{\mathbf{A}}_{1}\operatorname{Cov}\left[\hat{\mathbf{p}}(n)|H_{i}\right] \\ \times \overline{\mathbf{A}}_{1}^{T}\left(\mathbf{I} - \overline{\mathcal{M}}\right)\overline{\mathbf{A}}_{2}^{T} \\ + \overline{\mathbf{A}}_{2}\overline{\mathcal{M}}\overline{\mathbf{C}}\operatorname{Cov}\left[\mathbf{d}(n)|H_{i}\right]\overline{\mathbf{C}}^{T}\overline{\mathcal{M}}\overline{\mathbf{A}}_{2}^{T}.$$
(14)

where initial value is noted by $Cov [\hat{\mathbf{p}}(0)|H_i], i = 0, 1.$

The moments $E[\mathbf{d}(n)|H_i]$ and $Cov[\mathbf{d}(n)|H_i]$ of the measurements are provided in 3.2.

3.2. Statistical modelling of adaptive CM estimates

Based on 2.1, for the rank one observations $\mathbf{d}_{R,k}(n)$ under H_1 we have that

$$E\left[\mathbf{d}_{R,k}(n)|H_{1}\right] = \operatorname{vec}\left[\mathbf{R}_{s,k} + \sigma_{v}^{2}\mathbf{I}_{M}\right].$$
(15)

and the stacked $KM^2 \times 1$ vector $E[\mathbf{d}_R(n)|H_i]$ over $k = 1 \dots K$ and for i = 0, 1 can be formed based on (15) respectively.

It can be shown, that the $k, j \in K$ blocks of the $KM^2 \times KM^2$ network-wise covariance matrix Cov $[\mathbf{d}_R(n)|H_1]$ are given as

$$\operatorname{Cov}\left[\mathbf{d}_{R(k,j)}(n)|H_{1}\right] = \begin{cases} \left[\left(\bar{\mathbf{\Sigma}}_{k}\right)^{c} \otimes \bar{\mathbf{\Sigma}}_{k}\right], & k = j\\ \left[\left(\mathbf{R}_{s,k,j}\right)^{c} \otimes \mathbf{R}_{s,k,j}\right], & k \neq j \end{cases}$$
(16)

where $\bar{\boldsymbol{\Sigma}}_k = \mathbb{E}\left[|\alpha_k|^2\right] \boldsymbol{\Sigma}_s + \sigma_v^2 \mathbf{I}_M$ and where for $k \neq j \mathbf{R}_{s,k,j} = \mathbb{E}\left[\mathbf{y}_k(n)\mathbf{y}_j(n)^H\right] = \mathbb{E}\left[\alpha_k \alpha_j^c\right] \boldsymbol{\Sigma}_s$ and $(.)^c$ denotes a complex conjugate. Obviously the Cov $[\mathbf{d}_R(n)|H_0]$ is given as

$$\operatorname{Cov}\left[\mathbf{d}_{R,k}(n)|H_{0}\right] = \sigma_{v}^{4}\mathbf{I}_{M^{2}}.$$
(17)

Thus the $E[\mathbf{d}(n)|H_i]$ for (12) and $\operatorname{Cov}[\mathbf{d}(n)|H_i]$ for (14) can be given for i = 0, 1 as

$$E\left[\mathbf{d}(n)|H_i\right] = \left[\mathbf{T}^{-1} \otimes \mathbf{I}_{M^2}\right] E\left[\mathbf{d}_R(n)|H_i\right],\tag{18}$$

and

$$\operatorname{Cov}\left[\mathbf{d}(n)|H_{i}\right] = \left[\mathbf{T}^{-1} \otimes \mathbf{I}_{M^{2}}\right] \\ \times \operatorname{Cov}\left[\mathbf{d}_{R}(n)|H_{i}\right]\left[\left(\mathbf{T}^{H}\right)^{-1} \otimes \mathbf{I}_{M^{2}}\right].$$
(19)

When the $\hat{\mathbf{R}}_k(n) = \text{vec}^{-1} [\mathbf{T}\hat{\mathbf{p}}_k(n)]$ is obtained by using the exponential type of averaging (as used in LMS type of algorithms), then it is not Wishart distributed [23, Theorem 3.3.1., 3.5.2.]. We propose the usage of Total Variance method [24] for approximating the $\hat{\mathbf{R}}_k(n)$ by conditional approximative Complex Central (Correlated) Wishart distributions (CC(C)W), for studying the conditional CDFs of LE of adaptively estimated CMs. Thus we use the approximation

$$\hat{\mathbf{R}}_k(n)|H_i \sim CW_M\left(\bar{N}_i, \bar{\boldsymbol{\Sigma}}_{k,i}\right),\tag{20}$$

for i = 0, 1 and where \sim denotes an approximative distribution, \bar{N}_i is the approximating DoF and $\bar{\Sigma}_{k,i}$ is the approximating population covariance matrix parameter of the corresponding CC(C)W distribution. The values for \bar{N}_i and $\bar{\Sigma}_{k,i}$ can be found by matching the mean and trace of the moments of $\hat{\mathbf{R}}_k(n)|H_i$ with the corresponding moments of the devectorized adaptive estimate $\operatorname{vec}^{-1}[\mathbf{T}\hat{\mathbf{p}}_k(n)]$. This gives (see [22] for details), by using the TV method,

$$\bar{\boldsymbol{\Sigma}}_{k,i} = \frac{1}{\bar{N}_i} \operatorname{E}\left[\hat{\mathbf{R}}_k(n) | H_i\right] = \frac{1}{\bar{N}_i} \left(\operatorname{vec}^{-1}\left[\mathbf{T} \operatorname{E}\left[\mathbf{p}_k(n) | H_i\right]\right]\right).$$
(21)

and

$$\bar{N}_{TV,i} = \left\lceil \frac{\operatorname{Tr}\left[\operatorname{E}\left[\hat{\mathbf{R}}_{k}(n)|H_{i}\right]^{c} \otimes \operatorname{E}\left[\hat{\mathbf{R}}_{k}(n)|H_{i}\right]\right]}{\operatorname{Tr}\left[\mathbf{T}\operatorname{Cov}\left[\mathbf{p}_{k}(n)|H_{i}\right]\mathbf{T}^{H}\right]}\right\rceil, \quad (22)$$

where $\operatorname{E}\left[\hat{\mathbf{R}}_{k}(n)|H_{i}\right] = \operatorname{vec}^{-1}\left[\mathbf{T} \operatorname{E}\left[\mathbf{p}_{k}(n)|H_{i}\right]\right]$

3.3. Detection Performance Analysis

Let the eigenvalues of $\bar{\Sigma}_{k,i}$ in (20) be denoted in non-increasing order as $\nu_{1,i} \geq \nu_{2,i} \geq \cdots \geq \nu_{M,i}$.

Based on the [5], [25], the $\hat{\mathbf{R}}_k(n)|H_0$ (20) is assumed to follow the CCW distribution and the eigenvalues of $\bar{\boldsymbol{\Sigma}}_{k,0}$ are $\nu_{1,0} = \cdots = \nu_{M,0} = \sigma_v^2/\bar{N}_0$. The $P_{FA,e}$, based on the non-asymptotic CDF model of the $\hat{\mathbf{R}}_k(n)|H_0$, is given as

$$F_{H_0,e}(x) = |\det(\hat{\mathbf{A}})|$$

$$P_{FA,e}(\gamma_{LE,e}) = 1 - F_{H_0,e}(\gamma_{LE,e})$$
(23)

where the $M \times M$ matrix $\hat{\mathbf{A}}_{i,j} = \left(\frac{\bar{N}_0 - j - i - 1}{i - 1}\right) \gamma_R(\bar{N}_0 + i - j, \frac{x}{\nu_{1,0}})$, for $i, j = 1, \ldots, M$ and where $\gamma_R(k, u) = \frac{1}{\Gamma(k)} \int_0^u x^{k-1} e^{-x} dx$ is the regularized incomplete Gamma function. The detection threshold $\gamma_{LE,e}$, based on the non-asymptotic model is given as

$$\gamma_{LE,e} = F_{H_0,e}^{-1} (1 - P_{FA,e}) \tag{24}$$

and can be evaluated in terms of a numerical inversion of the exact CDF formula at a desired $P_{FA,e}$ value.

Since the $\hat{\mathbf{R}}_k(n)|H_1$ is assumed to be distributed by a CCCW distribution, the P_D based on the non-asymptotic CDF $|H_1$ of the LE of a CCCW matrix $\hat{\mathbf{R}}_k(n)|H_1$ is given by [26] as follows

$$F_{H_{1},e}(x) = K_{CC} \left| \left\{ \nu_{i}^{\bar{N}_{1}-M+j} \bar{\Gamma} \left(\bar{N}_{1}-M+j, \frac{x}{\nu_{i,1}} \right) \right\}_{i,j} \right|,$$

$$K_{CC} = \left[\prod_{i=1}^{M} (\bar{N}_{1}-i)! \prod_{j=1}^{M} (M-i)! \right]^{-1} \prod_{k=1}^{M} (k-1)!,$$

$$P_{D,e}(\gamma_{LE,e}) = 1 - F_{H_{1},e}(\gamma_{LE,e}).$$
(25)

for i, j = 1, ..., M and where $\overline{\Gamma}(k, u) = \int_0^u x^{k-1} e^{-x} dx$ is the lower incomplete gamma function [27, 8.350].

4. SIMULATION RESULTS

In this section we investigate the probability of detection P_D of the the CTA type of distributed, adaptive LE detection algorithm. The performance of the algorithm is well illustrated by the P_D versus SNR analysis, where the change in the (network averaged) SNR is achieved by changing the noise power value σ_v^2 . The channel gains are assumed to be constant and are sampled for the CR node $k \in K$ as $\alpha_k \sim CN(0, 1)$. We assume to have one PU signal $\mathbf{s}(n) = \mathbf{s}(n)\mathbf{1}, \mathbf{s}(n) \sim CN(0, 1)$ and $\boldsymbol{\Sigma}_s = \mathbf{11}^H$. Obviously rank $(\mathbf{11}^H)=\mathbf{1}$. We select the $M = 2, N = 7000, \mu = 0.001$ and $P_{FA} = 10^{-2}$ for all the nodes. The thresholds of the LE detectors at nodes $k \in K$ are found by using (24) with the TV approximation. Also we select the diffusion topology of the estimates in the CR network, i.e the A matrix, as a combination of the local ($\mathbf{A}, \mathbf{C} = \mathbf{I}$) and ring-around ($\mathbf{A} = \mathbf{A}_{ring}^T, \mathbf{C} = \mathbf{I}$) topologies, similarly as in [19, Eq. 11].

In the following simulations the performance of 4 different network sizes: K = 1, 3, 10, 30 nodes are compared, while the comparable results are taken from the last node in the set. The Monte Carlo estimated P_D results (based on the adaptively estimated CMs and denoted as **Ad. Exp.** in the figures) are compared with the nonasymptotic theoretical model (25) (denoted as **Theory**) and with the P_D results based on approximately equivalent CCW matrices (denoted as **W. Exp.**). These latter matrices are generated based on the respective moments under H_1 . The P_D versus SNR results are given in Fig. 1 when TV approximation is used for the CTA algorithm.

It is seen that the non-asymptotic theoretical P_D model describes the detection performance of adaptively estimated CMs well, also when the noise power is high relative to the PU signal power (SNR). As the number of nodes in the network increases, the point where the P_D starts to decrease from one, converges to the left by equalizing and averaging the P_D on every CR node.

It can be concluded that the TV approximation for the nonasymptotic $\text{CDF}|H_1$ is usable for studying the performance of the LE detection of adaptively estimated CMs. When the nodes cooperate in estimating the network-wise CM (while nodes are able to communicate directly only with limited subset of neighbour nodes) then the resulting LE detection performance is equalized and stabilized over the individual CR nodes.

5. CONCLUSIONS

In this paper a distributed and adaptive, CTA diffusion LMS based LE detection algorithm was studied, which is applicable in CR net-



Fig. 1. Probability of detection, CTA, TV, Case 2

works for detecting the presence of a PU signal. We proposed a general framework for analysing the performance of the diffusion LMS based LE detection scheme and we demonstrated that the theoretical results are matching with the simulations. It was shown that the cooperative estimation and detection scheme enhances the detection performance.

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